

# How to Sell Renewable Electricity - Interactions of the Intraday and Day-ahead Market Under Uncertainty

## AUTHORS

Andreas Knaut

Frank Obermüller

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**Institute of Energy Economics  
at the University of Cologne (EWI)**

Alte Wagenfabrik  
Vogelsanger Straße 321a  
50827 Köln  
Germany

Tel.: +49 (0)221 277 29-100  
Fax: +49 (0)221 277 29-400  
[www.ewi.uni-koeln.de](http://www.ewi.uni-koeln.de)

**CORRESPONDING AUTHOR**

**Andreas Knaut**

Institute of Energy Economics at the University of Cologne  
Tel: +49 (0)221 277 29-306  
Fax: +49 (0)221 277 29-400  
[andreas.knaut@ewi.uni-koeln.de](mailto:andreas.knaut@ewi.uni-koeln.de)

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# How to Sell Renewable Electricity - Interactions of the Intraday and Day-ahead Market Under Uncertainty

Andreas Knaut<sup>a</sup>, Frank Obermueller<sup>a</sup>

<sup>a</sup>*Institute of Energy Economics, University of Cologne, Vogelsanger Strasse 321a, 50827 Cologne, Germany*

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## Abstract

Uncertainty about renewable production increases the importance of sequential short-term trading in electricity markets. We consider a two-stage market where conventional and renewable producers compete in order to satisfy the demand of consumers. The trading in the first stage takes place under uncertainty about production levels of renewable producers, which can be associated with trading in the day-ahead market. In the second stage, which we consider as the intraday market, uncertainty about the production levels is resolved. Our model is able to capture different levels of flexibility for conventional producers as well as different levels of competition for renewable producers. We find that it is optimal for renewable producers to sell less than the expected production in the day-ahead market. In situations with high renewable production it is even profitable for renewable producers to withhold quantities in the intraday market. However, for an increasing number of renewable producers, the optimal quantity tends towards the expected production level. More competition as well as a more flexible power plant fleet lead to an increase in overall welfare, which can even be further increased by delaying the gate-closure of the day-ahead market or by improving the quality of renewable production forecasts.

*Keywords:* Sequential Markets, Electricity Market, Competition Under Uncertainty, Strategic Interaction

JEL classification: D81, L13, L94, Q21

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*Email addresses:* andreas.knaut@ewi.uni-koeln.de, +49 22127729306 (Andreas Knaut), frank.obermueller@ewi.uni-koeln.de, +49 22127729308 (Frank Obermueller)

## 1. Introduction

The electricity sector is currently experiencing rapid changes, especially due to the deployment of large capacities for electricity generation from renewables with the aim of decarbonizing economies. This leads to a transformation of the producer side, away from conventional generation technologies (such as coal, gas, and nuclear) towards an increasing share of variable renewable electricity generation (especially wind and solar). Whereas these technologies were highly subsidized in the past and therefore not well integrated into the market, it is now high on the European Union's policy agenda to integrate renewable generation into the market (EU Commission (2009), EU Commission (2013)). This means in the future, renewable producers are expected to sell their entire production at the existing sequential wholesale electricity markets, e.g. the day-ahead and the intraday market.<sup>1</sup>

In electricity markets, demand and supply need to be balanced at all times. Therefore it is essential for all market participants to announce their foreseeable production and consumption in advance. The largest share of electricity is currently traded in the day-ahead market, which can be considered as a kind of forward market. Trading commonly takes place at noon one day before physical delivery. This is necessary to signal the regional supply and demand situations to the transmission system operators in advance, such that they can guarantee grid stability. In contrast, the intraday market provides the opportunity to trade electricity down to 30 minutes before physical delivery. Hence, adjustments to the day-ahead market clearing result can be traded which may occur due to (uncertain) short term deviations in electricity systems (e.g. demand forecast errors, renewable forecast errors, and unforeseen power plant shortages).

