Collusion in Supply Functions with Forward Markets*

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Abstract
This paper studies the effect of forward trading on the sustainability of collusion when spot market strategies take the form of supply functions. It is shown that the existence of forward markets enlarges the range of discount factors for which collusion can be sustained. Also the opposite effect might prevail when a potentially deviating firm holds a significant amount of forward contracts. The results are indifferent to the type of contract fulfilment (financial settlement or physical delivery), but long-term contracts are found to limit the possibilities of firms to collude.

1 Introduction
This paper studies the effect of forward trading on the range of collusive equilibria when spot market strategies take the form of supply functions.

A forward contract specifies the obligation of the seller to deliver a certain quantity of a good in the future for a price, which is fixed in the contract, but which is paid upon delivery. By selling forward, a firm can therefore change its strategic position in the following spot market. Allaz and Vila (1993) show how forward trading increases competition in a Cournot oligopoly by the simple fact that rival suppliers interact several times before the market finally clears. Each firm has an incentive to sell forward, locking in its quantity for the spot market and thus to achieve a kind of Stackelberg advantage over its competitor. Thus, total supply in the spot market is larger with forward trading and the equilibrium price is lower. Since both firms are allowed to trade forward, they both face the same incentive to expand their output through forward contracting while they would be better off if there was no contracting at all. The impact of this prisoner’s dilemma is amplified when there are several opening rounds of the forward market and therefore several interactions.

Mahenc and Salanie (2004) completely reverse the results of Allaz and Vila (1993) by considering the case of price competition in the spot market and the possibility of producers to buy forward their own production. By buying forward, the producer commits to have an interest in raising prices in the spot market, and because competition in the spot market is in prices, the rival firm will follow suit. Thus, equilibrium prices are higher and competition is softened by the effect of forward markets. A similar result, but for the case of Cournot oligopoly, is obtained by Ferreira (2003) if there are infinitely many openings of

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the forward market before the spot market clears. However, all these contributions, Allaz and Vila (1993), Ferreira (2003), and Mahenc and Salanie (2004) just consider one shot games with multiple rounds of forward trading and one opening of the spot market.

In a repeated game setting with several openings of the forward and the spot market Liski and Montero (2006) show that the possibility of forward trading can increase the range of discount factors for which a collusive agreement can be sustained. This result holds for price and quantity competition in the spot market, but similar to the formerly mentioned papers, the effect of forward positions of firms differs substantially between these two cases. For a price-setting oligopoly, selling forward locks in the corresponding quantities and thus reduces the size of the market which could be captured by a deviating firm. This contrasts with competition in quantities where a deviating firm can never capture the complete spot market because other firms set their quantities in advance and supply them anyway. In both cases, however, selling forward at a monopoly price locks in the collusive return, even in case of deviation. So Liski and Montero (2006, p.214) conclude:

when firms compete in prices in the spot market, the critical discount factor above which firms can sustain maximal collusion is decreasing in firms’ short positions (up to a certain level). Conversely, when firms compete in quantities, the critical discount factor is increasing in firms’ short positions, and consequently, firms may well end up buying forwards in order to sustain collusion.

While Liski and Montero (2006) motivate their analysis with reference to electricity markets, most spot markets for electric power are neither characterised by a classic Bertrand or Cournot setting but rather by competition in supply functions. Ciarreta and Gutiérrez-Hita (2006) provide a model of collusion in supply functions, but do not consider forward trading. Green (1999) and Newbery (1998) show that the pro-competitive results of Allaz and Vila (1993) carry over to the case of supply function bidding in the spot market. However, their models do not extend to collusion in repeated games as in Liski and Montero (2006). Since the fundamental work of Klemperer and Meyer (1989), it is well known that a Supply Function Equilibrium (SFE) is located between the theoretical Cournot and Bertrand prices and quantities. One might therefore imagine that the effect of forward trading on collusion in supply functions will be somehow contained between the results for price and quantity competition. But in light of the contrary results for both cases in the literature, there is no evident intuition what effect forward positions should have. Moreover, Green (1999) and Newbery (1998), while not explicitly highlighting this aspect, both provide models where firms bid supply functions partly tracing below their marginal costs. This means that forward trading can make the SFE break out of the rigid ‘tunnel’ defined by Bertrand and Cournot solutions which was initially identified by Klemperer and Meyer (1989). Evidently, there is a gap in the theory of collusion concerning the effect of forward trading when the spot market clears in supply functions.

The empirical relevance of such a market setting is witnessed by Sweeting (2007) who shows that National Power and PowerGen, the two dominating electricity producers in the UK power market, exerted substantial market power in a period when the general market conditions became more competitive. He
discovers that both companies could have raised individual profits by increasing their output and concludes that the evidence is consistent with tacit collusion. The same UK market for electric power, however, has served as the major example for a real world market where firms bid supply functions in a short term market and can close contracts for difference several months ahead (see Green, 1999). But as pointed out before, a model of repeated interaction for such a market is missing in the literature.

In the spot market, an important aspect of the models of Allaz and Vila (1993), Mahenc and Salanie (2004), and Liski and Montero (2006) is that forwards lock in a specific quantity for the forward selling firm. In a supply function equilibrium in contrast, the firms’ output is stochastic and determined by a random demand shock after the firms decided on their spot market bids. Market clearing determines the equilibrium price and assigns the firms’ spot sales along their supply functions. Forward positions therefore determine the strategies in the spot market, but do not lock in a specific quantity.

For the forward market, Allaz and Vila (1993) study a sequence of distinct openings of contracting for physical delivery. The fully competitive outcome results as the limiting case for infinitely many openings of the forward market. The authors consider this as a valid representation of continuous trading which is indeed the rule for many forward markets. Liski and Montero (2006) model a one-by-one sequence of openings of the forward and the spot market, but allow forward contracting for several periods in advance. If every period in the future can be traded forward at every opening of the forward market, the limiting outcome is again equivalent to the result of Allaz and Vila, except for the additional possibility of collusive equilibria. Newbery (1998) and Green (1999) study the linear SFE model for the spot market and consider conjectural variations of firms about the rivals’ forward market response to their own forward sales. Moreover, they show that a firm which has contracted forward, bids indeed more competitive in the corresponding spot market. For the choice of forward positions, however, the strategic effect works through the conjectural variation which allows to model all possibilities from no contracting to the fully competitive case with complete contract coverage of the firm’s expected output. Table 1 in Appendix A.3 (page 25) summarises the models from the cited literature and their assumptions.

This paper seeks to fill the gap in the theory of tacit collusion by integrating major aspects from the models of Green (1999), Liski and Montero (2006) and Ciarreta and Gutiérrez-Hita (2006). Section 2 introduces the notation, the static model of Green (1999) and some results building upon Ciarreta and Gutiérrez-Hita (2006). Section 3 studies sustainability of collusion in a setting of repeated forward and spot market openings with competition in supply functions. It is shown that the mere existence of a forward market helps to sustain collusion when firms fear to be trapped in competition on both forward and spot market once the collusive agreement breaks. Section 4 studies the effect of demand uncertainty on firms’ expected profits and its implication for the range of discount factors for which collusion can be sustained. Section 5 discusses extensions of the model which represent typical features of real world markets. One of these aspects concerns the difference between financial versus physical contracting. Liski and Montero (2006, footnote 5) point out that for homogeneous goods, financial contracts might have substantially different effects on collusion compared to physical contracts. Subsection 5.1 shows that
this is not true for competition in supply functions and thus does not apply for most electricity wholesale markets which actually give a good example of a perfectly homogeneous good. Subsection 5.2 discusses how forward contracts which are spanning several openings of the spot market change the incentives to collude. Subsection 5.3 considers a generalisation of the functional form of firms’ strategies to non-linear supply functions, and Section 6 concludes.

