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Spatial dependencies of wind power and interrelations with spot price dynamics

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Abstract

Wind power has seen strong growth over the last decade and increasingly affects electricity spot prices. Generation from wind energy is stochastic, and if there is lot of wind, prices tend to be lower. Therefore, for an investor, but also for the whole electricity system, it is important to assess the value of wind power at different locations. In this paper, we develop a stochastic simulation model that captures the full spatial dependence structure of wind power by using copulas, incorporated into a structural supply and demand based model for the electricity spot price. This model is calibrated with German data. We find that the specific location of a turbine – i.e., its spatial dependence with respect to the aggregated wind power in the system – is of high relevance for its value. Many of the locations analyzed show an upper tail dependence that adversely impacts the market value. Therefore, a model that assumes a linear dependence structure would systematically overestimate the market value of wind power in many cases. This effect becomes more important for increasing levels of wind power penetration and may render the large-scale integration into markets more difficult.

Keywords: OR in Energy; Wind Power; Market Value; Copula; Stochastic Spot Price Model; Simulation

JEL classification: C15, C51, Q41

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1. Introduction

The amount of electricity generated by wind power plants has increased significantly during recent years. Due to the fact that wind power is stochastic, its introduction into power systems caused changes in electricity spot price dynamics: they have become more volatile and exhibit a correlated behavior with wind power fed into the system. In times of high wind infeed, spot prices are observed to be generally lower than in times with low production of wind power. Empirical evidence of this effect has been demonstrated for different markets characterized by high wind

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power penetration, e.g., by Jónsson et al. (2010) for Denmark, Gelabert et al. (2011) for Spain, Woo et al. (2011) for Texas or Cutler et al. (2011) for the Australian market. Due to the cost-free availability of wind energy, wind power plants are characterized by marginal costs of generation that are lower than for other types of power plants such as coal or gas. Hence – if the wind blows – wind power generation may replace other types of generation and thus lead to lower spot market prices in such hours. As a consequence, power plants are faced with increasingly difficult conditions and an additional source of price risk when participating in the market. Until now, fluctuating renewable energy technologies (including wind power itself) have often been exempted from this price risk by support mechanisms (e.g., by fixed feed-in-tariff systems) in order to incentivize investments.[1] However, their price risk draws more and more attention as they make up an increasing share of the generation mix and may at some point be fully integrated in the liberalized power market. Therefore, for an individual investor as well as for a social planner it becomes increasingly important to understand the value of wind generation and how it depends on the location of the wind turbine.

The purpose of this paper is to derive revenue distributions and the market value of wind at specific locations, i.e., the weighted average spot price wind power is able to achieve when selling its electricity on the spot market. It is clear that the value of a specific location depends on whether it can typically produce only in hours where many other wind generators at other locations can also produce, or not. To capture the full stochastic dependence structure of wind infeed we use copulas, and incorporate the stochastic wind generation in a structural supply and demand based model for electricity prices. We calibrate the model with German data, since Germany already has a high share of wind power.

We find that taking into account the entire spatial dependence structure is indeed necessary, and that considering only correlations between a specific turbine and the aggregate wind production would be misleading. Even if the correlation of a specific turbine is lower compared to another, the resulting market value may be lower due to a non-linear, asymmetric dependence structure. In fact, we find a pronounced upper tail dependence that adversely impacts the market value for many of the locations analyzed. Therefore, a model solely based on linear dependence measures would systematically overestimate the market value of wind power in many cases. Moreover, it is shown that this effect becomes increasingly important for higher levels of wind power penetration.

To derive these results, we need to take the following two steps. In a first step, we develop a stochastic simulation model for electricity spot prices that incorporates the aggregated wind power infeed as one of the determinants. Electricity spot prices are very volatile and follow daily, weekly and seasonal patterns due to a very price-inelastic fluctuating demand and limited storage possibilities. The market is designed such that in a competitive environment, suppliers who can offer electricity at lowest marginal generation costs will cover the demand. Hence, an increasing

[1]For a comprehensive overview of different renewable support mechanisms including their economic implications, e.g., refer to Green and Yatchew (2012).

demand comes along with the need for capacities characterized by higher marginal generation costs and generally leads to higher electricity prices. Since the marginal costs of wind power production are close to zero, available production quantities will always cover some part of the demand as long as prices are non-negative. The supply curve representing all available generation capacities ranked in ascending order of their short-run marginal costs of production is often referred to as the so called 'merit order'. In our model, we implicitly approximate the merit order by estimating an empirical function from hourly spot prices, demand and wind power. We then feed the price formation process with aggregated wind power series, and add a stochastic component in order to cover additional stochastic price movements, caused, e.g., by unplanned power plant outages, scarcity prices, speculation, or demand side management.

In the second step, we link the market's aggregated wind power to the wind power of single turbines in order to quantify their market value and the revenues depending on their specific location. We use copulas to model this interrelation. Why copulas are necessary is illustrated by the following thought experiment. There are two turbines A and B characterized by equal availability factors and equal correlation coefficients between their own infeed and the aggregated infeed of the turbines in the market. Turbine A follows the production pattern of all other turbines very closely at low infeed levels but is much less dependent at high infeed levels, and can therefore realize high prices when producing at full power. In contrast, turbine B faces the adverse situation of having a particularly high probability that every time it runs at full power, a large share of all the other turbines in the system is also running (i.e., a high correlation in the upper tail). Hence, weighted average prices gained by turbine B will be lower.

Our paper contributes to three lines of literature. First, our paper builds on the literature on structural demand and supply models. Within this class of models, Bessembinder and Lemmon (2002) were among the first to study the importance of demand and production costs for electricity prices. They develop a theoretical model for electricity derivatives and show that the level and variance of the electricity demand impact the forward premium. Motivated by these theoretical foundations, Longstaff and Wang (2004) provide empirical evidence for a significant forward premium in the PJM market. Barlow (2002) presents a different approach to model electricity prices based on an Ornstein-Uhlenbeck model for the demand process and a functional dependence between prices and demand. The model developed by Burger et al. (2006) follows the same conceptual approach by including a non-linear functional dependence of the electricity spot price on a stochastic demand process as well as a long-term non-stationarity. Their model is used to price derivatives via Monte Carlo simulation. Howison and Coulon (2009) deploy a stochastic electricity bid stack, i.e., detailed information on the supply curve. They further extend the number of state variables explaining the electricity spot price by including fuel prices. We extend this literature by including stochastic production quantities due to weather phenomena (e.g., wind) that may impact the supply side and hence electricity prices.

Secondly, we build on the literature using copulas. Copulas have first been identified by Papaefthymiou (2006) to be a suitable tool in modeling multivariate dependencies of wind power. Grothe and Schnieders (2011) model spatial dependencies of wind speeds in order to allocate wind farms in Germany such that an optimal reduction of power output fluctuations is achieved. Hagspiel et al. (2012) model European wind power based on copula theory and use the simulated data as an input for a probabilistic load flow analysis. In contrast to the existing literature, we apply conditional copulas to model the dependence structure between specific turbines and the aggregated wind power. The latter in turn is needed as an input for the spot price model.

Finally, our paper complements ongoing research on the valuation of power generation assets. So far, this line of research has mainly focused on conventional power (e.g., Thompson et al. (2004), Porchet et al. (2009) or Falbo et al. (2010)) and the optimization of hydro power schedules (e.g., García-González et al. (2007) or Densing (2013)). Also, a number of papers have valued wind power based on historical data (e.g., Green and Vasilakos (2012)). However, we know of no study presenting a model that fully captures the stochastics of wind power and interrelations with spot price dynamics.

The remainder of this article is organized as follows: Section 2 provides a short introduction to copula modeling with a particular focus on conditional copula sampling which we apply in our model. The model itself is presented in Section 3. Section 4 reports the results of the methodology applied to the case of wind power in Germany, namely the revenues and the market value of specific wind turbines. Section 5 concludes.

2. Stochastic dependence modeling using copulas

In this section, we briefly discuss the modeling of stochastic dependencies with the help of copulas. A more detailed introduction is provided e.g., in Joe (1997) or Nelsen (2006). For a comprehensive literature review of the current status and applications of copula models, the interested reader is referred to Genest et al. (2009), Durante and Sempi (2010) and Patton (2012).

2.1. Copulas and copula models

A copula is a cumulative distribution function with uniformly distributed marginals on $[0,1]$. Sklar's theorem is the main theorem for most applications of copulas, stating that any joint distribution of some random variables is determined by their marginal distributions and the copula (Sklar (1959)). The bivariate form of Sklar's theorem is as follows: For the cumulative distribution function $F : \mathbb{R}^2 \to [0,1]$ of any random variables X, W, with marginal distribution functions F_X, F_W, there exists a copula $C : [0,1]^2 \to [0,1]$ such that

$$F(x,w) = C\left(F_X(x), F_W(w)\right). \tag{1}$$

The copula function is unique if the marginals are continuous.[2] Conversely, if C is a copula and F_X and F_W are continuous distribution functions of the random variables X, W, then (1) defines the bivariate joint distribution function. From Sklar's theorem, it follows that copulas can be applied with any marginal distributions. Particularly, marginal distributions may differ for each of the random variables considered.