The characteristics of renewable electricity generation have increased the importance of sequential short-term trading and are affecting the competition in electricity markets. Renewable energy technologies differ in two important aspects from classic conventional technologies. First, renewables produce electricity at short run marginal costs of zero whereas conventional technologies have short run marginal costs greater than zero. Second, renewable electricity production depends on weather conditions that can only be predicted to a certain level. The uncertainty diminishes with a shorter time duration to the physical delivery. Thus, volatile renewable producers have a higher uncertainty if they trade in the day-ahead market. Therefore, the optimal bidding strategy for renewable energy producers in the intraday and day-ahead market under uncertainty is not clear and focus of the following investigations.

Electricity markets are known to be especially vulnerable to the potential abuse of market power (Green & Newbery, 1992; Borenstein et al., 2002). The demand can be regarded as very price inelastic in the

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<sup>1</sup>The forward market is not a relevant market for volatile renewables due to the uncertain production in the long run.

short-run and therefore participants may be able to increase prices above the competitive level. While this has been an issue of large conventional generators in the past, we also can expect large renewable producers as being able to act strategically in sequential electricity markets. The size of renewable aggregators who aggregate renewable generation plants and sell the production in the market is steadily increasing especially because they are able to lift significant scale effects by increasing their renewable portfolio (e.g. reduction in costs of trading and reduction of forecast uncertainty).

In this paper, we analyze the competition between conventional and renewable producers that interact in two sequential stages by using an analytic model. The first stage is considered as the day-ahead and the second as the intraday market. The electricity production of the renewable producer is uncertain in the first stage and is realized in the second stage. In particular, this affects renewable producers in choosing the optimal quantity to trade in both stages. Furthermore, we account for flexibility constraints of conventional power producing technologies, because not all conventional technologies are flexible enough to change their production schedules in short time intervals (e.g. 30 minutes before physical delivery). These flexibility constraints are included in our model to measure effects on profit maximizing quantities and prices. We analyze the results based on different levels of competition for the renewable producers, ranging from a monopoly to oligopolies under a flexible and less flexible power plant fleet.

Our investigation is strongly related to the branch of two-stage Cournot games as well as the literature of optimal bidding strategies for renewable producers. Concerning two-stage Cournot games, a fundamental work is given by Allaz (1992) and Allaz & Vila (1993) who investigate Cournot competition of a duopoly in sequential markets. Their subject of investigation is the forward market which, however, can be transferred to our idea of a day-ahead auction before the market is finally cleared in an intraday auction. The setting differs to our model with respect to the type of players. In Allaz & Vila (1993) both players have increasing marginal costs of production and no uncertainty associated with their level of production. In Allaz (1992), uncertainty is incorporated in the two-stage model such that risk hedging influences the optimal production. However, Allaz (1992) and Allaz & Vila (1993) assume implicitly infinite production possibility, which is not true for our renewable producer. Similar to Allaz & Vila (1993), Saloner (1987) developed an extension of the classical Cournot one-shot duopoly to a model with two production stages in which the market clears only once after the second stage. In this framework Saloner showed the existence of a unique Nash-Cournot equilibrium under the possibility of a second stage response action. Nevertheless, the model does not account for different player types or uncertainty of production. Twomey & Neuhoff (2010) consider the case of renewable and conventional producers that are competing in electricity markets. They analyze the

case when conventional players use market power to increase prices. With their model they are able to show that renewable producers are worse off in settings with market power. However, they do not consider the strategic behavior of renewable producers and abstract from uncertainty.