2 The Static Game

Definitions

Let \( i, j \) be two firms producing a homogeneous good. They compete on a spot market where firms bid supply functions \( q_i(p), q_j(p) \) with \( p \) denoting the spot market price. The strategy space is reduced to affine supply functions\(^1\) of the form \( q_i = \alpha_i + \beta_i p \). Subsection 5.3 discusses the impact of a generalisation to non-linear (non-affine) strategies. Demand is given by \( D(p) = A - bp + \varepsilon \), where \( \varepsilon \) is a zero mean random shift. After both firms chose their supply function, the demand shock \( \varepsilon \) realises and determines the market clearing price \( p \). Each firm then produces at a cost of \( C(q) = c_1q + c_2q^2 \) the quantity \( q(p) \) which correspond to its chosen supply function.

A necessary assumption is that of permanently positive supply. In developed electricity markets, one observes fluctuating demand, but there is no economic blackout in the form that the willingness to pay drops so low that production ceases. To represent this permanency of supply and demand, assume that the intercept of the inverse demand function is always strictly larger than the intercept of the firms’ marginal costs: \( (A + \varepsilon)/b > c_1 \). The following paragraphs first ignore the fact that the variance of \( \varepsilon \) affect expected profits in line with the model of Ciarreta and Gutiérrez-Hita (2006). Section 4 then determines the effect of positive variance on the previously established results.

Before bidding on the spot market, firms can close ‘contracts-for-difference’ (financial forwards) over quantities \( x_i, x_j \). The seller of the contract receives the forward price \( f \) times the contracted quantity from the buyer, and pays to the buyer the realised spot price \( p \) times the contracted quantity \( x_i \) or \( x_j \) respectively. The case of physical forward trading is economically almost equivalent but requires some more notation. Its discussion is postponed for the sake of clarity to Section 5.1. The forward market is assumed to be competitive and populated with risk-neutral speculators and customers which take the counter-position of firms. A no arbitrage condition therefore imposes \( f = p \) in expectation, while demand shocks might indeed result in differences between the forward and the realised spot price. In line with Green (1999), I do not consider producers going long in the forward market (e.g. buying forward their own output).

\(^1\)This is in line with a number of earlier contributions, e.g. Green (1999). Newbery (1998) and Ciarreta and Gutiérrez-Hita (2006) restrict strategies further to the strictly linear case with \( \alpha = 0 \).
The ex-post realised profit of firm $i$ writes

$$\pi_i = (f - p)x_i + pq_i(p) - \frac{c_2}{2}q_i(p)^2. \quad (1)$$

**Spot Market Strategies**

On the spot market firm $i$ maximises profits over its residual demand $q_i = D(p) - q_j(p)$. Residual demand is stochastic due to the fluctuation of $\varepsilon$ in $D(p)$, but the supply function framework allows firms to bid optimal prices for an infinite number of possible demand realisations. Note that the firm’s spot market sales vary with $p$ by $-(b + \beta_j)$ (see the definition of demand and the rival’s supply function). So the first order condition for a profit maximum of firm $i$ at a given level of demand is

$$\frac{d\pi_i}{dp} = D(p) - q_i(p) - x_i - (b + \beta_j)(p - c_1 - c_2(D(p) - q_j(p))) = 0 \quad (2)$$

The first order condition should hold for an arbitrary realisation of demand, thus for a number of possible equilibrium prices. Thus, it must hold as an identity, which allows to equal all factors multiplying $p$ at the same power.

Replace $i$’s residual demand $D(p) - q_i(p)$ in Equation (2) by its supply function $q_i(p) = \alpha_i + \beta_ip$. Equating all terms multiplying $p$ yields an equation including $\beta_i$. Cancelling all terms multiplying $p$ yields an equation including $\alpha_i$. Solving these two equations leads to the optimal slope $\beta_i^*$ and intercept $\alpha_i^*$ of the best response supply function of $i$:

$$\beta_i^* \equiv \frac{b + \beta_j}{1 + c_2(b + \beta_j)} \quad (3)$$

$$\alpha_i^* \equiv \frac{x_i - c_1(b + \beta_j)}{1 + c_2(b + \beta_j)} = \beta_i^* \left( \frac{x_i}{b + \beta_j} - c_1 \right) \quad (4)$$

which is equivalent to Equation 7 of Green (1999). The definition in (3) represents the well known slope of a linear supply function duopoly (see e.g. Green (1996); Ciarreta and Gutiérrez-Hita (2006)). It can be solved for the symmetric non-cooperative equilibrium which will be denoted by $\beta_n$ in the remainder of the paper.

$$\beta_n \equiv \sqrt{\frac{b^2 + \frac{4b}{c_2} - b}{2}} \quad (5)$$
Note that $\beta_n$ is independent of the firm’s forward positions. Equations (4) and (3) allow to write the best response supply function of $i$ given $\beta_j$ as:

$$q_i^* = \beta_i^* \left( p + \frac{x_i}{b + \beta_j} - c_1 \right)$$

The inverse of this supply function shows that the optimal price bid is equal to marginal costs at the level of forward contracts (see Newbery, 1998; Green, 1999). This effect is illustrated in Figure 1 and will be discussed further below.

The Choice of Forward Positions

Firms can sell forward before they submit their bids for the spot market. One could think of several possible designs of this forward market, such as exchange based continuous forward trading, bilateral contracting, repeated auctions and so forth. Some regulating agencies even imposed electricity producers to sell forward as a measure to increase competition. The focus of this paper is to determine the effect of firms forward positions on their incentives to collude. Section 3 therefore takes the forward positions of firms rather as a parameter resulting from the forward market design. They are determined in the following by conjectural variations of firms about the rival’s reaction to own forward market sales. The approach is adopted from Newbery (1998) and Green (1999) and allows to model a range of different levels of forward contracting.

A crucial difference of a supply function equilibrium compared to the models of Allaz and Vila (1993), Ferreira (2003), Mahenc and Salanie (2004) or Liski and Montero (2006) concerns the effect of forwards on the spot market outcome: Forward sales in the former mentioned articles are an instrument to lock in a certain quantity and to squeeze the rival firms sales in the spot market. This in contrast to supply functions where the final quantities of firms are always flexible and undetermined in advance. The linear model with the best response strategies in Equations (3) and (4) has two crucial features: first, forward positions of firm $i$ only affect its spot market strategy through the intercept $\alpha_i^*$. Second, because the slope of firm $i$ is unaffected by its contract positions, and the rival firm $j$ only reacts to changes in the slope of $i$ (see Klemperer and Meyer, 1989, Eq. 5), the spot market strategy of $j$ is independent of $i$’s forward positions. Phrased differently: In the linear SFE model, forward contracting of one firm does reduce the rivals quantity in the spot market equilibrium, but the rivals strategy in the spot market is unaffected (see the discussion of Green, 1999, pages 115-116). This allows to decompose the total derivative of profits with respect to forwards positions as follows:

$$\frac{d\pi_i}{dx} = \frac{\partial \pi_i}{\partial x_i} + \frac{d\pi_i}{d\alpha_i} \frac{d\alpha_i}{dx} + \frac{d\pi_i}{d\alpha_j} \frac{dx_j}{dx} \frac{dx_i}{dx}$$

where the first term is the direct effect of forwards on profit, the second term captures the effect of forwards on the optimal spot market strategy of firm $i$, and the last term mediates the effect of the rivals forwards through the conjectural variation $dx_j/dx_i$. When $\alpha_i$ is the best response $\alpha_i^*$ as defined in (4), one has $d\pi_i/d\alpha_i^* = 0$ given a predetermined forward price $f$. So by a standard envelope theorem condition $d\pi_i/d\alpha_i = df/d\alpha_i$ for the situation when the forward price is
not yet locked in. Equation (6) can be rewritten as
\[
\frac{d\pi_i}{dx_i} = (f - p) + x_i \frac{df}{dx_i} \frac{d\alpha^*_i(x)}{dx_i} + x_j \frac{d\pi_i}{dx_j} \frac{d\alpha^*_j(x)}{dx_j} dx_i
\] (7)

A basic no-arbitrage condition\(^2\) requires \(f = p\) in expectation, and the first term in (7) vanishes. Moreover, \(\frac{df}{dx_i} = \frac{dp}{dx_i}\) and \(\frac{d\pi_i}{dx_j} = \frac{dp}{dx_j}\). Setting the derivative in (7) to zero and rearranging yields Green’s Equation (14):
\[
x_i = -q_i \left( \frac{(b + 2\beta_n) \frac{dx_j}{dx_i}}{b + \beta_n \left(1 - \frac{dx_j}{dx_i}\right)} \right)
\]

When firms have Bertrand conjectures of the type \(\frac{dx_j}{dx_i} = -1\) they will cover all of their output through the contract market. For the spot market, this implies that firms offer their expected total output at a price equal to marginal costs. Other than for the model of Newbery (1998), however, full contracting here does not result in zero profits for two reasons: first, marginal costs are increasing so firms still earn inframarginal profits when they sell all their output at the marginal cost of the marginal unit. Second, demand is fluctuating and supply functions are steeper than marginal costs. So for any realisation of demand off the expected level, firms earn a margin on top of their marginal costs.