In our application we are interested in the dependence structure of the market's aggregated wind power W and a single turbine's wind power X. The copula captures the complete dependence structure of X and W. The selection of an appropriate copula model can be made independent from the choice of the marginal distribution functions. Taking advantage of this, the joint distribution of W and X is determined in a two stage process: First, the marginal distribution functions F_W and F_X are determined, followed by the selection of the most appropriate copula model.

Copula functions are mostly determined in a parametric way. There are different types of parametric copula models that can be used to capture the pairwise dependence. In many applications – such as ours – it is particularly important to differentiate between symmetric or asymmetric, tail or no tail, and upper or lower tail dependence structures. Therefore, one can test several parametric copula models that are able to capture these characteristics: The Gaussian copula is symmetric and has zero or weak tail dependence (unless the correlation is 1). In contrast, the symmetric Student-t copula has a relatively strong symmetric tail dependence. Whereas the Frank copula is another symmetric copula with particularly low tail dependence, Clayton and Gumbel copulas incorporate an asymmetric tail dependence. Lower tail dependence is captured by the Clayton copula, while the Gumbel copula incorporates an upper tail dependence.[3] These copulas are listed in Table 1.[4]

Table 1: Copula models

Copula family	**Copula function** $C(u,v)$
Gaussian	$\Phi_\Sigma\left(\Phi^{-1}(u), \Phi^{-1}(v)\right)$
Student-t	$t_{\Sigma,\nu}\left(t_\nu^{-1}(u), t_\nu^{-1}(v)\right)$
Clayton	$\left(\max\left\{u^{-\theta}+v^{-\theta}-1, 0\right\}\right)^{-1/\theta}$
Frank	$\frac{-1}{\theta}\ln\left(1+\frac{\left(e^{-u\theta}-1\right)\left(e^{-v\theta}-1\right)}{e^{-\theta}-1}\right)$
Gumbel	$e^{-\left((-\ln(u))^\theta+(-\ln(v))^\theta\right)^{1/\theta}}$

[2]Sklar's theorem also holds for the multivariate case of $n > 2$ dimensions.

[3]Gaussian and Student-t copulas belong to the group of Elliptical copulas, whereas Frank, Gumbel and Clayton copulas belong to the group of Archimedean copulas. For a more extensive discussion of different copula families, see, e.g., Nelsen (2006)

[4]u and v can be interpreted as $F_X(x)$ and $F_W(w)$, respectively. Φ_Σ denotes the multivariate normal distribution function with covariance matrix Σ and $t_{\Sigma,\nu}$ the multivariate Student-t distribution with ν degrees of freedom and covariance matrix Σ.

The copula parameters can be estimated based on observed data by optimizing the log-likelihood function:

$$\hat{\theta} = \max_{\theta} \sum_{t} \ln c\left(F_X\left(x_t\right), F_W\left(w_t\right); \theta\right) \tag{2}$$

where θ denotes the parameter vector and c the copula density. The selection of the most appropriate copula model can then be determined based on the Akaike Information Criteria (AIC).

2.2. Conditional copula and simulation procedure

Like any ordinary joint distribution function, copulas have conditional distribution functions. The conditional copula can be calculated by taking first derivatives with respect to each variable, i.e., for $u = F_X(x)$ and $v = F_W(w)$ we have

$$C(u|v) = \frac{\partial C(u,v)}{\partial v} \quad \text{and} \quad C(v|u) = \frac{\partial C(u,v)}{\partial u}. \tag{3}$$

For the application presented in this paper, there is one inherent advantage of using conditional copulas rather than sampling directly from the bivariate copula distribution: Samples can be conditioned on time series that may serve as inputs to the simulation procedure.[5] The time series characteristics can thus be preserved during the simulation process.

We consider the stochastic processes $(X_t)_{t\in\mathbb{N}}$ and $(W_t)_{t\in\mathbb{N}}$. $F_{X_t}(X_t), F_{W_t}(W_t)$ are uniformly distributed random variables on $[0,1]$. For random variables $U_t, V_t \sim U\left(0,1\right)$, $F_{X_t}^{-1}\left(U_t\right)$ and $F_{W_t}^{-1}\left(V_t\right)$ thus follow the distributions of X_t and W_t, respectively. It is important to notice that by applying the inverse distribution functions, the dependence structure is not influenced, i.e., U_t and V_t as well as $F_{X_t}(X_t)$ and $F_{W_t}(W_t)$ have the same copula C.

The conditional sampling procedure can be summarized as follows:

1. Apply the marginal distribution function F_{W_t} to the time series of the market's aggregated wind power $(w_1, w_2, w_3, ...)$ in order to get $(v_1^*, v_2^*, v_3^*, ...)$.
2. Simulate $(u_1, u_2, u_3, ...)$ from independent uniformly distributed random variables.
3. For each observation $F_{W_t}\left(w_t\right) = v_t^*$, apply the inverse conditional copula $C^{-1}_{F_{W_t}(W_t),F_{X_t}(X_t)}\left(\cdot|v_t^*\right)$ to translate u_t into u_t^* by:

$$u_t^* = C^{-1}_{F_{W_t}(W_t),F_{X_t}(X_t)}\left(u_t|v_t^*\right) \tag{4}$$

4. Apply the inverse marginal distribution functions to $(u_1^*, u_2^*, u_3^*, ...)$ in order to obtain the corresponding simulations of the random variable X_t: $\left(F_{X_1}^{-1}\left(u_1^*\right), F_{X_2}^{-1}\left(u_2^*\right), F_{X_3}^{-1}\left(u_3^*\right), ...\right)$.

[5] We use time series of the market's aggregated wind power as an input variable for the spot price model.

3. The Model

The main goal of our quantitative analysis is to simulate the power generation of a single turbine and simultaneous spot prices. These two quantities enable us to analyze the price effect of wind power as it is realized by single turbines and hence to determine the respective market value. To this end, we develop a stochastic simulation model for the single turbine wind power and electricity spot prices, including a precise representation of their interrelations. The interrelation is established by the aggregated wind power that is related to both the electricity spot prices as well as the single turbine wind power. Hence, we set up a model that represents these two relationships: First, a structural supply and demand based model that takes, among others, the aggregated wind power as an input. Second, a stochastic dependence model that links the single turbine wind power to the aggregated wind power. These two parts of the model can be summarized by the following two equations:

$$S_t = h_t\left(D_t - W_t\right) + Z_t \tag{5}$$
$$X_t = F_{X_t}^{-1}\left(C^{-1}_{F_{X_t}(X_t),F_{W_t}(W_t)}\left(U_t | F_{W_t}\left(W_t\right)\right)\right) \tag{6}$$

where S_t is the hourly stochastic spot price and X_t the hourly single turbine wind power. The spot price S_t is determined by two components: First, the function h_t describes the dependence of the spot price on the residual demand that is determined by the difference of the electricity demand level D_t and the stochastic aggregated wind power W_t. Second, a short term stochastic component adds to the spot price that is denoted by Z_t. As operators of wind power plants are able to curtail their power output in case of negative spot prices, their price is non-negative, i.e., $S_t^W = \max\{0, S_t\}$.

The second part of the model links the hourly single turbine wind power X_t to the aggregated wind power W_t at time $t \in \mathbb{N}$. F_{X_t} and F_{W_t} denote the corresponding marginal distribution functions. The joint distribution function of these two random variables is determined by the corresponding copula, i.e., $F_{X_t,W_t}\left(x_t, w_t\right) = C\left(F_{X_t}\left(x_t\right), F_{W_t}\left(w_t\right)\right)$. Due to the copula's ability to separate marginal distribution functions and the dependence structure, the joint distribution function can be modeled in a two-step process: First, the marginal distribution functions $F_{X,t}$ and $F_{W,t}$ are determined. Second, the appropriate copula $C_{F_{X_t}(X_t),F_{W_t}(W_t)}$ is selected and estimated. We deploy the conditional copula in order to keep the time series properties of the stochastic process $\left(W_t\right)_{t\in\mathbb{N}}$. For the simulation procedure, independent $[0,1]$-uniformly random variables U_t are needed.[6] Note that the marginal distribution functions are the same within a month m, i.e., $F_{X_i} = F_{X_j}$ if $i, j \in m$. The same holds for F_{W_t}, h_t and $C_{F_{X_t}(X_t),F_{W_t}(W_t)}$.

Based on Equations (5) and (6), the hourly single turbine wind power and the spot prices can be simulated from stochastic processes. These cover the stochastic nature incorporated in the price

[6]For a more detailed theoretical description of the simulation procedure see section 2.2.

determinants as well as in the dependence structure between the single turbine's wind power and the aggregated wind power.

3.1. The data

Different data sets are deployed in order to calibrate and estimate the different parts of the model. In the following, we explain the content and origin of these sets, as well as the way in which the data are preprocessed.

Prognosis of the German aggregated wind power: For the estimation of the appropriate copula (C) as well as for the structural supply and demand model (represented by h_t in Equation (5)), data is needed on the effectively delivered day-ahead prognosis of the German aggregated wind power in 2011. This is provided by the transmission system operators and published on the EEX Transparency Platform (EEX Transparency Platform (2012)). Note that the day-ahead prognosis – and not the actual aggregated wind power – is used, since this is the relevant information for the day-ahead market (Jónsson et al. (2010)).