The other branch of relevant literature covers optimal bidding strategies under uncertain production of one single player. Many papers in this field analyze numerical models from a price taker perspective and focus on wind power producers. For instance, Botterud et al. (2010) numerically analyze the optimal bidding for a wind power producer in a two-stage market (day-ahead and real time market) under certain risk assumptions.<sup>2</sup> They find that the optimal bid on the day-ahead market depends on risk behavior and the respective market prices. Furthermore, it tends towards the expected production as a deviation penalty between the day-ahead and the real time market is introduced. Botterud et al. (2010) focus on one specific wind power producer without considering the implications of adjusted bidding strategies on the market equilibrium. Those effects can influence the optimal bidding strategy as we will show in the investigated oligopoly cases. Further literature similar to Botterud et al. (2010) can be found in Bathurst et al. (2002), Usaola & Angarita (2007), Pinson et al. (2007), and Morales et al. (2010).

To the best of our knowledge there is no literature that focuses on competition between producers with marginal costs of zero and uncertain production levels on one side and producers with strictly increasing marginal costs of production on the other side in a generalized model framework. With this paper we intend to close this gap in the literature and facilitate the discussion about optimal market designs for short-term electricity markets.

The latter of the paper is organized as followed: In section 2 we develop the basic model framework. Section 3 analyzes the Cournot competition and the basic model is applied to the monopolistic as well as the oligopolistic case. Section 4 focuses on the impact of flexibility constraints for conventional power technologies. Section 5 sheds light onto the incentives of renewable producers to withhold capacity in the intraday market. In section 6 we show the effects on welfare, producer and consumer surplus by numerical examples. In section 7 we conclude our results and discuss possible policy implications.

## 2. The Model

We consider two players that interact at two stages in the wholesale market for electricity, namely, conventional producers ( $c$ ) and renewable producers ( $r$ ). The consumers are assumed to behave completely price-inelastic in the short-run and demand a quantity  $D$ . The demand of consumers is satisfied already

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<sup>2</sup>Here, *real time market* means the ancillary grid services for balancing supply and demand.

in the first stage, since we assume consumers as being myopic and risk-averse. On the supply side, we distinguish between conventional producers and renewable producers.

Conventional producers in the model are represented as competitive fringe. They are able to produce electricity at total costs of  $C(q_c)$  where  $q_c$  is the quantity produced. These quantities are sold into the market at a uniform price of the marginal production costs. The conventional producers also act as market makers which means they always satisfy the residual demand in both stages<sup>3</sup>.

Renewable producers produce electricity at zero marginal costs. Their final production level  $Q$  is uncertain in the first stage with the probability density function  $f(Q)$ . The uncertainty about the production level resolves over time (from stage 1 to 2; cf. Figure 1).

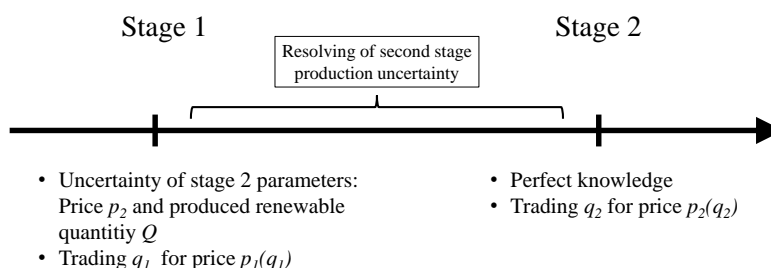


Figure 1: Basic two-stage model

Throughout our analysis we assume the probability function  $f(Q)$  as symmetric. In our view this assumption is reasonable, since well behaved forecasting models should be able to produce a symmetric distribution.<sup>4</sup>

Conventional and renewable producers can trade electricity in the two stages ( $t = 1$  and  $t = 2$ ). For the conventional producers quantities are denoted by  $q_{ct}$  and for the renewable producer by  $q_{rt}$ . Here, we allow for  $q_{ct}$  and  $q_{rt}$  to be positive or negative. This allows producers, e.g. to sell too much production in the first stage and buy back quantities in the second stage. As already mentioned, we assume the demand of consumers ( $D$ ) to get satisfied in the first stage. In the second stage, conventional and renewable producers can adjust their positions, e.g. conventional producers buy quantities from the renewable producer in order to replace their more expensive conventional production with renewable electricity. In this setting it is unclear

<sup>3</sup>Conventional producers have a strong incentive to sell their production in a market as long as the price is above their marginal production costs. This makes it seem to be a reasonable assumption that conventional producers always satisfy the residual demand when prices are above or equal to their marginal generation costs.