With Cournot conjectures, \(\frac{dx_j}{dx_i} = 0\), Green’s Equation (14) results in no forward sales at all. This is due to the strategic neutrality of forward positions on the rivals spot market supply function. It is in clear contrast to the case of Allaz and Vila (1993), where firms use forward sales to curtail the rivals quantity, which is equivalent to the rivals strategy on the spot market. The linear SFE model, however, allows for a strategic effect of forwards which is channelled through conjectural variations in the forward market. Section 3 studies the effect of forward positions on sustainability of collusion, and takes the forward positions of firms rather as a parameter. This approach is in line with the models of Newbery (1998) and Green (1999) which allow a range of different levels of forward contracting. Moreover, it compares well to the results of Liski and Montero (2006), who obtain different levels of forward contracting from different market designs and deduce the effect on sustainability of collusion depending on these contracting levels. Section 5.3 discusses the case of non-linear SFE which provides endogenous incentives for strategic forward contracting.

### Collusion in supply functions

Colluding firms can optimally implement the monopoly solution. While a cartel can control all supply in the spot market, there still are competitive speculators in the forward markets which impose equality of the price for forwards and the expected spot market price. Knowing that \(f = p\) in expectation, forward

\(^2\)This condition is rather strict (although it is very common) because it implies total flexibility of the forward price \(f\). To meet this condition, forward contracts need to be traded in infinitely small quantities and speculators must be fully informed so that any marginal increase of one firms forward position immediately decreases the offered forward price to adjust for the expected increase of competition in the spot market. A departure of this assumption, however, would either imply arbitrage opportunities at the cost of traders or a completely illiquid forward market without any transaction.
positions completely cancel from the profit function (1). The optimal strategies for joint profit maximisation are therefore equivalent to the situation without forward trading and can be adopted from Ciarreta and Gutiérrez-Hita (2006).\textsuperscript{3}

\[\beta_c \equiv \frac{b}{2 + c_2 b}\]  
\[\alpha_c \equiv \frac{-bc_1}{2 + c_2 b} = -\beta_c c_1\]  

So, as long as there is no concern about deviation, the colluding firms are indifferent about the forward positions taken on the contract market. Figure 1 summarizes the different spot market strategies.

![Figure 1: Collusive and Nash spot market supply with and without financial contracting (see Ciarreta and Gutiérrez-Hita (2006) and Green (1999))](image)

The very first unit of collusive supply is priced at marginal cost. The same is true for non-collusive supply when firms do not hold any forwards. Larger quantities are then supplied with an increasing mark-up. Obviously, the collusive supply function being much steeper exhibits increasingly higher mark-ups compared to its non-collusive counterpart. For the non-collusive case with forward sales, the spot market supply function shifts outwards such that it crosses marginal costs at the level of the contracted quantity. Larger quantities are sold above marginal costs, smaller quantities even below marginal costs.

\textsuperscript{3}Note that Ciarreta and Gutiérrez-Hita (2006) normalise \(c_1\) to zero and omit the supply function intercept \(\alpha\). Note further that, indeed, a monopoly which has contracted forward at the monopolistic price would have an incentive to decrease prices in the spot market. This, however, can never be an equilibrium, because speculators would not be willing to buy at the monopolistic price in the first place. Thus, in a one shot game, no speculator would be willing to buy forward from the monopolist. In a repeated game setting, however, a situation where the monopolists sells forward and sticks to the monopolistic supply function on the spot market can be an equilibrium without contradicting the assumption of rational agents.
3 The Repeated Game

Consider now the case of infinitely many interactions of the same two firms on, first, the forward market for the coming spot market, than the corresponding spot market, then the forward market for the next opening of the spot market and so on. It is well-known that in such a setting cooperative (meaning collusive) solutions can be sustained when firms succeed to implement incentives which deter defection. This paper sticks to the commonly studied trigger strategies where firms revert permanently to the one-shot Nash equilibrium once the collusive agreement breaks. A deviating firm can thus at best earn deviation profits once.

Sustainability of collusion in the spot market

Suppose first, that deviation only occurs in the spot market. Spot market profits are maximal when the deviating firm bids its best response given by (3) and (4) to the collusive supply function of its competitor as defined in (8) and (9). Let \( \alpha_d \) and \( \beta_d \) denote the deviating strategy parameters with \( \beta_d = \beta^c(\beta_c) \) and note that \( \beta_n > \beta_d > \beta_c \). Furthermore, let \( \pi^d \) designate deviation profits. The incentive constraint in it’s classic form is

\[
\frac{1}{1-\delta} \pi^c > \pi^d + \frac{\delta}{1-\delta} \pi^n
\]

(10)

where \( \delta \) designates the discount factor, and \( \pi^c, \pi^n \), are the profits which arise when both firms choose supply functions with intercept \( \alpha_c \) and slope \( \beta_c \) or intercept \( \alpha_n \) and slope \( \beta_n \) respectively. (The same type of subscripts are used later on for prices \( f \), and \( p \), and quantities \( q \).) Again, it is obvious that \( \pi^d > \pi^c > \pi^n \). One can thus determine the minimum discount factor \( \delta \) for which the incentive constraint just binds as an equality.

\[
\delta \equiv \frac{\pi^d - \pi^c}{\pi^n - \pi^d}
\]

(11)

So how does forward trading affects the possibility to sustain collusion? The key result is summarised in Lemma 1.

Lemma 1. Forward markets enlarge the range of discount factors for which collusion can be sustained, when firms contract sufficiently less forward when colluding than they do during the punishment phase. Colluding firms optimally do not contract forward at all.

Proof. A sufficient condition for Lemma 1 is Property 1.

Property 1. For a joint variation of forward positions \( x_i = x_j = x \)

(a) collusive profits \( \pi^c \) are unaffected by the forward positions held by firms.

(b) non-cooperative profits \( \pi^n_i \) of a firm \( i \) are concave and decreasing in the number of forwards \( x \) hold by both firms.

(c) Profits \( \pi^d_i \) of a deviating firm \( i \) are convex and increasing in the number of its forward positions \( x_i \) irrespective of the forward position of its rival. Precisely,

\[
\frac{d \pi^d_i}{dx} = f_c - p_d.
\]
The proof of Property 1.a is trivial. Obviously, collusive profits are completely unaffected by the forward positions of firms because the spot strategies (8) and (9) are constant in \( x_i \) and the forward price equals the expected spot price. For the proof of Property 1.b, note that non-cooperative profits are decreasing the more the firms contract forward, because the spot market becomes more competitive as shown by Green (1999). A detailed proof of Property 1.b is in Appendix A.1.2. Deviation profits in contrast are increasing in the number of forwards held by firms. This mirrors the fact that a firm which deviates in the spot market earns additional revenues for the difference of the monopoly price, which is contracted in advance, and the spot price, which is lower than the contracted price because of deviation. This effect is described by Property 1.c and proved formally in Appendix A.1.1.