Wind speeds: Hourly mean wind speeds for various measurement stations in Germany are provided via the national climate monitoring of the German Weather Service for the years 1990-2011 (Deutscher Wetterdienst (DWD) (2012)). The measurement data for 19 locations are used in this project to determine the corresponding power output series of wind turbines.[7] Wind speeds are scaled to the hub height of currently installed wind turbines (100 meters).[8]

Wind power capacities: The development of currently installed wind power capacities per federal state between 1995 and 2011 is available from the German Wind Energy Association (German Wind Energy Association (BWE) (2012)). In 2011, installed wind power capacities in Germany amounted to 27.1 GW.

Electricity demand levels: Hourly electricity demand levels for the German market in 2011 — used as one of the explaining variables for spot prices and denoted by D_t in Equation (5) — are provided by ENTSO-E (2012).

Spot prices: EPEX day-ahead prices from 2011 are deployed for the calibration of the spot price model (Equation (5)). The EPEX day-ahead market is organized by an auctioning process that matches supply and demand curves once a day, thus determining prices at which electricity is exchanged in each respective hour.

3.2. Derivation of synthetic aggregated wind power

As an important input for the model, curves are needed that describe the wind power infeed that the currently installed wind power capacities would have produced during the last decades

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[7] Missing data are interpolated based on the previous and next available value if the missing gap is not exceeding 12 hours. If the gap is longer, the values are replaced by data of the same station and same hours of the previous year.

[8] As wind speeds are measured only a few meters above the ground, they are scaled to the hub height of modern wind turbines (100 meters) assuming a power law: $v_{h_1} = v_{h_0}(h_1/h_0)^{\alpha}$, where h_0 is the measurement height, h_1 the height of interest and α the shear exponent. According to Firtin et al. (2011), α is assumed to be 0.14.

(i.e., the long-term stochastic behavior of aggregated wind power in the power system). In the model, the curve is needed for the estimation of the marginal distribution $F_{W,t}$ of the aggregated wind power W_t. It is important to notice that this curve has to be derived synthetically, as wind power capacities changed significantly during the last years.

Based on wind speeds and wind power capacities, the synthetic German aggregated wind power is generated as follows: By applying a power curve capturing the characteristics of the transformation process from wind energy to electrical power, wind turbine power generation profiles can be derived. In this study, the power curve is assumed to be one of a GE 2.5 MW turbine (General Electric (2010)). Alternatively, one could use an average taken from multiple turbines. The transformation is based on a look-up table derived from the power curve and linear interpolation. Furthermore, electrical output is determined as a ratio of installed wind power capacity (i.e., scaled to $[0, 1]$). Multiplying this ratio with the wind power capacity installed in the corresponding federal state yields the wind power infeed. The above steps are repeated for 16 locations (one for each federal state) and all available years (1990–2011), resulting in a time series for what would have been produced during the last 22 years with current wind power capacities. In order to check the plausibility of this approach, historical wind energy time series and volumes can be compared to the model estimates. The comparison for the 2011 time series yields high conformity with an R^2 of 0.84. Another check of consistency is done by calculating the accumulated aggregated wind power production volumes for the past 10 years from the synthetically generated curves, and comparing them to the overall wind power production as reported in Eurostat Database (2012). We find the deviations to be less than 12%.

3.3. Structural supply and demand based model for the electricity spot price

We develop a structural supply and demand based model to derive electricity spot prices dependent on the level of wind power infeed. In electricity markets, the supply curve representing all available sources of electrical generation ranked in ascending order of their short-run marginal costs of production is often referred to as the merit order. In our model, we implicitly approximate the merit oder of all technologies except for wind energy by estimating an empirical function from hourly spot prices and residual demand.[9] Since in electricity markets demand is very inelastic in the short term, the merit order determines the prices to a large extent.[10] We then introduce wind power as a generation technology that has marginal costs of production close to zero. Hence - if available - it will always cover some part of the demand as long as prices are non-negative.

We describe the non-linear relation between residual demand and spot prices (i.e., h_t in Equation (5)) by an empirical function estimated from historical hourly spot prices, demand and wind infeed data. To derive a functional form for h_t we use spline fits which are suitable to capture the

[9] A similar approach has been applied in Burger et al. (2006)

[10] For an empirical investigation of the short-term elasticity of electricity demand, e.g., refer to Liejsen (2007)

non-linearities in the demand-price dependence. The parameters of h_t are estimated from historical data for the reference year 2011 on a monthly basis in order to capture seasonal differences and variations on the supply side that occur, e.g., because of planned outages, variations in fuel costs, etc.

The data and the corresponding spline fit are shown in Figure 1 for the month of February 2011. All other months of 2011 are presented in Figure B.10 in the Appendix. As can be observed, the dependence between residual demand levels and prices is characterized by steep ends and a comparatively flat part in between (i.e., for the residual demand ranging between 40 and 70 GW). The steeper part in the lower tail is generally more pronounced than the price increase for higher residual demand levels. Rather moderate price increases in the upper tail may be interpreted by prevailing excess capacity in the German power market, leading to very few instances at which scarcity prices occur.

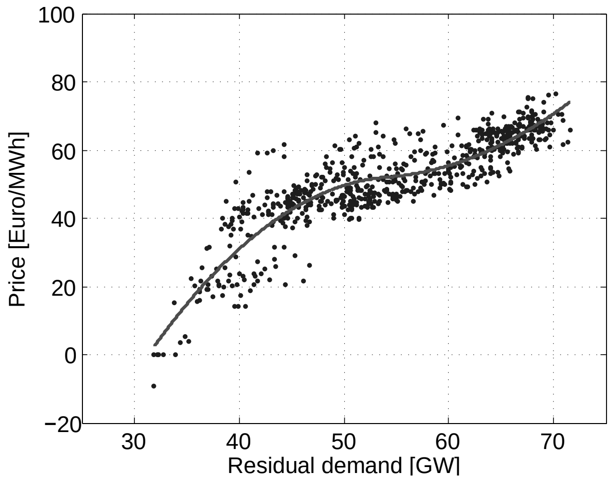


Figure 1: Demand-price dependence in February 2011 and spline fit

Besides the functional dependence on (residual) demand, additional stochastic factors influence spot market prices such as speculation, unplanned power plant outages, scarcity prices or demand side management.[11] These effects are lumped together and captured by the short-term stochastic process Z_t in Equation (5). In the following, we aim at finding a model for Z_t that is capable of capturing the characteristics observed in the data. After having determined h_t, we can derive the observed residual short-term stochastic component based on the observations of residual demand and spot prices from $z_t = s_t - h_t(d_t - w_t)$, and use the result for the calibration of the stochastic process $(Z_t)_{t\in\mathbb{N}}$. The time series z_t is visually observed to be stationary within the considered time frame, which is confirmed by an augmented Dickey-Fuller test that indicates that the null

[11] All forms of demand response and load reduction are often referred to as demand side management (DSM). As such, the dependence of demand on prices is one of the factors captured by Z_t. Note that we hereby assume that DSM is independent of the wind power infeed that we use as an exogenous simulation input.

hypothesis of a unit root can be rejected at the 95% level.

The empirical auto-correlation function of z_t decays slowly, however, with an apparent dependence at a lag of 24 hours. We therefore choose to model Z_t as a seasonal ARIMA (SARIMA) model with a 24 hour seasonality. In order to do so, the ARIMA model needs to be extended to include non-zero coefficients at lag s, where s is the identified seasonality period. SARIMA models can be specified in a multiplicative form, resulting in a more parsimonious model than simply extending ARIMA to s lags.[12]

As the Engle's ARCH test indicates that there is conditional heteroscedasticity in the data, we extend the SARIMA by a GARCH component. GARCH-type models are able to capture conditional heteroscedasticity by splitting the error term ϵ_t into a stochastic component η_t and a time-dependent standard deviation σ_t. The latter can then be expressed dependent on lagged elements of ϵ and σ_t (Engle (1982), Bollerslev (1986)).

Various specifications of SARIMA-GARCH models are estimated and evaluated. Based on the AIC, a GARCH(1,1)-SARIMA(2,0,2)$\times$(1,0,1)$_{24}$ model is found to perform best. The inclusion of additional parameters hardly improves the fit. Note that no constant needs to be added to the model of Z_t due to the fact that the process has been already centered by applying a spline fit.

Comparing the residual's distribution to the normal distribution yields unsatisfactory results (Figure 2, left hand side). Thus, alternatively, the error term can be specified as a t-distribution which leads to an improved match of the distributional shapes (Figure 2, right hand side). Instead of $\eta_t \sim \mathcal{N}(\mu, \sigma^2)$ we therefore use $\eta_t \sim t(\nu)$, with ν being the t-distribution's degrees of freedom that are estimated from the data.