<sup>4</sup>Of course the distribution would not be symmetric in cases where production is expected to be extreme in the sense of a very low (close to zero) or very high (close to the capacity limit) production. Further information on wind forecasts and uncertainty can be found in Zhang et al. (2014).

what quantity ( $q_{r1}^*$  and  $q_{r2}^*$ ) is optimal to trade in the first and second stage for the renewable producer.

The market clearing conditions at both stages can be written as

$$\text{Stage 1:} \quad D = q_{c1} + q_{r1} \quad (1)$$

$$\text{Stage 2:} \quad D = q_{c1} + q_{c2} + q_{r1} + q_{r2}. \quad (2)$$

The conventional producers produce electricity based on linear increasing marginal cost functions in both stages. A linear marginal cost abstracts from real cost functions in electricity markets in two important assumptions. The first model assumption is the linearity. In reality, the cost function is usually a monotonic increasing function (with a mainly stepwise convex-similar shape). Therefore, in theory, a usual simplifying assumption is a convex cost function. In contrast to this, we assume linearity since it simplifies the theoretical analysis. Similar results can be obtained with a convex cost function (e.g. arbitrary second order quadratic functions monotonic increasing in  $\mathbb{R}^+$ ). However, this increases the complexity without generating significant further insights.

Second, in reality, marginal costs of production may change with time, which can have multiple reasons. In electricity markets this may be due to technical constraints of power plants (start-up costs, minimum load restrictions or partload-efficiency losses) or due to transaction costs of participants that do not engage in short-term trading in short intervals before production. In the end, this may lead to a reduction of electricity supply that is available on short notice.

We account for a change of the supply side by considering two different marginal cost functions  $MC_1(q)$  and  $MC_2(q)$  with different inclinations  $a_1$  and  $a_2$ . Since the number of flexible power plants is lowered the closer we get to physical delivery (or less power plant operators participate in the second market),  $a_2$  has to be greater than  $a_1$ . As explained before, the supply curve may change due to two reasons. First, technical constraints of power plants which are not able to adjust their power output in short intervals before production can lead to reduced supply. Second, there may be transaction costs for power plant operators to participate in the intraday market which is why supply is also reduced. This approach is similar to Henriot (2014) and has been empirically verified for the German intraday market by Knaut & Paschmann (2016).

For the analysis we have to define the properties of the marginal cost function in the second stage. Besides the increase of the slope to  $a_2$ , the whole curve needs to cross the market clearing point from the first stage. Because if there are no adjustments in quantities the price of the first and second stage are identical. Thus the marginal cost function for the second stage can be obtained by a rotation around the market clearing point from stage 1 (cf. Figure 2). This means an increase in production comes at additional



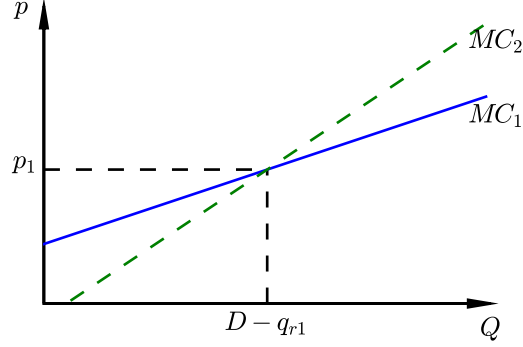


Figure 2: Marginal cost function in the first and second stage

costs and a decrease in production at less savings of production costs. In combination with the market clearing conditions, this leads to the following two equations for price formation in the two stages:

$$p_1(q_{r1}) = a_1(D - q_{r1}) + b_1 \quad (3)$$

$$\mathbb{E}[p_2(q_{r1}, q_{r2})] = a_2(D - q_{r1} - q_{r2}) + b_1 + (D - q_{r1})(a_1 - a_2). \quad (4)$$