Property 1 implies that \( \delta \) is increasing in the level of contracts held by a firm during collusion and decreasing in the number of contracts which firms expect in the punishment phase. This ends the proof for Lemma 1.

Figure 2 illustrates this reasoning by depicting collusive, non-cooperative and deviative profits over different levels of contracted quantities. The critical discount factor \( \delta \) has a direct graphical interpretation: at a given level of contracting, \( \delta \) is equivalent to the distance between deviation profits and collusive profits, divided by the distance between deviation profits and Nash profits (see Equ. 11).

![Graph showing profits over different levels of (symmetric) forward contracting](image)

Take as an example the case without any forward trading as studied by Ciarreta and Gutiérrez-Hita (2006). The discount factor is represented in Figure 2 by \( \frac{BC}{BD} \). Of course, the level of contracting might be different during periods of collusion and in the punishment phase with permanent reversion to the one-shot Nash equilibrium. Suppose colluding firms do not hold forwards, but fear to fall back into a more competitive situation with half of their and their rivals output contracted forward. The critical discount factor is depicted
in Figure 2 as the fraction of distances $\frac{BC}{BC + EF}$, which is obviously smaller than $\frac{BC}{BD}$. The result established here for competition in supply functions therefore resembles the result of Liski and Montero (2006) for quantity competition: collusion is easier to sustain if firms don’t hold any forwards, but fear to be trapped in a situation with large forward sales and consequently a very competitive spot market.

For ‘deviation in forward markets’ there is no clear definition what such a strategy would be, and the next paragraphs will explain why Lemma 1 remains valid when non-cooperative behaviour of firms in forward markets is considered. Before turning to this aspect, note the following implication of Property 1 which will be needed for the discussion in Section 4.

**Corollary 1**. There are infinitely many combinations $(x_d, x_n)$ of forward positions $x_d$ held by a deviating firm, and the forward positions $x_n$ held by both firms during the punishment phase, such that the critical discount factor $\delta$ is equivalent to the case without forward markets (e.g. $x_d = x_n = 0$). This constant level of $\delta$ is in the following denoted $\delta_{x=0}$.

**Proof.** Immediate from Property 1 and the definition of $\delta$. □

**Observability and deviation in forward markets**

The definition of deviation in the forward market is not evident when the spot market clears in supply functions. Liski and Montero (2006) consider the case where a deviating firm sells more than the monopoly quantity forward. In the model studied here, there is no fixed monopoly quantity which can be surpassed by a deviating firm. Rather, there is the monopolistic supply function which guarantees that the optimal monopoly price prevails whatever demand shock occurs. Collusive spot sales are unaffected by the forward contracted quantity.

So, what can a deviating firm achieve in the forward market? At best, it can cheat on speculators or customers by selling forwards at the monopolistic price, and make a margin by depressing the spot price to earn the difference between spot and forward. Property 1.c specifies, that there is no optimal level of forward sales for the deviating firm. A deviator has the incentive to sell as many forwards as possible. Speculators, in contrast, will not accept to buy forwards at the monopolistic price when there is a doubt about the sustainability of collusion. Consider two opposite cases:

(1) Firms individual forward positions are fully observable and the sustainability of collusion is straightforward to detect by the contracting counterparts of a potential deviator. The counterpart will accept to buy forwards at the monopolistic price as long as the incentive constraint can be expected to hold.

(2) If individual forward positions are private information, speculators will infer that firms have an incentive to sell forwards at a monopolistic price and later to depress the spot price. With this expectation, the counterpart will not accept any price higher than the expected spot price in case of deviation. The deviating firm, in contrast, cannot earn any additional profit from selling forward (see Property 1.c). Thus, the firm will either sustain collusion if the incentive constraint holds, or deviate in the spot market. In either case, ‘deviation in the forward market’ is not a relevant or attractive strategy.
4 Uncertainty of Demand

The preceding analysis just considered firms profits ($\pi_i$, $\pi_j$) at a fixed level of demand ($\varepsilon = 0$). This is in line with the majority of the literature, where some demand variation is necessary to identify the optimal slope of supply functions, but the profit effect of that variation is largely ignored, mainly because uncertainty enters linearly and would not alter the general results. Even Ciarreta and Gutiérrez-Hita (2006), analysing the critical discount factor for collusion in supply functions, do not discuss the effect of demand shocks on expected profits although the authors assume demand uncertainty for the definition of the equilibrium strategies. This is surprising at a first glance because the variance of demand shocks has a different impact on collusive compared to deviation profits. The critical discount factor $\delta$ is defined by the ratio of additional profits of deviation compared to sustained collusion. It is not obvious that different levels of demand variation do not change this ratio. The following lemma therefore complements both the results from the preceding section and those of Ciarreta and Gutiérrez-Hita (2006).

Lemma 2.

a. When firms are not contracting forward during collusion, deviation or Nash reversion, the critical level of the discount factor $\delta_{x=0}$ is unaffected by the variance $\sigma^2$ of the demand shock $\varepsilon$.

b. The critical level of the discount factor $\delta$ is decreasing in $\sigma^2$ when it is above the level of $\delta_{x=0}$. It is increasing in $\sigma^2$ if forward positions are such that $\delta < \delta_{x=0}$.

Proof. Let $\hat{\pi}_i, \hat{\pi}_j$ denote the expected profits of firms, in contrast to $\pi_i, \pi_j$, the profits at the expected level of demand. Recall the definition of $\pi_i$ in Equation (1) and rewrite it in terms of powers of $p$ with $q_i = \alpha_i + \beta_i p$.

$$\pi_i = (fx - c_1 \alpha_i) + p(-x + \alpha_i - c_1 \alpha_i - \frac{c_2}{2} \alpha_i^2) + p^2 \left(\beta_i - \frac{c_2}{2} \beta_i^2\right)$$

Demand shocks affect the firms profit through fluctuations of the market clearing price. As the market clearing price is linear in the shock $\varepsilon$ and the expected value function is linear in its arguments, the variance of $\varepsilon$ affects expected profits only through the last term multiplying $p^2$. This allows to write expected profits as $\hat{\pi}_i = \pi_i + \text{Var}(p)(\beta_i - \frac{c_2}{2} \beta_i^2)$. With the definition of the market clearing price $p$ one obtains

$$\hat{\pi}_i = \pi_i + \sigma^2 \left(\frac{\beta_i - \frac{c_2}{2} \beta_i^2}{b + \beta_i + \beta_j}\right).$$

which yields a handy property for the proof of Lemma 2:

Property 2. The derivatives of expected profits of deviation, collusion and Nash-reversion with respect to $\sigma^2$ are all positive, and can be ordered as follows:

$$\frac{d\hat{\pi}^d}{d\sigma^2} > \frac{d\hat{\pi}^c}{d\sigma^2} > \frac{d\hat{\pi}^n}{d\sigma^2}$$

So the expected profits $\hat{\pi}_i$ are linear in the variance of demand shocks $\sigma^2$ and the expression in (12) allows to compare the effect of uncertainty on profits of
collusion, deviation, and Nash reversion through the specific strategy parameters \( \beta^d, \beta^c \) and \( \beta^n \). A full proof of Property 2 is in Appendix A.2.

For an illustration, note that Property 2 states that the ordering of derivatives is the same as the order of the levels: profits of deviation are larger than collusive profits, which again are larger than non-cooperative equilibrium profits. Figure 3 depicts expected profits over different levels of \( \sigma^2 \) in a similar manner as Figure 2 depicted profits over different levels of forward contracting. As before, the critical discount factor can be expressed in terms of profits as 

\[
\delta = \frac{\pi^d - \pi^c}{\pi^d - \pi^n}.
\]

In Figure 3 these differences in profits are visualised for the case without any forward trading by the distances \( BC/BD \) at some arbitrary level of \( \sigma^2 \).