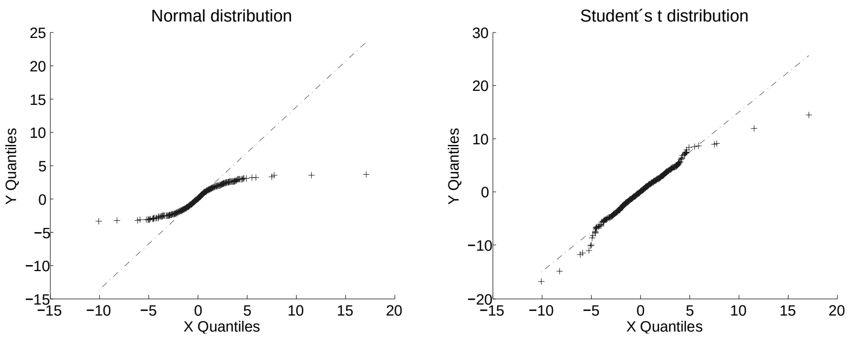


Figure 2: QQ-plots of the 2011 residuals compared against a normal distribution and a Student-t distribution

[12]For more details about SARIMA models, the reader is referred to, e.g., Box et al. (2008).

Written explicitly, the model for Z_t now takes the following form:

$$
\begin{aligned}
Z_t =& \phi_1 Z_{t-1} + \phi_2 Z_{t-2} + \Phi_1 Z_{t-24} + \Phi_1(\phi_1 Z_{t-25} - \phi_2 Z_{t-26}) \\
& + \epsilon_t + \theta_1 \epsilon_{t-1} + \theta_2 \epsilon_{t-2} + \Theta_1 \epsilon_{t-24} + \Theta_1(\theta_1 \epsilon_{t-25} - \theta_2 \epsilon_{t-26})
\end{aligned} \tag{7}
$$

$$\epsilon_t = \sigma_t \eta_t \tag{8}$$

$$\sigma_t^2 = \alpha + \beta_1 \epsilon_{t-1}^2 + \gamma_1 \sigma_{t-1}^2 \tag{9}$$

$$\eta_t \sim t(\nu) \tag{10}$$

The parameters for the above model are estimated from the short-term stochastic process z_t by optimizing the log-likelihood function. The estimates are presented in Table 2.

Table 2: Parameter estimates for the short-term stochastic process model

Parameter	ϕ_1	ϕ_2	Φ_1	θ_1	θ_2	Θ_1	α	β_1	γ_1	ν
Estimate	0.366	0.359	0.965	0.566	0.074	-0.845	2.955	0.295	0.466	3.610
Std. Error	(0.146)	(0.120)	(0.002)	(0.146)	(0.019)	(0.005)	(0.032)	(0.027)	(0.272)	(0.156)

3.4. Estimation and selection of copula models

The aggregated wind power exhibits a strong impact on spot market prices. In contrast, as long as the single turbine's capacity is sufficiently small, it impacts prices only marginally. However, we find a distinctive stochastic dependence between the single turbine wind power and the aggregated wind power that establishes a relationship between the single wind turbine and spot prices. This dependence is found to be of particular importance for the market value, as will be shown in Section 4.

In this section, we select and estimate models for the joint distribution of a single turbine wind power and the German aggregated wind power for 19 wind power stations in Germany[13]. We apply the two-stage process introduced in Section 2: First, the marginal distributions are determined, followed by the selection and estimation of the copula model that best describes the dependence structure.

In order to determine the marginal distributions, we consider the hourly synthetic wind power data for the years 1990–2011 for the different stations as well as for the German aggregated wind power. The yearly data is split into monthly intervals in order to capture seasonal differences. We thus obtain 22x12 subsamples from which we get 22x12 empirical distribution functions. With 22 years, the data covers a wide range of weather uncertainties that largely determine the quantity risk of wind power. Furthermore, the extensive database allows us to use the empirical distribution

[13] We determine the models for the joint distribution functions between the German aggregated wind power and the following stations: Aachen, Angermünde, Augsburg, Bremen, Dresden, Emden, Erfurt-Weimar, Idar-Oberstein, Kahler Asten, Kleiner Feldberg, Konstanz, Leipzig-Halle, Magdeburg, Münster-Osnabrück, Oldenburg, Potsdam, Rostock, Saarbrücken and Schleswig.

functions as marginal distribution functions (F_{W_t}, F_{X_t}) of the two variables of interest, namely the single turbine wind power and the aggregated wind power.[14]

In contrast to the marginal distribution functions, the copula model $C_{F_{X_t}(X_t),F_{W_t}(W_t)}$ is estimated from the data of the effectively delivered day-ahead prognosis of the German aggregated wind power in 2011 and the corresponding hourly single turbine wind power. Even though 22 years would be available when using the synthetic aggregated wind power, we argue that for estimating, the copula model it is important to rely on observed rather than synthetically generated data. This is motivated as follows: First, a source of imprecision would be incorporated due to the fact that the synthetic aggregated wind power represents the actual power delivery, whereas the day-ahead prognosis is the relevant quantity for the spot market activities. Even though wind power forecasts have become more reliable over the last years (Foley et al. (2012)), we want to avoid this imprecision in the estimation of the copula models. Second, subsamples consisting of approximately 700 observations are sufficiently large for a reliable estimation of the copula parameters. Just as the empirical distribution functions, the copula models are selected and estimated on a monthly basis.

To find the most appropriate copula model, various types are fitted to the data based on the procedure introduced in Section 2.1.[15] Table A.6 in Appendix Appendix A report the copulas that provide the best fit to the data in terms of AIC for all stations that are considered in this paper. In the following, we will first concentrate on particular stations (namely *Bremen*, *Kleiner Feldberg* and *Augsburg*) in order to point out the most important aspects with respect to the dependence structure and the effect on the results. *Bremen* is located in northern Germany where most of the current wind capacity is installed due to generally high average wind speeds. *Kleiner Feldberg* is a mountain in central Germany, also characterized by comparatively favorable wind speeds but less surrounded by other wind turbines. Finally, we analyze *Augsburg*, which is located in southern Germany and far away from most wind power capacities. *Augsburg* has the fewest full load hours among the three stations considered. Table 3 lists the copulas providing the best fit to the data (in terms of AIC) for these three locations in every month. [16] For *Bremen* and *Augsburg*, the copula that provides the best fit in almost every month is the Gumbel copula. For these locations, there is a distinctive asymmetric upper tail dependence in the dependence structure of the single turbine wind power and the aggregated wind power. In contrast, there is hardly any tail dependence for the turbine located at *Kleiner Feldberg*. Here, most of the copulas that best fit the data are symmetric (Gaussian, Student-t and Frank copula).

Once the marginal distributions and copulas are estimated, the conditional copula model can be used to simulate the single turbine wind power conditional on the German aggregated wind

[14]Instead, a parametric distribution, e.g., a beta distribution, could be assumed or estimated. This becomes particularly attractive when there is a lack of data.

[15]The following copula models are tested: Gaussian copulas, Frank copulas, Clayton copulas, Gumbel copulas and Student-t copulas for ν=1,2,3,4,5,10,20,30,40,50.

[16]The table reporting the AIC values for all months and all copulas fitted to the data of the three stations considered is provided in Appendix A.

Table 3: Copula selection for the three stations of interest

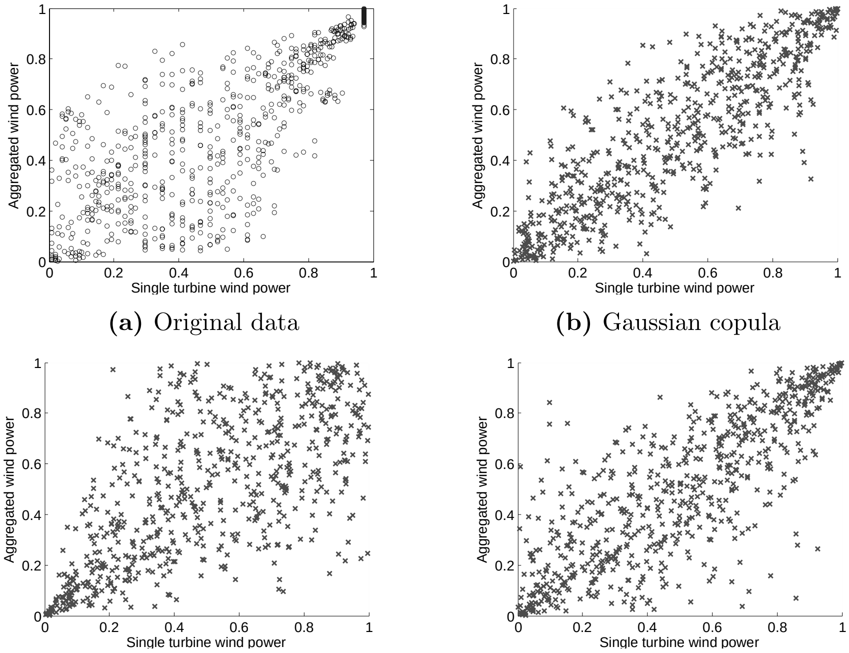
Month	Augsburg	Bremen	Kleiner Feldberg
January	Normal	T40	Normal
February	Gumbel	Gumbel	Normal
March	Gumbel	Gumbel	Frank
April	Gumbel	Gumbel	Frank
May	Gumbel	Gumbel	Frank
June	Gumbel	Gumbel	Clayton
July	Frank	Gumbel	Frank
August	Gumbel	Gumbel	Frank
September	Gumbel	Gumbel	T10
October	Gumbel	Gumbel	Normal
November	Gumbel	Gumbel	Frank
December	Frank	T10	Normal

power, based on the the sampling procedure that was introduced in Section 2.2. We loop through the 22 years and the 12 months of data and draw $n = 10000$ samples of the single turbine wind power for each point of the aggregated wind power curve, while applying the corresponding single turbine marginal distribution out of the 22x12 available.