In a next step, we will derive the respective profit functions for the conventional and renewable producer. The conventional producer's profit function is defined as

$$\Pi_c(q_{c1}, q_{c2}) = p_1(q_{r1})q_{c1} + p_2(q_{r1}, q_{r2})q_{c2} - C_1(q_{c1}) - C_2(q_{c1} + q_{c2}) + C_2(q_{c1}). \quad (5)$$

Revenues in both stages are the products of the respective prices and quantities. Production costs depend on the power plants utilized for production. Since the marginal costs of production may change with time, the costs consist of the sum of quantities planned for production in each stage.

The profit function of the renewable producer

$$\Pi_r(q_{r1}, q_{r2}) = p_1(q_{r1})q_{r1} + p_2(q_{r1}, q_{r2})q_{r2} \quad (6)$$

consists of the quantities traded at the respective prices in the first and second stage without associated production costs.

We are able to show how competition between renewable producers and conventional producers can be modeled by applying this framework to different settings. In this paper, we will consider three cases:

- Competition in the first stage with identical cost functions:  $q_r = Q$ ,  $a_1 = a_2 = a$

- Competition in the first stage with changing cost functions:  $q_r = Q$ ,  $a_2 > a_1$
- Competition in the first and second stage with changing cost functions:  $q_r \leq Q$ ,  $a_2 > a_1$ .

### 3. Cournot Competition of Renewable Producers Competition in the First Stage with Identical Cost Functions

Throughout this paper we focus on a linear marginal cost function which can be regarded as the simplest case.<sup>5</sup> In this section, we will first give an intuition for the results of the model based on the simple case of identical cost functions and a renewable monopolist who acts strategically in the first stage. For this part of the analysis, we assume that the renewable producer sells the complete remaining production in the second stage, meaning  $q_r = Q$ .<sup>6</sup> In a next step, we will extend the analysis from the renewable monopoly to an oligopoly.

We can parametrize the linear marginal cost function  $MC(q_c) = aq_c + b$  by the gradient  $a \in \mathbb{R}_{\geq 0}$  and an offset  $b \in \mathbb{R}_{\geq 0}$  with variable  $q_c \in \mathbb{R}_{\geq 0}$  as the produced quantity from conventional producers. Because demand is assumed to be price inelastic, we can write the prices in both stages a function of renewable quantities:

$$p_1(q_{r1}) = a(D - q_{r1}) + b \quad (7)$$

and

$$p_2(Q) = a(D - Q) + b. \quad (8)$$

#### 3.1. Renewable Producer Monopoly

First, we look at the simple case in which all renewable production is traded by one firm. From economic literature it is well known that under the assumptions of Cournot competition, the monopolist has incentives to deviate from welfare optimal behavior in order to maximize its own profits. In our sequential market setting, this can be observed as well. By proposition 1 we show that the optimal bidding strategy for a renewable producer under a monopoly is to bid half the expected production in the first stage.

**Proposition 1.** *The profit maximizing quantity for a renewable monopolist is  $q_{r1}^* = \frac{\mu_q}{2}$  with  $\mu_q$  the expected renewable production.*

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<sup>5</sup>The main results also hold for convex second-order cost functions. However, the exact results may slightly deviate (i.e. it has a slightly shifting influence to the profit maximizing bidding strategy, but comparable small impact on the main results).

<sup>6</sup>Note that we assume additionally  $Q \leq D$ . If  $Q > D$  and renewable producers have to sell their whole production in stage 2, we would force producers to bid negative prices. In such cases, we would expect that renewable producers reduce their production to avoid too low prices, e.g. below 0. This will be discussed in section 5 in which we extend the model and allow for  $q_r \leq Q$ .