![Figure 3: Expected profits over variance of demand shocks \( \sigma^2 \)](image)

As profits are linear in \( \sigma^2 \), one can use the intercept theorem\(^4\) to analyse the incentive constraint for different levels of \( \sigma^2 \). When the three lines have a common point of intersection, the intercept theorem applies and the fraction \( BC/BD \) will be constant for any positive level of \( \sigma^2 \). In Figure 3, this intersection occurs at

\[\sigma^2 = -(A-bc_1)^2.\]

A negative value for the variance obviously has no reasonable interpretation. It just provides the joint zero of the three lines, and due to this common point, the proportions of \( \pi^d - \pi^c \) and \( \pi^d - \pi^n \) do not change over different levels of \( \sigma^2 \).

The following paragraphs show that the intercept theorem applies generally for the case without forward trading as in Ciarreta and Gutiérrez-Hita (2006). Consider the definition of profits in Equation (1). With \( x_i = 0 \) firm \( i \)’s profit \( \pi_i \) can be written as

\[\pi_{i,x_i=0} = q_i(p) \left( P - c_1 + \frac{c_2}{2} q_i(p) \right)\]

With \( x_i = 0 \), the supply function intercept \( \alpha_i \) reduces to \( \alpha_i = -\beta_i c_1 \), irrespective of firm \( i \) being engaged in collusion or playing its best response. Thus the firms

\(^4\)Also known as “Thales theorem”, but the latter name is imprecise in its distinction to another geometric theorem. In French the theorem is referred to as “théorème d’intersection”, in German its name is “Strahlensatz”.

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quantity can be written as \( q_i = \beta_i (p - c_1) \). Substituting \( q_i \) and the spot market clearing price \( p \) into the expression for profit gives

\[
\pi_{i,x_i=0} = \left( \frac{A - bc_1}{b + \beta_i + \beta_j} \right)^2 (\beta_i - \frac{c_2}{2} \beta_j^2).
\]

Reconsider the definition of \( \hat{\pi}_i \) in Equation (12). Replace \( \pi_i \) in Equation (12) with \( \pi_{i,x_i=0} \) as defined above. It is obvious that \( \hat{\pi}_i \) has a null for \( \sigma^2 = -(A - bc_1)^2 \), no matter what strategy \( \beta \) the firm plays. Thus, without forward trading, \( \hat{\pi}^d \), \( \hat{\pi}^n \) and \( \hat{\pi}^c \) have a common point of intersection in the plane spanned by \( \pi_i \) and \( \sigma^2 \), so the intercept theorem applies. Therefore, \( \delta \) is constant in \( \sigma^2 \) for the case without forward contracts, which ends the proof of Lemma 2.a.

Now, suppose a deviating firm has sold forward a quantity \( x_d > 0 \). By Property 1 its ex-post profits will be higher than without forward contracting, but by Property 2, the slope of profits with respect to \( \sigma^2 \) is unchanged. Therefore, the line defining its expected profits \( \hat{\pi}^d \) over \( \sigma^2 \) is shifted upwards as depicted in Figure 3. The three lines for collusive, non-cooperative and deviation profits do not intersect in a common point. Expected profits of deviation instead intersect non-cooperative profits at some level of \( \sigma^2 \) which is larger compared to the intersection with collusive profits. Therefore, \( \hat{\pi}^d - \hat{\pi}^n \) increases in larger proportion with \( \sigma^2 \) than \( \hat{\pi}^d - \hat{\pi}^c \) for any \( \sigma^2 > 0 \). This implies that \( \delta \) is now decreasing with \( \sigma^2 \). The reverse argument can be made for the case when there are positive levels of forward sales in the punishment phase: \( x_n > 0 \). So it is evident, that there are combinations of \( x_d \) and \( x_n \) which shift deviation profits upwards and punishment profits downwards such that the three lines still keep a common point of intersection. These combinations of \( x_d \) and \( x_n \) therefore imply that the ratio of \( \hat{\pi}^d - \hat{\pi}^n \) and \( \hat{\pi}^d - \hat{\pi}^c \) is constant for all non-negative levels of \( \sigma^2 = 0 \). Take \( \sigma^2 = 0 \), and we are back to the case without uncertainty which is described in Corollary 1. Therefore one obtains:

\[
(x_d, x_n) \text{ such that } \delta = \delta_{x=0} \iff \frac{d\delta}{d\sigma^2} = 0
\]
\[
(x_d, x_n) \text{ such that } \delta > \delta_{x=0} \iff \frac{d\delta}{d\sigma^2} < 0
\]
\[
(x_d, x_n) \text{ such that } \delta < \delta_{x=0} \iff \frac{d\delta}{d\sigma^2} > 0
\]

This ends the proof of Lemma 2.b and completes the proof of Lemma 2. \( \square \)

5 Extensions of the Model and Their Implications

There are several examples of markets in reality which resemble the stylised setting described in the previous sections. The following pages describe a number of further aspects of these markets, and how integrating these aspects affects the model results. Many real world electricity markets nowadays allow to trade forward, both with physical and financial contracts. Moreover, forward contracts typically span more than just one opening of the spot market. Finally, while the strategic effect of forward trading largely cancels in the linear supply function equilibrium model, a more general model of supply function bidding might yield a different result.
5.1 Physical vs. Financial Forwards

In the early days of electricity sector deregulation, most countries, such as the UK, favoured the pool model which bundles all supply in a short term market. Nowadays real world electricity markets typically provide a free choice of the market platform. Electricity exchanges co-exist with energy brokers and bilateral over-the-counter sales of electricity products. Still, the reference price is typically provided by the day-ahead auction of an exchange market where firms bid some sort of supply or demand function. Strategies obviously change when firms have already sold part of their output physically in advance. The following paragraphs show that the result is economically equivalent in terms of equilibrium price and revenues to the case of only financial forward contracting. However, the market volumes and the seller/buyer positions on the spot market might change. This is of relevance when studying real world situations where a-priori firms can be financially or physically contracted.

The notation in the following is almost identical to the notation before. However, some more specifications have to be made. Forward positions \( x_i, x_j \) are now to be distinguished between financial and physical forward contracts. Financially contracted quantities \( x_{i,f}, x_{j,f} \) are cleared by a balancing payment of the (possibly negative) difference \( f - p \) from the buyer to the seller at the time when the spot market clears. Physical forward sales \( x_{i,\phi}, x_{j,\phi} \) oblige the seller to produce the corresponding amount of electric power at the contracted time and allow the buyer to consume this energy without bidding for it on the corresponding spot market. The buyer pays the contracted price to the seller upon delivery. A basic no-arbitrage condition imposes equality of the forward price for physically and financially settled contracts which is jointly denoted by \( f \). Spot market demand is now \( D(p) - x_{i,\phi} - x_{j,\phi} \) because some demand has already been contracted in advance. Moreover, quantities \( q_i, q_j \) still denote total output of firms, but this is not necessarily equal to spot market supply any more. Let \( s_i(p), s_j(p) \) denote the spot market supply functions of the linear form, \( s_i = \alpha_i + \beta_i p \), and note that \( q_i = x_{i,\phi} + s_i(p) \) which is restricted to be positive.5 Note that, while total output must be non-negative, supply in the spot market can be positive or negative. In other words, firms might buy back on the spot market what they have sold forward before. The profit function becomes

\[
\pi_i = (f - p)x_{i,f} + fx_{i,\phi} + ps_i(p) - c_1(s_i(p) + x_{i,\phi}) - \frac{c_2}{2}(s_i(p) + x_{i,\phi})^2
\]

The derivation of optimal strategies follows the same procedure as in Section 2. The first order condition for profit maximisation over firm \( i \)'s residual demand is,

\[
\frac{d\pi_i}{dp} = s_i - x_{i,f} - (b + \beta_j)(p - c_1 - c_2(x_{i,\phi} + s_i)) = 0,
\]

where firm \( i \)'s spot market residual demand \( D(p) - x_{i,\phi} - x_{j,\phi} - s_j(p) \) has already been substituted by its spot market supply \( s_i \). Using \( s_i = s_i(p) = \alpha_i + \beta_i p \) and equating all factors multiplying \( p \) yields the same result for the optimal slope \( \beta_i \) as in (3) which is independent of the firms forward positions.