Example: Figure 3 shows the dependence structure of the original data as well as simulations from three different types of copula models for a wind turbine in *Bremen*. Visually, the Gumbel copula provides the best fit to the data, which is confirmed by the comparison of the AIC. It can be observed that there is a distinctive upper tail dependence between the single turbine wind power and the German aggregated wind power. It should be noted that this type of dependence is generally undesirable for wind turbines selling their power on the spot market, as there is a high probability that spot prices are low in case of high power generation.

Figure 4 shows the original data together with simulations from the Gumbel copula for the single turbine wind power located in *Bremen* and the aggregated wind power, transformed back to their marginal distributions.[17] As can be seen, simulations match the original data very well.

[17] The station is assumed to consist of 4 turbines with 2.5 MW each, thus having a rated power of 10 MW



(a) Original data **(b)** Gaussian copula

(c) Clayton copula **(d)** Gumbel copula

Figure 3: Dependence structure of the original data and simulations from three copula models

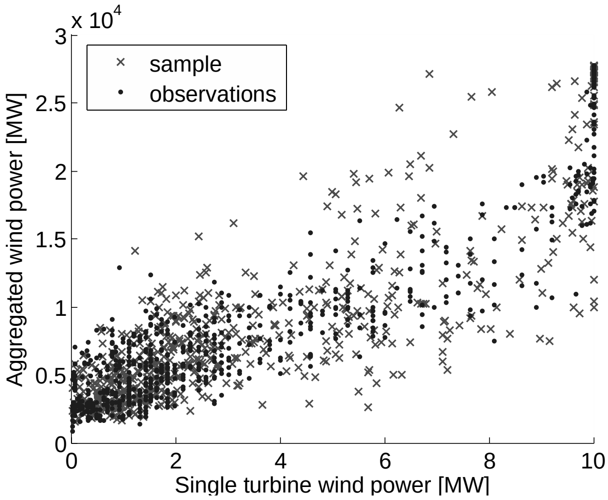


Figure 4: Observations and sample of the single turbine wind power and the aggregated wind power

4. Results

After having specified the entire model and estimated the necessary model parameters, samples of the electricity spot price S_t and single turbine wind power X_t can be obtained. We sample from the model Equations (5) and (6) in a Monte Carlo simulation ($n = 10000$) in order to investigate revenue and market value distributions as well as the relevance of the dependence structure with the German aggregated wind power. All stations are assumed to have a rated power of 10 MW.

While the revenue is simply the sum of the products of power production and prices, the market value of a wind turbine is the average spot price weighted with the power production of the respective wind turbine:

$$MV = \frac{\sum_t X_t S_t}{\sum_t X_t}. \tag{11}$$

Figure 5 presents the yearly revenue distribution for a wind turbine located in *Bremen* together with the 5% value at risk. The expected revenue amounts to 0.82 Mio. Euro, with a standard deviation of 0.038 Mio. Euro and a slightly negative skew. The 5% value at risk is found to be 0.75 Mio. Euro. Note that the distribution of absolute revenue is determined by both the number of full load hours that can be achieved at the specific site of interest and the corresponding market value. However, the scope of this paper lies on the dependence structures of different sites and their impact on the market value, which is thus the main focus in the following analysis. In particular, we demonstrate the relevance of the market value for the revenues of wind power in today's context, as well as for higher wind power penetration levels for which the effect becomes increasingly important.

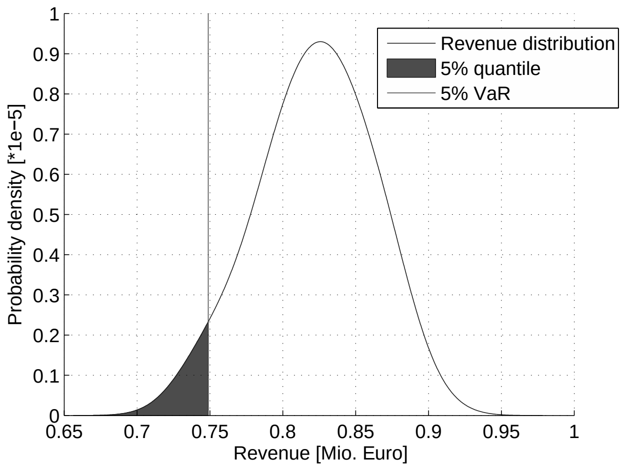


Figure 5: Yearly revenue distribution of the *Bremen* station and the 5% value-at-risk

4.1. Revenues and market value of different wind turbines

To quantify the effect arising from the dependence structures, distribution functions of the market value are determined and compared for the three stations *Augsburg*, *Bremen* and *Kleiner*

Feldberg. Table 4 lists the main results for these three stations for the month of February. The expected average spot price of the simulations is 48.52 Euro/MWh. In contrast, the expected market value of the wind turbines is much lower for all turbines due to the dependence between the single turbine wind power and the aggregated wind power, which in turn has a price damping effect. From only the correlation coefficient ρ, one would have anticipated the expected market value of a turbine in *Augsburg* ($\rho = 0.37$) to be much higher than the expected market value of a turbine in *Kleiner Feldberg* ($\rho = 0.51$) which in turn should have a higher market value than a turbine in *Bremen* ($\rho = 0.75$). However, this is not the case: Although the correlation coefficient for a turbine in *Kleiner Feldberg* is much higher than that of a turbine in *Augsburg*, the expected market value is also higher. The reason lies in the dependence structure. As shown in Section 3.4, the dependence structure for *Augsburg* in February is best described by a Gumbel copula, thus incorporating an upper tail dependence between the single turbine wind power and the aggregated wind power. In contrast, the dependence structure between the single turbine wind power in *Kleiner Feldberg* and the aggregated wind power is modeled most accurately by a symmetric Gaussian copula. Therefore, *Kleiner Feldberg* benefits from an advantageous dependence structure when selling its wind power at the spot market.

Table 4: Main results for the month of February

	Augsburg	*Bremen*	*Kleiner Feldberg*
Expected average spot price [Euro/MWh]	48.52	48.52	48.52
Correlation coefficient	0.37	0.75	0.51
Selected copula model	Gumbel	Gumbel	Gaussian
Expected market value [Euro/MWh]	43.10	41.31	44.33
Standard deviation [Euro/MWh]	5.98	6.63	5.63

The distributions of the yearly market value for the three stations considered are shown in Figure 6. Following the same logic as discussed for the specific month of February, the yearly market value of a turbine in *Kleiner Feldberg* is higher than the market value for *Augsburg*. As can be seen in Table 3, the dependence structure for *Augsburg* is modeled with a copula incorporating an upper tail dependence in almost every month, whereas the one for *Kleiner Feldberg* is mostly symmetric. Consequently, for the three distributions that are shown in Figure 6, the dependence structure reduces the expected yearly market value of the turbines by 3.54, 4.97 and 2.63 Euro/MWh, respectively, compared to the expected average spot price level (49.80 Euro/MWh).

4.2. Market value variations in Germany

Germany is characterized by a surface area of 357,021 km^2 and a maximum horizontal width and vertical length of 642 km and 833 km, respectively. Furthermore, there are several diverse geographical regions, suggesting that meteorological conditions may vary substantially when analyzing different locations throughout the country.

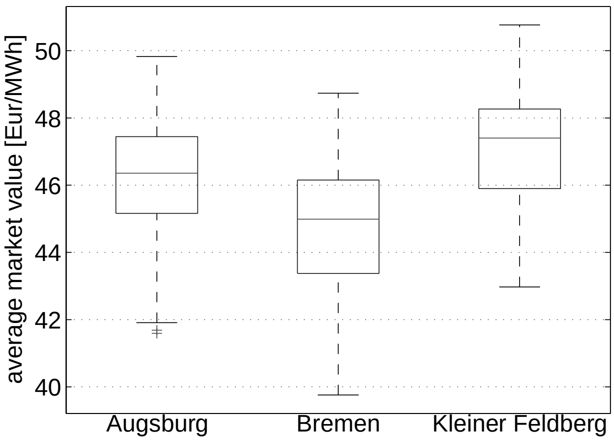


Figure 6: Yearly market value of the three turbines

With the model developed, we analyze the market value for 19 different stations in Germany, as depicted in Figure 7. As the analyzed stations differ with respect to their exact location (and thus with respect to their dependence structure related to the aggregated German wind power), we expect market values to differ as well. Specifically, we expect the market value to be lowest for the stations that are closest to the majority of installed wind power. Indicated by different colors, Figure 7 shows the expected market value of the stations that were considered.