*Proof.* The basic profit function of a renewable producer in our theoretical model framework is described in (6). For identical marginal cost functions, we derive the following expected profit function

$$\mathbb{E}[\Pi_r(q_{r1})] = q_{r1} (a(D - q_{r1}) + b) + \int (Q - q_{r1})f(Q) (a(D - Q) + b) dQ. \quad (9)$$

Where the first derivative results in

$$\frac{d}{dq_{r1}} \mathbb{E}[\Pi_r(q_{r1})] = a(D - q_{r1}) + b - aq_{r1} - Da \int f(Q) dQ + a \int Qf(Q) dQ - b \int f(Q) dQ. \quad (10)$$

Since  $f(Q)$  is symmetric and the marginal cost function is linear, we can further simplify the expected profit function by the following substitutes:

$$\text{Expected value for } Q: \quad \int Qf(Q) dQ = \mu_q \quad (11)$$

$$\text{Distribution function has a total probability of 1:} \quad \int f(Q) dQ = 1 \quad (12)$$

This leads to the simplified necessary condition for the profit maximizing quantity  $q_{r1}^*$  as

$$\frac{d}{dq_{r1}} \mathbb{E}[\Pi_r(q_{r1})] = -Da + a\mu_q - aq_{r1} + a(D - q_{r1}) \stackrel{!}{=} 0. \quad (13)$$

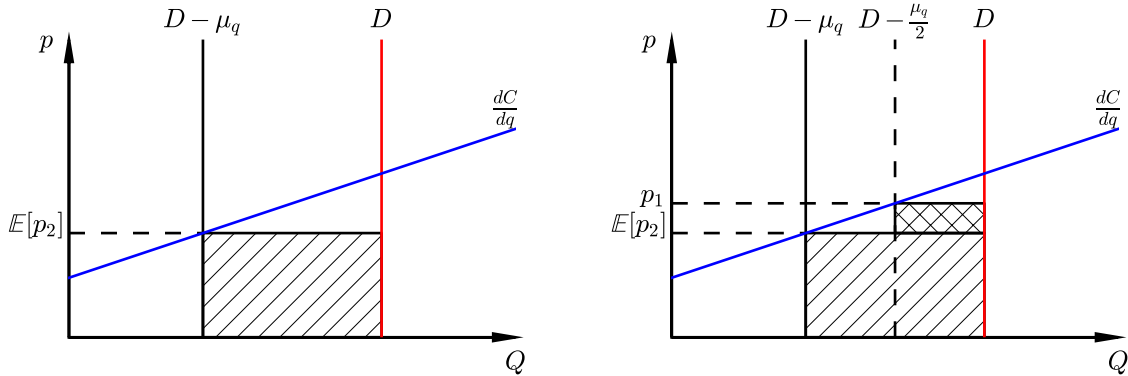
Now we can solve this equation for  $q_{r1}$  which results in the profit maximizing quantity

$$q_{r1}^* = \frac{\mu_q}{2}. \quad (14)$$

In order for this being a maximum the second derivative has to be negative. This can easily be checked by calculating

$$\frac{d^2}{dq_{r1}^2} \mathbb{E}[\Pi_r(q_{r1})] = -2a. \quad (15)$$

Since  $a$  is defined as the slope of the marginal cost function and is positive by definition,  $q_{r1}^* = \frac{\mu_q}{2}$  indeed describes the profit maximizing quantity for the renewable producer.  $\square$



(a) Lower bound for the profit of the renewable producer (b) Profit of the renewable producer from the first and second stage

Figure 3: Optimal quantity of the renewable monopolist

The motivation of the renewable producer to bid half her expected quantity in the first stage becomes clear by analyzing Figure 3. Since we consider a linear marginal cost function, we can abstract from the uncertainty in renewable production  $f(Q)$  and only consider the expected production  $\mu_q$ . The profit of the renewable producer can be split into two parts. One part stems from selling the expected production into the market, as can be seen in Figure 3(a) (single hatched area). This part can be considered as a lower bound to the profit of the renewable producer and does not depend on the strategy of the renewable producer because she has to sell all production to the market in the final stage. The resulting price in the second stage is thus given by  $\mathbb{E}[p_2]$ . The second part of the renewable producer profit can be obtained by selling a quantity forward in the first stage at a price  $p_1$ . In order to increase her profit, the quantity in the first stage needs to be between  $D - \mu_q$  and  $D$  to obtain a higher price compared to  $\mathbb{E}[p_2]$ . Since the marginal cost function is linear and we have a monopolist selling forward, it is optimal to sell half her expected production because it maximizes the additional profit in Figure 3(b) (cross hatched area).