5Non-negativity of firms output is still secured by the assumption, \((A + \varepsilon)/b > c_1 \) \( \forall \varepsilon \), but the notation makes it less evident here.
The intercept, however, now accounts for the physically forward contracted quantities. Definition (4) is generalised as follows:

$$\alpha^*_i \equiv \beta_i \left( \frac{x_{i,f}}{b + \beta_j} - c_1 - c_2 x_{i,\phi} \right) \quad (4')$$

The market clearing condition determines the spot market price:

$$p = \frac{A - \alpha_i - \alpha_j - x_{i,\phi} - x_{i,\phi} + \varepsilon}{b - \beta_i - \beta_j},$$

Take the definition in (4’) and note that $\beta_i/(b+\beta_j)$ can be expressed as $1 - \beta_i c_2^6$. The market clearing price becomes:

$$p = \frac{A + \varepsilon - (x_{i,\phi} + x_{j,\phi} + x_{i,f} + x_{j,f})(1 - \beta_n) + 2\beta_n c_2}{b - 2\beta_n},$$

which shows that physical $(x_{i,\phi}, x_{j,\phi})$ and financial forward positions $(x_{i,f}, x_{j,f})$ have an identical effect on the equilibrium price in the spot market. With speculators and rational expectations, the forward price must equal the expected spot market price, thus $f = (p|\varepsilon = 0)$, and therefore conjectural variations of firms will not distinguish between physical or financial forward contracts. The strategic effect of either form of contracting is equivalent. Moreover, the optimal collusive strategy obviously does not change through the mere possibility to sell physically forward, and the same applies for the incentives to deviate.

The only difference of physical compared to financial forwards is that physical forward sales diminish the traded volumes in the spot market compared to financial forwards. Firms profits, however, are just the same for either form of contracting. Figure 4 shows how firms best response supply functions vary for the case of physical and financial contracting respectively. The ‘supply function’ now extends to the range of negative quantities. In other words, firms which have sold physically forward their output will buy on the spot market for prices below the marginal cost of their contracted output and sell additional units when the realised price exceeds marginal costs. Physical forward trading can thus flip the producers position from a seller to a buyer on the spot market.

To conclude: We have seen that financial forward contracting can make firms bid prices below their marginal costs, possibly even negative prices, while physical forward sales can make firms flipping their position on the spot market from seller to buyer depending on the price. So one might observe equilibrium ‘supply’ functions for positive and negative prices as well as positive and negative quantities, thus, in each of the quadrants of the price-quantity space. Indeed, this kind of bidding behaviour has a direct representation in a real world market design: The rules of the EPEX Day-Ahead Auction for France, Germany, Austria, and Switzerland require bids to be made as monotone piecewise linear functions mapping prices from $[-300000€/MWh, 300000€/MWh]$ to any positive or negative quantity. It is the auctioneer who splits each individual bid into a supply and a demand part and forms one aggregate supply and one aggregate demand function. But independent who buys and who sells in this auction, total system output must equal total system demand, and the difference between

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6By the definition of the optimal slope of the supply function: $\beta_i = \frac{b + \beta_j}{1 + c_2(b + \beta_j)} = \frac{1}{\frac{b}{\beta_j} + c_2}$. Multiply the denominator of the right hand side and subtract $\beta_i c_2$. 

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no forward trading
\[ x_\phi = 0, \ x_f = 0 \]

physical forward trading
\[ x_\phi > 0, \ x_f = 0 \]

financial forward trading
\[ x_\phi = 0, \ x_f > 0 \]

Figure 4: Spot supply of firms with physical vs. financial forward positions

physical and financial forwards is just a shift of traded volumes between the spot and the forward market. Moreover, physical and financial forwards are strategically equivalent. Prices, output levels and incentives are the same for either type of contract. The previously established results are therefore unaffected by the type of contract which is considered.

5.2 Length of Contracts

In reality, it is typically the day-ahead market for electricity which requires supply function bidding, thus a short term forward market with daily deliveries compared to long term markets for future months, seasons, or years. Long term contracts are then settled day-by-day against the prices prevailing on the day-ahead market over the span of the delivery period. Green and Le Coq (2010) study the effect of such contracts which cover more than one period on the sustainability of collusion in a repeated Bertrand oligopoly. They find that the sustainability of collusion decreases in the length of the contracts. However, they determine two countervailing effects: the ‘gain-cutting’ and the ‘protection’ effect of long term forward contracts.

Gain-cutting by long-term contracts

The gain-cutting effect as described by Green and Le Coq (2010) results from the lock-in of quantities through forward contracts. Consider a Bertrand oligopoly which is engaged in collusion, and has sold physically forward the monopoly quantity at a monopoly price. The contracts cover several openings of the spot market. Between these spot openings, there is no re-opening of the forward market, so firms cannot adjust their contract cover within this period. One firm
deviates at the first opening of the spot market by bidding a price slightly below
the monopolistic price. Other than in a usual Bertrand oligopoly, the deviating
firm cannot capture the whole market but just the part which has not been sold
forward. This is what Green and Le Coq (2010) call the gain-cutting effect. It
occurs only once, independent of the length of the contract and obviously works
pro-collusive.

The protection effect in the model of Green and Le Coq (2010)
Consider the situation as described for the gain-cutting effect. After one firm
deviated at the first opening of the spot market in the delivery period, several
other openings of the spot market take place which still are covered by the
same long term contract. For the length of the delivery period, all firms therefore
profit from forward contracts which were fixed at a collusive price. This includes
the deviating firm, which is thus not suffering the full punishment for defection
from its rivals. This protection effect identified by Green and Le Coq (2010)
works against collusion.

No gain-cutting when firms bid supply functions
Coming back to the case of supply function bidding, the collusive strategies
identified in (9) and (8) are invariant to a change in forward positions. A
deviating firm would not see any difference in cheating against a colluding rival
when this rival holds contracts or not, and therefore, no gain cutting effect
occurs. The protection effect, however, does not disappear: Suppose firms have
closed long term contracts at a collusive price for several openings of the day-
ahead market where they compete in supply functions. Suppose further, that
one firm deviates at the first opening of the day-ahead market. It will gain profit
\( \pi_d \) as identified in Section 3. For the remaining openings of the period covered
by the long term contract, both firms will bid non-cooperative supply functions,
but in addition to their usual gains \( \pi_n \) they will earn revenues from the difference
between the collusive forward contracted price and the competitive day-ahead
price. This is just the protection effect as identified by Green and Le Coq (2010).
Therefore, collusion will be harder to sustain when contracts cover longer time
spans.

5.3 Non-Linearity and the Choice of Forward Positions
The linear SFE model is widely used in the literature and has also proven to
work well in empirical applications. For the forward market, however, it leads
to an equilibrium behaviour which seems to be heavily driven by the somewhat
strict assumption of only linear supply curves: In a Nash equilibrium, when
firms take the rival’s choice of forward positions as given, there is no incentive
to sell forward. Real world markets suggest rather the opposite, namely that
most of the sales are made forward and the spot market market volumes are
small compared to the transactions made on forward markets. While this could
be related to risk aversion on the demand side (see the discussion in the online
Appendix of Green, 1999), forward contracts indeed allow for strategic impacts
on the rival’s spot market strategy when supply functions are assumed to deviate
from the strictly linear case.
The effect of forwards on capacity constraint, non-linear SFE

Klemperer and Meyer (1989) have shown that there generally exists a multitude of supply function equilibria which are bound from above by the supply function which finally becomes steep and hits the Cournot price and quantity for the maximum realisation of demand. The multitude of SFE is bound from below by the supply function which finally becomes flat and hits the Bertrand solution for the maximum realisation of demand (see Figure 2 in Klemperer and Meyer, 1989). There is only one central SFE which is monotonically increasing and well-defined for any positive quantity. In the case of linear demand and linear marginal costs, this unique solution is the well-known linear SFE. In reality, however, demand shocks are unlikely to become infinitely large, therefore non-linear SFE are feasible even with linear marginal cost and demand functions. Green and Newbery (1992) discuss how capacity constraints can narrow this range of equilibria, because the supply curves necessarily become steep once the capacity limit is reached. They therefore consider the upward bending, convex supply function to be more realistic compared to a concave solution which would imply lower margins for larger realisations of demand.