Results indicate that the expected market value ranges from 42 to 48 Euro/MWh for the analyzed stations, compared to an expected average spot price level of 49.80 Euro/MWh. Hence, the market value lies between 6 and 15% lower than the average spot price. As expected, lowest values are found for the stations that are closest to the majority of currently installed wind power, i.e., mainly in the area of *Magdeburg* and *Münster-Osnabrück*. For stations in this area, the dependence structure shows a pronounced asymmetric upper tail dependence. It is observed that expected market values are similar for all stations located in the so called 'North German Plain', which is a geographical region in Northern Germany characterized by constant lowlands and hardly any hills. Note that *Aachen* is at the far end of the North German Plain and, as such, equally characterized by comparatively low expected market values of 43.47 Euro/MWh. In contrast, *Kahler Asten* is located in Germany's Central Uplands, where meteorological conditions are different (e.g., due to pronounced thermals), which is reflected by higher values. Other stations in or south of the Central Uplands show higher expected market values as there are very few installed wind power capacities.

Kahler Asten and *Kleiner Feldberg* are special cases, as they are characterized by advantageous, symmetric dependence structures, resulting in expected market values that are the highest compared to the other stations considered. Similarly, *Emden* and *Rostock* – both located at the seashore – show higher values, compared to other stations in the North German Plain, due to comparatively advantageous dependence structures.

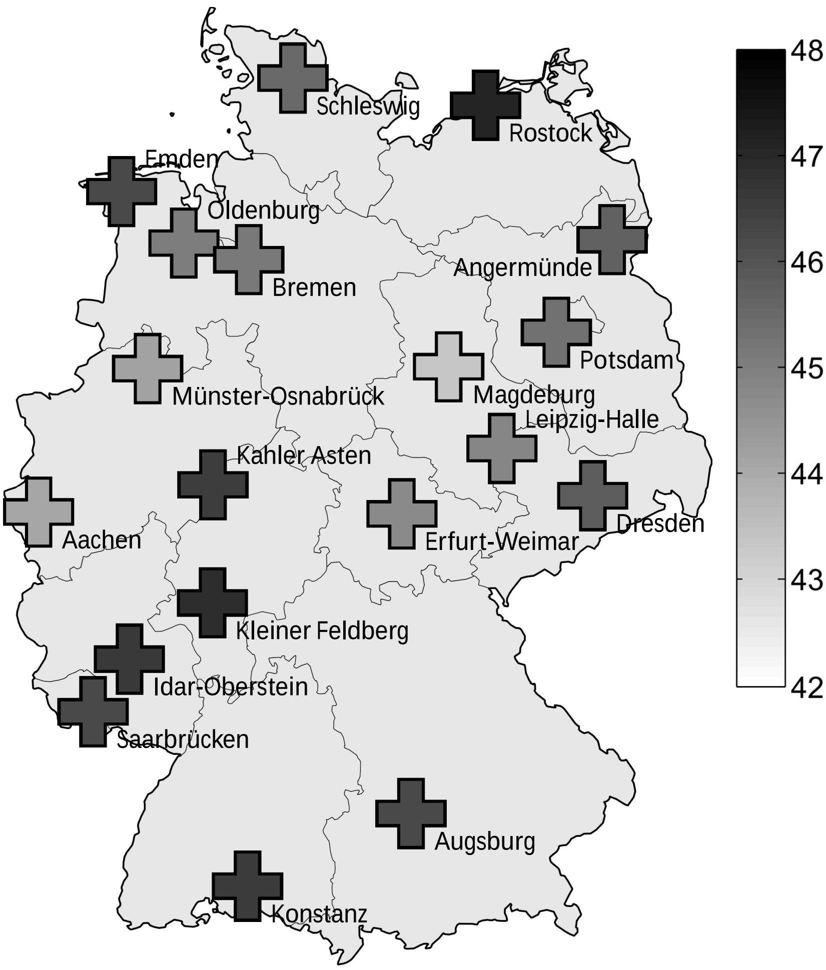


Figure 7: Expected market value for 19 stations in Germany

4.3. The impact of changing wind power penetration levels

In the previous section, model parameters were set and estimated to reflect the current environment with respect to the physical generation mix and the market conditions. In this section, some of the model parameters are modified to analyze their impact on the outcome. As has been clarified, the effect of wind power on spot market prices largely depends on the quantities of wind power being integrated in the market. With the help of the model presented in this paper, the aforementioned effect is quantified for the case of changing wind power penetration levels in Germany. First, we scale up the wind power penetration up to two times the capacity that is currently installed. Note that this is roughly in line with targets envisaged by the German government, which wants to further extend wind power to 45.8 GW in 2020 (installed capacity was 27.1 GW in 2011). Second, we compare the impact of today's wind power penetration to a situation with no wind power installed. For the analysis, installed wind power capacities are scaled-up stepwise and simulation runs are repeated for each of these steps. The underlying assumptions of this approach are as follows:

- The proportionate geographic distribution of wind power capacities within Germany remains the same. Note that due to the linear up-scaling, the dependence structure is preserved. Alternatively, region-specific changes in installed capacities could be implemented, e.g., for testing the effect of an increased wind power extension in some specific area.
- The functional dependence between residual demand levels and spot prices is again estimated from 2011 data, as explained in Section 3.3. This is certainly a strong assumption, as the conventional power sector will dynamically develop with increasing wind power penetration. However, it should be kept in mind that current wind power capacities are being rapidly expanded, whereas the conventional power sector seems to be behind in terms of capacity adjustments. Also note that the functional dependence could also be altered (e.g., by shifting or assuming a different shape). However, this was not implemented in order to focus on the specific impact of the wind power penetration levels.
- The parameter estimates for the short-term stochastic spot price process remain the same. Here again, the model could be adjusted in order to represent expectations regarding future short-term stochastic price movements.

The resulting distributions of the yearly market value of the *Bremen* station under increasing wind power penetration ranging from 100-200% are shown in Figure 8. As can be observed, the market value distribution is highly affected both in average level and variance. While the expected market value is at 44.83 Euro/MWh at 100% scaling, it decreases to 30.13 Euro/MWh at a scaling of 200%. At the same time, its standard deviation increases from 1.94 to 3.40 Euro/MWh, respectively.

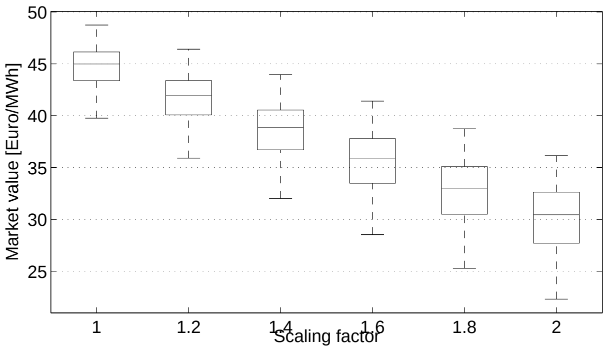


Figure 8: Yearly market value of the *Bremen* station under increasing wind power penetration

To achieve further insights regarding the effect of the wind power penetration level, we repeat the simulation for all three stations considered in Section 4.1 and a wind penetration level ranging from 0-200%. The relative change in expected values of the resulting market value distributions are presented in Figure 9. For completeness, the expected average spot price level is also included. Compared to an expected average spot price of 56.70 Euro/MWh at 0% scaling, the level is reduced by 12% to 49.80 Euro/MWh for today's penetration level. Hence, provided that the rest of the system remains the same, the spot price level would be 7 Euro/MWh higher with no wind power penetration. In this case, resulting market values are above average spot price levels (due to higher wind power infeeds during wintertime when overall demand as well as prices tend to be also higher) and almost equal for any single wind turbine as spot prices are only marginally affected by wind power. Just as average spot price levels, expected market values decrease as the penetration level increases, however, at very different slopes. Whereas the average spot price itself is affected the least, the expected market value decreases corresponding to their dependence structure. They drop below average spot price levels at penetration levels as low as around 30% of today's capacities. A scaling of 100% corresponds to the current situation described in detail in Section 4.1. As can be observed, the difference between the average spot price and the market value further increases as the scaling factor approaches 200%, reaching levels of 8.34, 11.63, and 6.20 Euro/MWh for *Augsburg*, *Bremen* and *Kleiner Feldberg*, respectively.

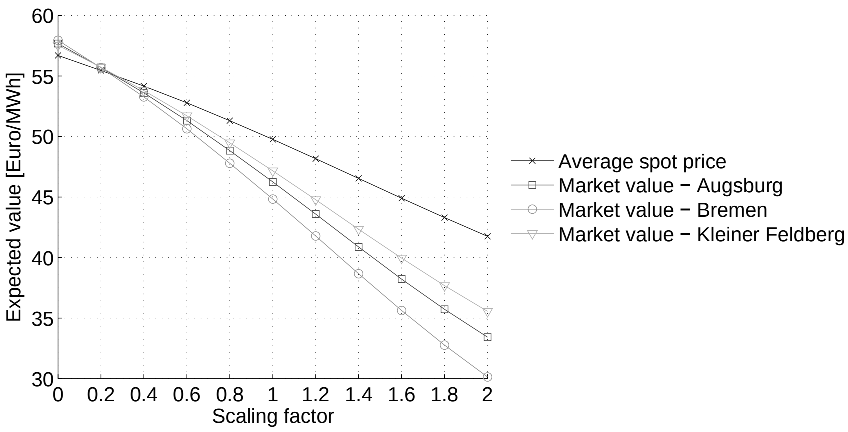


Figure 9: Relative change in expected values of the spot price and the market value under changing wind power penetration levels

5. Conclusions

This paper presents a model for the simulation of single turbine wind power and electricity spot prices, including a precise representation of their interrelations. Copula theory is applied to model single turbine wind power and aggregated wind power, thus allowing to decouple their dependence structure from their marginal distributions. The formation of prices is formulated as a function of the aggregated wind power in a structural supply and demand based model. As such, the model extends formerly known modeling approaches through the ability to simulate and quantify the price effect of wind power, and hence to determine their market value.