**Proposition 2.** *The optimal strategy of a renewable monopolist selling its renewable production in sequential markets with multiple stages is to sell it in small quantities at decreasing prices.*

*Proof.* The triangle in Figure 3(a) can be considered as the maximum profit which can be gained by selling the expected production of the renewable producer. When the renewable producer is able to sell this production in multiple stages, it is optimal to sell it little by little in order to maximize her profit. This means prices in multiple sequential market stages would be declining until the price of  $\mathbb{E}[p_2]$  is reached in the final stage. In this case, the renewable producer would be able to increase its profit by the triangle in Figure 3(a) compared to selling the expected quantity already in the first stage.  $\square$

For the case of multiple market stages also conventional producers would be able to increase their profit. In this case, they would be able to obtain a higher profit in the first stage, where they can sell a larger quantity at a higher price. On the other hand, consumer surplus would be lowered due to higher prices.

This leads us to the conclusion that with a renewable monopolist, different market designs can have a large impact on distributional effects between producers and consumers. Consumers lose if producers trade electricity in multiple stages. Thus, continuous trading in short-term markets lowers consumer surplus. From the view of consumers, a few separate auctions should be preferred to a continuous auction since this limits strategic behavior of a renewable monopolist.

Strategic production withholding is commonly observed by market participants at the margin (see, for instance, Fabra et al. (2006), Ausubel et al. (2014)). The reason is that it is most profitable to reduce the production at the margin if the corresponding price increase overcompensates the production withholding.<sup>7</sup>

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<sup>7</sup>Additional to pure production withholding, strategic behavior at the margin can also be exerted with bids above marginal costs to increase the market clearing price.

The production close to the margin has generally the lowest profits and thus the profit for the whole production fleet can be increased. In contrast to this, our results show that strategic production withholding may also occur for infra-marginal production with our underlying model assumptions (two-stage trading possibility, zero marginal costs for the renewable producers, positive marginal costs for the perfect competitive conventional producers). Unlike usual, it is not dependent on a higher steepness of the cost function for extra-marginal production but also holds for the basic case of a linear cost function. This spans a new dimension of strategic behavior and could also be investigated in further research.

### 3.2. Renewable Producer Oligopoly

In this section, we extend the monopoly case to the case of multiple symmetric renewable producers that form an oligopoly. The symmetry implies that the renewable producers have perfectly correlated generation as well as forecast errors. The remaining approach and notation are similar to previous sections. As we learned from before, the conventional producer reacts to the decision of the renewable producers and can be considered as a price taker. So we can focus on the optimal quantities of the renewable producers. We still consider a linear marginal cost function  $MC(q) = aq + b$  and define the players  $i = 1, \dots, N$  with their corresponding quantities in stage 1 as  $q_{ir1}$ . Furthermore, we define the sum of the quantities of all players but  $i$  as  $q_{-ir1} = \sum_{j \neq i} q_{jr1}$ . We find that the optimal bid of a renewable producer in the first stage is still driven by strategic behavior but tends towards the expected production level as the number of producers increases.

**Proposition 3.** *The optimal quantity traded in the first stage for each player is  $q_{ir1}^* = \frac{1}{N+1} \mu_q$  with  $\mu_q$  the total expected renewable production of all players.*

*Proof.* See AppendixA. □

As a direct implication from the optimal first stage bid we see that for the linear marginal cost function, the optimal strategy is still independent of the gradient or the uncertainty of production.