Coming back to the effect of forward contracts, consider an equilibrium where firms are bidding convex supply functions which are steeper than the linear SFE, such that the function finally becomes vertical at some quantity $q_{\text{max}}$. When firms have limited production capacity, it is natural to assume that $q_{\text{max}}$ corresponds to their technical capacity constraint. Without forwards, firms fix the lower end of their supply function ($q = 0$) at a price equal to marginal costs and bid above marginal costs for all positive quantities. Now, when firm $i$ sells $x_i$ forward contracts it will offer the forward contracted quantity $q_i = x_i > 0$ at a price which is equal to marginal costs, while the new supply function will still become steep at $q_{\text{max}}$. Therefore, firm $i$ will bid a steeper supply function for every positive quantity [see the discussion of] pages 115-116]. This implies that forward contracting can alter the slope of supply functions in the spot market, and because the supply function of firm $j$ depends on the slope of firm $i$’s supply function, $i$ has now an instrument to affect firm $j$’s strategy. Note that steeper supply functions imply higher markups on marginal costs. Therefore, firms now have an incentive to sell forward to manipulate the rival’s spot market supply function. This is clearly different to the neutrality of forwards in the linear case. Green (1999, page 116) therefore concludes:

“The ‘upper limit’ of supply function competition in a spot market gives the same price-quantity results as Cournot competition, and adding a contract market (with zero conjectural variations) would be equivalent to Allaz and Vila’s model.”

Non-linear SFE and collusion

So how would considering non-linear supply functions affect the incentives for collusion in a repeated game? While the results from Section 3 are obtained from the linear model, the same reasoning applies when non-linear SFE are possible. Consider Property 1: Collusive profits are unaffected by the possibility to sell forward because the joint profit maximum does not change by the mere possibility to trade forward. Colluding firms always implement the joint profit
maximising supply function, independent of their forward positions. Profits of deviation are increasing in the number of forwards because the deviating firm earns both on the expense of its rivals and on the expense of speculators. Finally, the profits of Nash-reversion obviously change if the equilibrium is non-linear. However, forward contracting still increases competition in the spot market, which is the necessary property to prove Lemma 1. Moreover, convex SFE (as discussed above) endogenously yield an incentive to contract forward, thus to increase competition in the spot market which leads to lower overall profits during the punishment phase. With the Allaz and Vila (1993) model as the limiting case (as pointed out by Green, 1999), we are back to the case of Cournot competition discussed in Liski and Montero (2006). Indeed, this reinforces the generality of Lemma 1 which already states an equivalent result for supply function bidding compared to the result of Liski and Montero (2006) for Cournot competition.

Lemma 2 is less straight-forward to generalise. When Nash-reversion results in non-linear SFE, profits during the punishment phase become non-linear in the demand shock. Suppose, for example, that the collusive and deviation strategies are linear as before. Suppose further that Nash reversion results in an equilibrium with convex supply functions with a globally larger slope compared to the linear SFE. Thus, increasing demand uncertainty will progressively increase profits during the punishment phase (instead of linear as before) and therefore potentially increase the critical discount factor for stable collusion.

In summary, considering non-linear supply functions introduces new aspects to the model. Non-linear convex SFE provide an endogenous incentive for forward contracting which is not present in the linear case discussed in Sections 2 to 4. With this endogenous incentive to sell forward, the possibility to be trapped in a prisoner's dilemma with both firms contracting forward and therefore bidding competitive once collusion breaks down becomes a credible punishment. Therefore, the core result of the linear model survives: Forward markets can increase the range of discount factors for which collusion can be sustained.

6 Conclusion

This paper has studied the effect of forward contracts on the sustainability of collusion when firms’ strategies in the spot market take the form of supply functions. It extends the existing literature which is mainly focusing on either Bertrand or Cournot models and which has obtained substantially different results for either case. It is this aspect which makes the supply function setting specifically interesting because the usual notion of supply function equilibrium suggests that it is located somewhere between the Cournot and the Bertrand solution. Indeed, the analysis of a repeated game with forward markets and supply functions in the spot market yields a result similar to the one of Liski and Montero (2006) for the Cournot model: Forward markets can increase the range of discount factors for which collusion can be sustained because they drive up competition during the punishment phase. The contrary result might prevail as well, because a deviating firm can boost its profit from deviation by selling forwards at a monopolistic price and expanding output in the spot market. This raises the question of observability of firms’ forward positions and the willingness of speculators to buy forwards at a collusive price when they expect deviation
in the coming spot market. One would thus conclude that firms which engage in collusion do not sell forward, or that they carefully control the volumes of forward sales in relation to the overall output of firms.

While collusion is best to be sustained without any forward positions taken, one might wonder if this is feasible in reality. Empirically, there could be two reasons to sell forward some contracts when being engaged in collusion: First, there might be a risk-margin to gain from the demand side as indicated in the Appendix to Green (1999). Secondly, not selling forward could trigger suspicions of the competition authority which has to decide on what sectors to focus its attention. Investigations of the competition authority raise the probability of anti-collusive measures and a forced break of the collusive agreement. A profit maximising oligopoly could thus agree to sell forward to mimic a competitive market as long as this does not endanger the incentive constraint (10) to hold. An empirical investigation should bear these possibilities in mind, but a detailed treatment of these issues is beyond the scope of this study.

Another contribution of this paper compared to the existing literature is that the notion of physically versus financial forward contracting is irrelevant for the presented results. Most of the literature which addresses the strategic use of contracts considers physical forwards which lock in quantities at a fixed price (Allaz and Vila, 1993; Ferreira, 2003; Liski and Montero, 2006; Green and Le Coq, 2010). Such a design would be infeasible for electricity markets because supply and demand must match continuously on a very fine scale.

The stochastic nature of demand is another important feature of electricity markets and it has been shown that it might have ambiguous effects on the sustainability of collusion. A detailed discussion of non-linear SFE and capacity constraints shows that the equivalence to the case of Cournot competition in the spot market survives. Finally, the effect of long term contracts is discussed and it is argued that long term contracting further decreases the possibility to maintain collusion. Future research would need to address the price formation on forward markets and its interaction with more flexible supply function settings on the spot market in more detail.
A  Appendix

A.1  Proof of Property 1

A.1.1  The effect of forward positions on deviation profits (Property 1.c)

Consider the incentive for firm $i$ to deviate in the spot market. The preceding contract market is still characterised by the expectation that collusion holds. So the deviating firm can sell forward at a monopolistic price. The effect of forward contracts $x_i$ on profits can be decomposed as in in Equation (6) which is reproduced here:

$$\frac{d\pi_i}{dx_i} = \frac{\partial \pi_i}{\partial x_i} + \frac{d\pi_i}{d\alpha_i} \frac{d\alpha_i}{dx_i} + \frac{d\pi_i}{d\alpha_j} \frac{d\alpha_j}{dx_j} \frac{dx_j}{dx_i}$$

A deviating firm plays its (spot market) best response to the collusive strategy of its rival $j$. By the definition of the best response, the derivative of profits (with fixed forward prices) to the firms strategy $d\pi_i/\alpha_i^*$ is zero. Moreover, the collusive strategy $\alpha_c$ of $j$ is constant in $x_j$. Therefore, the above given decomposition of $d\pi_i/dx$ reduces for the deviating firm to:

$$\frac{d\pi_i}{dx} = \frac{\partial \pi_i}{\partial x_i} = f - p_d$$

which is positive, because the collusive forward price $f$ is larger than the realised spot price $p_d$ when $i$ deviates.