The model is calibrated for the case of Germany, where wind power today already makes up a significant share of the power mix. Nineteen locations are analyzed in detail, for which it is shown that the expected market value is reduced by up to 8 Euro/MWh compared to average spot price levels. However, the market value highly depends on the specific location and the corresponding dependence structure between the single turbine wind power at this location and the aggregated wind power. Whereas most locations are found to be characterized by rather adverse asymmetric dependence structures, some of the analyzed locations are identified as being related to the aggregated wind power such that their realizable selling prices are comparatively high.

Moreover, our results indicate that, in case of increasing wind power capacities, the adverse upper tail dependence structure of many locations has a negative impact on the market value, which makes market integration of wind power even more difficult. Nevertheless, integrating wind power into the market would allow market prices to reveal their key function by indicating the actual value of electricity and thus triggering investments in wind power projects characterized by high realizable spot prices. These projects would deploy balancing potentials much better and reduce the volatility in the electricity spot market as well as in the physical system.

Although a powerful tool to analyze the market value of wind power in a predefined setting, the model reveals its limitations in not being able to determine the dynamic reaction of the power system development in response to changing levels of wind power penetration. Further research could be done by extending the model in order to use it as a forecasting and derivative pricing tool, or by applying the modeling approach developed in this paper to other forms of renewable energy, e.g., solar power.

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Appendix A. Copula models and model selection

Table A.5: Copula model selection based on AIC for the Stations *Augsburg*, *Bremen* and *Kleiner Feldberg*.

	Copula	**Jan**	**Feb**	**Mar**	**Apr**	**May**	**Jun**	**Jul**	**Aug**	**Sep**	**Oct**	**Nov**	**Dec**
Augsburg	Clayton	-13.7	-49.4	-15.9	-20.5	-55.5	-103.7	-36.0	-10.8	-6.2	-73.0	-20.9	-112.0
	Frank	-34.7	-99.9	-54.8	-107.4	-103.4	-168.5	-67.1	-80.8	-55.8	-145.9	-46.3	-198.2
	Gumbel	-19.4	-145.4	-65.1	-172.3	-135.9	-194.0	-43.4	-133.4	-111.4	-163.3	-83.5	-146.6
	Normal	-36.6	-109.1	-64.2	-117.5	-120.0	-182.5	-59.2	-95.0	-66.7	-156.5	-55.0	-174.0
	T1	482.0	48.2	432.2	154.4	191.2	101.1	402.3	245.2	221.6	197.6	167.4	306.7
	T2	158.2	-102.8	109.4	-54.8	-40.9	-112.3	92.9	1.9	3.2	-43.8	-26.2	-13.7
	T3	77.8	-124.9	32.9	-95.2	-87.3	-155.7	21.0	-49.1	-39.2	-98.4	-58.0	-89.9
	T4	43.5	-129.5	1.6	-108.7	-103.6	-170.8	-7.6	-67.8	-53.6	-119.9	-66.4	-120.5
	T5	24.8	-129.9	-14.8	-114.5	-111.0	-177.6	-22.2	-76.8	-59.9	-130.9	-68.8	-136.2
	T10	-8.4	-124.8	-42.7	-120.4	-120.0	-185.2	-45.3	-89.5	-67.2	-147.8	-67.2	-160.7
	T20	-23.1	-118.5	-54.3	-120.3	-121.4	-185.4	-53.5	-93.2	-68.1	-153.5	-62.8	-169.0
	T30	-27.7	-115.8	-57.8	-119.7	-121.3	-184.8	-55.7	-94.0	-67.9	-154.8	-60.6	-171.1
	T40	-30.0	-114.3	-59.4	-119.3	-121.1	-184.4	-56.7	-94.3	-67.7	-155.4	-59.4	-172.0
	T50	-31.3	-113.3	-60.4	-119.0	-120.9	-184.1	-57.3	-94.5	-67.5	-155.7	-58.6	-172.5
Bremen	Clayton	-370.7	-309.2	-263.6	-273.2	-309.4	-193.8	-292.3	-247.0	-224.9	-357.1	-390.3	-652.9
	Frank	-511.6	-538.0	-502.5	-641.1	-530.0	-329.1	-685.1	-498.5	-596.2	-664.1	-640.0	-787.9
	Gumbel	-539.1	-700.2	-610.7	-746.7	-604.6	-420.5	-692.0	-667.4	-702.0	-727.5	-788.9	-826.3
	Normal	-554.9	-594.2	-533.6	-614.4	-560.9	-377.4	-637.5	-547.3	-571.0	-668.8	-698.1	-867.0
	T1	-333.6	-480.8	-322.3	-402.2	-305.6	-143.3	-283.6	-368.5	-303.5	-435.9	-563.0	-708.9
	T2	-477.1	-576.0	-469.5	-565.0	-480.3	-303.6	-500.5	-497.6	-488.1	-591.4	-666.7	-815.8
	T3	-514.1	-594.2	-505.8	-602.7	-521.8	-339.2	-559.5	-526.3	-532.2	-631.2	-689.8	-846.6
	T4	-530.1	-600.0	-520.4	-616.6	-538.3	-353.3	-585.6	-537.3	-549.6	-647.8	-698.2	-860.1
	T5	-538.6	-602.2	-527.6	-622.7	-546.6	-360.5	-599.8	-542.5	-558.2	-656.3	-701.9	-867.1
	T10	-551.9	-602.4	-536.8	-627.7	-558.8	-371.7	-623.6	-549.3	-570.0	-668.5	-705.1	-875.8
	T20	-555.4	-599.7	-537.5	-624.5	-561.6	-375.4	-632.3	-549.9	-572.3	-670.9	-703.5	-875.1
	T30	-555.8	-598.2	-536.8	-622.1	-561.9	-376.3	-634.5	-549.5	-572.3	-670.8	-702.2	-873.6
	T40	-555.8	-597.4	-536.2	-620.5	-561.9	-376.7	-635.4	-549.2	-572.2	-670.6	-701.4	-872.4
	T50	-555.8	-596.8	-535.8	-619.5	-561.8	-376.8	-636.0	-548.9	-572.0	-670.3	-700.9	-871.6
Kleiner Feldberg	Clayton	-76.5	-142.7	-158.9	-142.0	-252.9	-235.2	-202.5	-79.8	-94.8	-141.6	-77.6	-230.5
	Frank	-88.3	-189.8	-194.6	-231.9	-308.9	-187.9	-211.1	-110.1	-96.9	-166.3	-80.7	-269.4
	Gumbel	-58.2	-164.3	-115.3	-187.4	-197.9	-89.1	-101.3	-60.1	-114.1	-142.5	-32.4	-285.1
	Normal	-88.7	-199.8	-192.3	-227.0	-288.8	-188.8	-181.4	-100.4	-115.7	-192.4	-70.7	-318.5
	T1	288.9	136.8	321.1	173.3	190.2	238.2	286.6	405.0	123.2	275.6	327.8	-1.8
	T2	24.8	-79.2	6.0	-80.3	-106.3	-36.4	-20.4	78.8	-65.7	-15.5	56.7	-209.5
	T3	-32.8	-131.8	-75.0	-143.6	-181.9	-103.6	-94.8	-0.5	-101.1	-88.0	-5.1	-257.5
	T4	-54.6	-153.9	-110.2	-170.5	-214.5	-131.7	-125.2	-33.2	-112.6	-119.2	-29.4	-277.2
	T5	-65.4	-165.8	-129.5	-184.9	-232.2	-146.5	-140.9	-50.5	-117.3	-136.3	-41.6	-287.6
	T10	-81.2	-185.9	-163.9	-209.4	-263.5	-171.5	-166.2	-79.5	-121.2	-166.8	-60.5	-305.3
	T20	-86.1	-193.8	-178.9	-219.2	-276.9	-181.3	-175.3	-91.0	-120.0	-180.2	-66.8	-312.6
	T30	-87.3	-196.1	-183.6	-222.1	-281.1	-184.1	-177.7	-94.4	-118.9	-184.4	-68.4	-314.7
	T40	-87.7	-197.1	-185.8	-223.4	-283.1	-185.4	-178.8	-96.0	-118.3	-186.5	-69.1	-315.7
	T50	-88.0	-197.7	-187.2	-224.2	-284.3	-186.1	-179.4	-96.9	-117.8	-187.7	-69.5	-316.3

