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## **On Growth and the Direction of Technological Change**

by

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# On Growth and the Direction of Technological Change

von

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## Abstract:

Recent econometric analyses of growth in industrialized countries reveal that energy's elasticity of production systematically exceeds its factor cost share, whereas for labor the opposite holds. The paper reviews these analyses that reflect the observed direction of technological change towards increasing automation.

*Keywords:* Production Factor Energy; Economic Growth; Technological Change; Progress of Automation

*JEL-classification:* D30; O30; Q43

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## I. INTRODUCTION

In the industrialized countries the cost shares of the production factors are typically 0.7 for labor, 0.25 for capital, and 0.05 for energy. As is well known, by using these cost shares as technological factor-input weights, i.e. as values for the elasticities of production, neither the recessions during the energy crises in the 1970s, nor long-term economic growth can be explained. Large residuals, interpreted as the effects of ‘technical progress’, remain. “This has led to a criticism of the neoclassical model: it is a theory of growth that leaves the main factor in economic growth unexplained” (Solow, 1994). One response to the ‘Solow residual’ was the emergence of new growth theories initiated by Romer (1986). While the new theories have enriched the perspectives on growth in many ways, e.g. by introducing endogenous innovation, imperfect competition, or the accumulation of human capital, their shortcomings include the problem of “growth on the knife’s edge”, e.g., tiny deviations from the constant-returns-to-capital assumption result in either a loss of permanent growth or infinite growth in finite time (Solow 1994), and the almost entire abstraction from the physical sphere of production. This Letter reviews a complementary approach to production and growth theory, which reproduces empirical growth with small residuals while keeping the conventional diminishing-returns-to-capital assumption: it takes into account the production factors capital, labor, and energy, and technology parameters whose time-changes model innovation and technological change.

## II. GROWTH

We derive production functions  $q=q(k,l,e,t)$  from the following growth equation:

$$\frac{dq}{q} = \alpha \frac{dk}{k} + \beta \frac{dl}{l} + \gamma \frac{de}{e} + \delta \frac{dt}{t}. \quad (1)$$

Thereby,  $\alpha \equiv (k/q)/(\partial q/\partial k)$ ,  $\beta \equiv (l/q)/(\partial q/\partial l)$ , and  $\gamma \equiv (e/q)/(\partial q/\partial e)$  are the elasticities of production of capital  $k$ , labor  $l$ , and energy  $e$ , respectively. Note, that the inclusion of energy is necessary, if

production functions are to have a physically sound interpretation.<sup>1</sup> All quantities are normalized to their absolute values  $Q_0, K_0, L_0, E_0$  in a base year '0', i.e.,  $q=Q/Q_0, k=K/K_0, l=L/L_0, e=E/E_0$ .<sup>2</sup> As long as  $\delta=0$ , technical causality of work performance and information processing in production by capital, labor, and energy uniquely determines the output  $q$ , and  $k, l$ , and  $e$  are, by definition, all factors of production. Thus, we have constant returns to scale:  $\alpha+\beta+\gamma=1$ . A non-zero  $\delta$  will represent time-changes of technology parameters in the production functions, see below.

The requirement that the second-order mixed derivatives of  $q$  with respect to  $k, l, e$  are equal results in a set of partial differential equations for the elasticities of production. Due to the constant returns to scale, one of the elasticities can be eliminated. If one eliminates  $\gamma$ , the resulting equation for  $\alpha$  is  $k(\partial\alpha/\partial k)+l(\partial\alpha/\partial l)+e(\partial\alpha/\partial e)=0$ , the one for  $\beta$  has identical structure, and the coupling equation reads  $l(\partial\alpha/\partial l)=k(\partial\beta/\partial k)$ . The most general solution of the first two equations are  $\alpha=f(l/k, e/k)$  and  $\beta=g(l/k, e/k)$ , with arbitrary differentiable functions  $f$  and  $g$ . The trivial solutions are constant elasticities  $\alpha_0, \beta_0, \gamma_0=1-\alpha_0-\beta_0$ , from which the Cobb-Douglas function  $q_{CDE} = q_0 k^{\alpha_0} l^{\beta_0} e^{1-\alpha_0-\beta_0}$  results. This function allows the thermodynamically impossible (asymptotically) complete substitution of energy by capital and must therefore be avoided in scenarios for the future. In analyses of the past, however, it works satisfactorily (Kümmel et al. 2000, Lindenberger 2000), if its elasticities of production are close to the time-averages of the elasticities belonging to the production functions derived below.

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<sup>1</sup> The Laws of Thermodynamics imply that no production process can be driven without energy conversion.

<sup>2</sup> Energy is taken from energy balances in energetic units, e.g. petajoule per year, labor from labor statistics in hours worked per year, capital and the output of value-added in constant currency from the National Accounts. Ideally, one would like to measure capital by the amount of work performance and information processing that capital is able to deliver when fully activated by energy and labor. Likewise, the output might be measured by the work performance and information processing necessary for its generation. The detailed, quantitative technological definitions of capital and output are given by Kümmel (1982). However, since these measurements are not available, proportionality between them and the constant currency data is assumed.