For the proof of convexity, it is sufficient to note that $f$ is the collusive price and therefore unaffected by forward positions, and $p_d$ is decreasing in $x$. Rearrange the market clearance condition to obtain

$$p_d = \frac{A - \alpha_d - \alpha_c}{b + \beta_d + \beta_c}$$

The second derivative of $\pi^d$ is just

$$\frac{d^2\pi^d}{dx^2} = -\frac{dp_d}{dx_i}$$

which is positive because the spot price is decreasing in $\alpha_i$, and $\alpha_i = \alpha_i^*$ which is increasing in $x_i$ by Equation (4). This establishes convexity of the profits of a deviating firm in its forward positions, independent of the forward position of the rival firm.

A.1.2  The effect of forward positions on non-cooperative profits (Property 1.b)

Note that for the repeated non-cooperative game, other than for the case of deviation, the price dampening effect of forward positions also works through market expectations. Speculative traders will impose $f_n - p_n = 0$, thus $\partial \pi^n_i/\partial x_i = 0$. Note further, that the spot market best response takes the forward price as fixed. Therefore, even when both firms play their best response in the spot market, the overall derivative of profits with respect to their own spot market strategy...
is not zero. Instead it is ‘zero plus the effect which works through the forward price and is omitted in the spot market best response’. By the no-arbitrage condition the forward price equals the expected spot price. Therefore,

\[ \frac{dx_i^n}{dx_i} = x_i \frac{dp_n}{dx_i} \]

and for the joint variation of \( x_i = x_j = x \)

\[ \frac{d\pi_n}{dx} = \frac{\partial \pi_n}{\partial x_i} + \frac{\partial \pi_n}{\partial \alpha_i} \frac{dx_i}{dx} + \frac{\partial \pi_n}{\partial \alpha_j} \frac{dx_j}{dx} = 0 + x_i \frac{dp_n}{dx_i} \frac{dx_i}{dx} + x_j \frac{dp_n}{dx_j} \frac{dx_j}{dx} + \left( q_i + \frac{dq_i}{dp_n} (p_n - c_1 - c_2 q_i) \right) \frac{dp_n}{dx} \frac{dx}{dx} = x \left( x + q_i + \beta \left( p + \frac{c_1 - c_2 q_i}{b + \beta_n} \right) \right) \]

Using \( \alpha_i = \alpha_j = \alpha_n \), the expression above can be reduced to

\[ \frac{d\pi_n}{dx} = \frac{dp_n}{dx} \left( x + q_i + \beta (p_n - c_1 - c_2 q_i) \right). \]

Take Equation (2) to express the profit margin \( (p_n - c_1 - c_2 q_n) \) as \( (q_i - x)/ (b + \beta_n) \).

Take the derivatives of the definition of \( \alpha^* \) and the equilibrium price and obtain

\[ \frac{d^2 \pi_n}{dx^2} = \frac{\partial^2 \pi_n}{\partial x_i \partial x_i} + \frac{\partial^2 \pi_n}{\partial \alpha_i \partial x_i} \frac{dx_i}{dx} + \frac{\partial^2 \pi_n}{\partial \alpha_j \partial x_i} \frac{dx_j}{dx} = x \left( \frac{dq_i}{dx} + \frac{b}{b + \beta_n} \right) \]

which can be reduced to

\[ \frac{d^2 \pi_n}{dx^2} = \frac{-\beta_n}{(b + \beta_n)^2} \left( q_i + \frac{b}{b + \beta_n} \right). \]

which is obviously negative for any positive level of contracting.

For the second derivative of profits, use the definition of the best response \( q_i = \beta \left( p + \frac{c_1 - c_2 q_n}{b + \beta_n} - c_1 \right) \) which has the derivative

\[ \frac{dq_i}{dx} = \beta_i \left( \frac{1}{b + \beta_n} + \frac{dp}{dx} + \frac{dp}{dx} \right) = \beta \left( \frac{1}{b + \beta_n} + \frac{-2\beta_n^2}{(b + \beta_n)(b + 2\beta_n)} \right) = \frac{b \beta_n}{(b + \beta_n)(b + 2\beta_n)} \]

And for profits one obtains

\[ \frac{d^2 \pi_n}{dx^2} = \frac{-\beta_n}{(b + \beta_n)^2} \left( \frac{dq_i}{dx} + \frac{b}{b + \beta_n} \right) = \frac{-\beta_n}{(b + \beta_n)^2} \left( \frac{b \beta_n}{(b + \beta_n)(b + 2\beta_n)} + \frac{b}{b + 2\beta_n} \right) = \frac{-b \beta_n}{(b + \beta_n)^2(b + 2\beta_n)} \left( \frac{\beta_n}{(b + \beta_n)} + 1 \right) \]

which is negative and therefore \( \pi_n \) is proved to be concave and decreasing in \( x \).
A.2 Proof of Property 2

Consider the definition in Equation (12). The derivative of expected profits with respect to $\sigma^2$ is

$$\frac{d\hat{\pi}_i}{d\sigma^2} = \frac{\beta_i - \frac{c_2}{2} \beta_i^2}{(b + \beta_i + \beta_j)^2}$$

This derivative is positive for $\beta_i < \frac{2}{c_2}$ which is true for every profit maximising supply function, irrespective of whether it is a collusive or a best response bid. The difference between collusive, deviation, and non-cooperative profits can be captured easily in terms of the underlying strategies of firm $i$ and its rival $j$.

To compare the effect of uncertainty on expected profits between collusion and deviation, take the following cross derivative:

$$\frac{d^2\hat{\pi}_i}{d\sigma^2 d\beta} = \frac{(1 - c_2 \beta_j)(b + \beta_i + \beta_j)^2 - 2(b + \beta_i + \beta_j)(\beta_i - \frac{c_2}{2} \beta_i^2)}{(b + \beta_i + \beta_j)^4}$$

The latter expression is positive whenever $\beta_i$ is lower than the best response $\beta_i^*$ given in Equation (3). We know that $\beta_d = \beta^*(\beta_c) > \beta_c$, therefore

$$\frac{d\hat{\pi}_d}{d\sigma^2} > \frac{d\hat{\pi}_c}{d\sigma^2}.$$ 

Now, consider the effect of a joint variation of slopes $\beta_i = \beta_j = \beta$.

$$\frac{d^2\hat{\pi}_i}{d\sigma^2 d\beta} = \frac{(1 - c_2 \beta_j)(b + 2\beta)^2 - 4(b + \beta)(\beta - \frac{c_2}{2} \beta^2)}{(b + 2\beta)^4}$$

This is zero for $\beta = \beta_c$ given in Equation (8) and negative for all $\beta > \beta_c$.

Knowing that in the punishment phase $\beta_i = \beta_j = \beta_n$ and $\beta_n > \beta_c$, one obtains

$$\frac{d\hat{\pi}_n}{d\sigma^2} > \frac{d\hat{\pi}_n}{d\sigma^2}.$$ 

This ends the proof of Proposition 2. □
A.3 Cited Literature by Author, Year, and Model Assumptions

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<tr>
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<td>This paper</td>
<td>supply functions</td>
<td>repeated</td>
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Table 1: Summary of reviewed theoretical literature. **This paper includes the linear model with forwards of Green (1999), as well as the discussion of the non-linear model, collusion in supply functions as in Ciurrieta and Gutiérrez-Hita (2006), the effect of forwards on collusion as in Liski and Montero (2006), a discussion of uncertainty, and of physical vs. financial forwards.**
References