Table A.6: Selected copula models and rank correlation coefficients

Station	**Jan**	**Feb**	**Mar**	**Apr**	**May**	**Jun**	**Jul**	**Aug**	**Sep**	**Oct**	**Nov**	**Dec**
Aachen	Frank	Gumbel	Gumbel	Frank	Normal	Gumbel	Frank	Frank	Gumbel	Gumbel	Gumbel	T30
	0.63	0.54	0.55	0.53	0.69	0.44	0.58	0.57	0.69	0.61	0.51	0.65
Angermünde	Gumbel	Gumbel	Normal	Gumbel	T10	Frank	Gumbel	Gumbel	Gumbel	Normal	Gumbel	Normal
	0.58	0.61	0.56	0.57	0.49	0.52	0.64	0.59	0.58	0.67	0.53	0.66
Augsburg	Normal	Gumbel	Gumbel	Gumbel	Gumbel	Gumbel	Frank	Gumbel	Gumbel	Gumbel	Gumbel	Frank
	0.23	0.37	0.28	0.39	0.37	0.46	0.30	0.35	0.30	0.44	0.26	0.50
Bremen	T40	Gumbel	Gumbel	Gumbel	Gumbel	Gumbel	Gumbel	Gumbel	Gumbel	Gumbel	Gumbel	T10
	0.71	0.75	0.70	0.79	0.73	0.61	0.80	0.71	0.78	0.78	0.78	0.81
Dresden	Frank	Gumbel	Frank	Gumbel	T10	Gumbel	Frank	Gumbel	Gumbel	Normal	Frank	Clayton
	0.68	0.64	0.59	0.63	0.50	0.58	0.72	0.67	0.61	0.61	0.55	0.60
Emden	Normal	Gumbel	Normal	Gumbel	Normal	Gumbel	Frank	Gumbel	Gumbel	Normal	Gumbel	Normal
	0.62	0.50	0.68	0.67	0.70	0.49	0.76	0.61	0.73	0.72	0.71	0.68
Erfurt-Weimar	Frank	Gumbel	Gumbel	Gumbel	Gumbel	Gumbel	Frank	Gumbel	Gumbel	Gumbel	Gumbel	T50
	0.74	0.67	0.61	0.70	0.64	0.61	0.78	0.70	0.81	0.74	0.55	0.75
Idar-Oberstein	Frank	Gumbel	Gumbel	Gumbel	Gumbel	Gumbel	Frank	Gumbel	Gumbel	Gumbel	Gumbel	Frank
	0.42	0.55	0.49	0.37	0.46	0.35	0.39	0.47	0.55	0.57	0.40	0.68
Kahler Asten	Frank	Normal	Gumbel	Frank	Frank	T50	Frank	Frank	Frank	T20	Gumbel	Frank
	0.72	0.69	0.65	0.82	0.76	0.71	0.80	0.74	0.76	0.80	0.72	0.74
Kleiner Feldberg	Normal	Normal	Frank	Frank	Frank	Clayton	Frank	Frank	T10	Normal	Frank	Normal
	0.34	0.51	0.50	0.53	0.61	0.49	0.52	0.39	0.35	0.46	0.33	0.56
Konstanz	Frank	Gumbel	Normal	Gumbel	Gumbel	Gumbel	Frank	Gumbel	Gumbel	Gumbel	T30	Frank
	-0.06	0.22	0.19	0.21	0.24	0.39	0.25	0.27	0.41	0.35	0.14	0.37
Leipzig-Halle	Frank	T40	Gumbel	Gumbel	T40	Gumbel	Frank	Gumbel	Gumbel	Normal	Gumbel	T5
	0.80	0.78	0.71	0.75	0.70	0.69	0.83	0.77	0.79	0.80	0.70	0.70
Magdeburg	Gumbel	Gumbel	Gumbel	Gumbel	Gumbel	Gumbel	Gumbel	Gumbel	Gumbel	Gumbel	Gumbel	Normal
	0.72	0.72	0.63	0.66	0.57	0.65	0.76	0.71	0.67	0.70	0.65	0.76
Münster-Osnabrück	Frank	Gumbel	Gumbel	Gumbel	Gumbel	Gumbel	Frank	Gumbel	Gumbel	Gumbel	Gumbel	T40
	0.67	0.63	0.56	0.66	0.62	0.53	0.73	0.59	0.74	0.74	0.66	0.80
Oldenburg	Normal	Gumbel	Gumbel	Gumbel	Gumbel	Normal	Gumbel	Gumbel	Gumbel	Gumbel	Gumbel	Normal
	0.63	0.56	0.64	0.69	0.65	0.48	0.75	0.56	0.69	0.69	0.75	0.71
Potsdam	Frank	Gumbel	Normal	Gumbel	T30	T20	Frank	Frank	Frank	Frank	Normal	Normal
	0.83	0.76	0.78	0.82	0.72	0.76	0.85	0.81	0.82	0.84	0.78	0.69
Rostock	Gumbel	Gumbel	Clayton	Gumbel	Normal	Normal	Normal	T40	Frank	Frank	Gumbel	Clayton
	0.30	0.52	0.36	0.61	0.56	0.39	0.55	0.43	0.57	0.66	0.70	0.47
Saarbrücken	Gumbel	Gumbel	Normal	Gumbel	Gumbel	Gumbel	Frank	Gumbel	Gumbel	Gumbel	Gumbel	Frank
	0.27	0.42	0.48	0.06	0.47	0.39	0.34	0.35	0.47	0.56	0.28	0.64
Schleswig	Frank	Gumbel	Gumbel	Gumbel	Normal	Frank	Frank	Gumbel	Gumbel	Normal	Gumbel	Gumbel
	0.66	0.60	0.58	0.73	0.63	0.53	0.66	0.58	0.72	0.72	0.71	0.54

Appendix B. Demand-price dependence

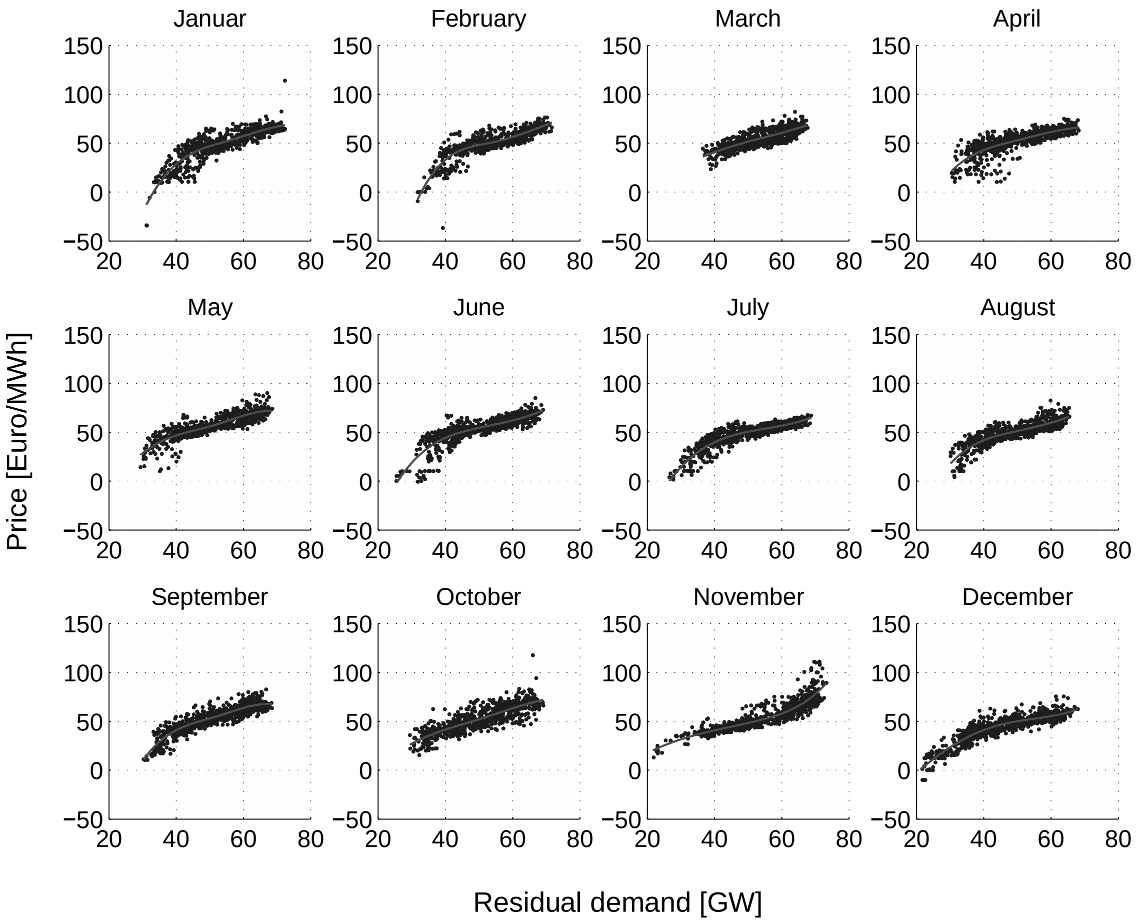

Figure B.10: Demand-price dependence and spline fits for all months of 2011