## 1. Industrial production

Simple, factor-dependent solutions of the differential equations that satisfy technologically reasonable asymptotic boundary conditions for industrial production are  $\alpha=a_0(l+e)/k$ ,  $\beta=a_0(c_0l/e-l/k)$ , and  $\gamma=1-\alpha-\beta$  with parameters  $a_0$  and  $c_0$ . The capital-efficiency parameter  $a_0$  gives the weight with which labor/capital and energy/capital combinations contribute to the productive power of capital, and  $c_0$  indicates the energy demand  $e_t=c_0k_t$  of the fully utilized capital stock  $k_t$  that would be required in order to generate the industrial output totally automated (Kümmel 1982). If one inserts these elasticities of production into eq. (1) and integrates, with  $\delta=0$ , one obtains the (first) LINEX production function:

$$q_{L1}(k, l, e) = q_0 e \exp \left\{ a_0 \left( 2 - \frac{l+e}{k} \right) + a_0 c_0 \left( \frac{l}{e} - 1 \right) \right\}, \quad (2)$$

which depends LINearly on energy and EXponentially on factor ratios. Innovations and structural change make the technology parameters  $a_0$ ,  $c_0$ , and  $q_0$  time-dependent.

## 2. Service production

In the service sector, by its very nature, the potentials of automation are more limited than in manufacturing. However, it is still possible to substitute human labor –to some extent– by energy-driven and increasingly information processing capital.<sup>3</sup> We incorporate these production possibilities by using the law of diminishing returns: We assume that the approach toward the limiting state of maximum automation in service production is associated with decreasing returns to energy utilization. The simplest corresponding elasticity of production for energy is:  $\gamma=a_0(c_m-e/k)$ , where  $c_m=e_m/k_m$  measures the energy demand of the maximum automated capital stock

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<sup>3</sup> In fact, in the medium term most progress of automation by computer-based information processing is expected in trade, banking, insurance, and public administration (Thome, 1997).

(Lindenberger 2000). Using  $\alpha=a_0(l+e)/k$ , as above, and  $\beta=1-\alpha-\gamma$ , integration of eq. (1), with  $\delta=0$ , yields the service production function  $q_{SI}$ :

$$q_{SI}(k, l, e) = q_0 l \left( \frac{e}{l} \right)^{a_0 c_m} \exp \left\{ a_0 \left( 2 - \frac{l+e}{k} \right) \right\}, \quad (3)$$

where, again, innovation and structural change make the technology parameters  $c_m$ ,  $a_0$ , and  $q_0$  time-dependent.

It is important to note that  $\alpha$ ,  $\beta$ , and  $\gamma$  must be non-negative in order to make sense technologically. For instance, the non-negativity of  $\gamma=a_0(c_m-e/k)$  implies that one cannot feed more energy into the energy conversion devices of the capital stock than they can receive according to their technical design. The requirement of non-negative  $\alpha$ ,  $\beta$ , and  $\gamma$  imposes restrictions on the admissible factor quotients in the elasticities of production and the production functions, and incorporates the thermodynamic and technical limits to substitution in the model.

### 3. *Empirical results*

Application of the above and related production functions to actual growth experience involves the determination of the corresponding technology parameters by fitting the functions to empirical time-series data of value added, capital, labor, and energy by non-linear OLS, subject to the constraints of non-negative elasticities of production. A number of such studies have been carried out for various sectors of the US, the Japanese, and the German economy (Ayres, 2001; Ayres and Warr, 2003; Beaudreau, 1998; Kümmel et al., 1985, 2000, 2002; Lindenberger, 2000; Lindenberger et al. 2001). Their findings can be summarized as follows:

- Observed economic growth is reproduced with **minor residuals**. Even at constant technology parameters over periods of one-and-a-half decades, including the recessions after the oil crises in the 1970s, value-added is reproduced well. Thus, by taking into account the indispensable production factor energy appropriately besides capital and labor, the activation

of the increasingly automated capital stock can be modelled endogenously, and the Solow residual is mostly resolved.

- Modelling empirical growth over periods of two decades and more requires an explicit treatment of **innovation and structural change**. The capital stocks' energy-demand parameters ( $c_t$  and  $c_m$  in eqs. 2 and 3) decrease and the capital-efficiency parameters ( $a_0$  in eqs. 2 and 3) increase, particularly after the oil-price hikes in the 1970s. These parametric shifts reflect the massive investments into more energy-efficient technologies and structural changes after the energy crises.
- The (time-averaged) **elasticities of production** of energy exceed the cost share of energy considerably, whereas for labor the opposite holds, and only for capital, elasticities and shares are roughly in equilibrium. The discrepancies reflect the observed direction of technological change towards increasing automation, whereby costly routine labor is substituted by energy and increasingly information processing capital, as technical progress makes the corresponding factor combinations accessible.

### III. CONCLUSIONS

From the perspective of the outlined growth theory and its empirical results, the evolution of production appears as a non-equilibrium process: This process is characterized by the permanent incentive to enhance substitution possibilities in order to continually replace costly labor by energy and (more and more information processing) capital, thus increasing automation in production. Therefore, the accumulation and continuous restructuring of the capital stock relies both on appropriately qualified and creative labor and the increasing and increasingly efficient utilization of the natural resource energy. The latter constitutes an essential and major driver of growth and technological change.

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