**Corollary 1.** *The profit maximizing traded quantity in stage 1 of the above setting is identical for all players. Furthermore,  $q_{ir1}^*$  is independent of the steepness  $a \in [0, \infty)$  of the marginal cost function, the offset  $b \in [0, \infty)$  of the marginal cost function, and the probability distribution function  $f(Q_i)$ .*

According to Proposition 3 it is optimal for renewable producers to always trade less than the expected production in the first stage since this maximizes their profits. The overall quantity tends towards the overall expected quantity as the number of players increases.<sup>8</sup>

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<sup>8</sup>Note that, for the moment, we assumed a linear marginal cost function which does not change between the first and the second stage.

In stage 1, this leads to an overall traded quantity of renewable production of

$$q_{r1} = \sum_{j=1}^N q_{jr1} = \frac{N}{N+1} \mu_q \quad (16)$$

with  $\mu_q := \sum_{j=1}^N \mu_{jq}$ . In two sequential markets, renewable producers have an incentive to trade less than the total expected renewables production in the first stage. The more players enter the market the stronger the competition and thus the traded amount in the first stage tends towards the expected production. Our results of the first stage show that, under the described setting, a renewable producer acts exactly as predicted in a standard one-shot oligopolistic Cournot game.

#### 4. Flexibility and its Role in Short-term Markets Competition in the First Stage with Changing Cost Functions

In this section we shed light on the implications of changing cost functions in short-term markets. As mentioned before, this can happen for essentially two reasons. One reason is that not all conventional power plants are flexible enough to adjust their production capacity in stage 2 in the short run. The second reason is that there can be transaction costs for power plant operators associated with the trading in the intraday market.

The difference between the cost function of the first and second stage has implications for the optimal quantity of the renewable producers in the first stage, which we analyze here in more detail. The nomenclature corresponds to the previous sections.

**Proposition 4.** *The optimal quantity traded in the first stage for each renewable player is  $q_{ir1}^* = \frac{1}{N+1} \mu_{iq} (N + 1 - \frac{a_1}{a_2})$ , with the ratio  $\frac{a_1}{a_2}$  representing the degree of flexibility of the supply side in both stages.<sup>9</sup>*

*Proof.* In a first step we will derive the optimal quantity of a player  $i$  who competes against  $N - 1$  identical players<sup>10</sup>. According to the setup, the prices in the first and second stage can be defined as:

$$p_1(q_{ir1}, q_{-ir1}) = a_1(D - q_{ir1}) - q_{-ir1} + b_1 \quad (17)$$

$$p_2(q_{ir1}, q_{-ir1}) = a_2(D - Q_i N) + b_1 + (a_1 - a_2)(D - q_{ir1} - q_{-ir1}). \quad (18)$$

Again, we can define the expected profit function for player  $i$ , take the first derivative and integrate over  $f_i$  (which is assumed as being identical for all players). Setting the first derivative equal to zero leads us to the necessary condition for an optimal quantity:

$$-a_1 \mu_{iq} + a_2 \mu_{iq} N + a_2 \mu_{iq} - a_2 q_{-ir1} - 2a_2 q_{ir1} \stackrel{!}{=} 0 \quad (19)$$

Under the assumption that all players are identical we can set  $q_{-ir1} = (N - 1)q_{ir1}$  and solve for  $q_{ir1}$  which leads to:

$$q_{ir1}^* = \left(1 - \frac{1}{N+1} \frac{a_1}{a_2}\right) \mu_{iq}. \quad (20)$$

<sup>9</sup>Small values of  $\frac{a_1}{a_2}$  represent a very inflexible supply side in the second stage.

<sup>10</sup>The sum over all other players is still denoted by the quantity  $q_{-ir1} = \sum_{j \neq i} q_{jr1}$

The second derivative of the expected profit function is negative, which proves  $q_{irr1}^*$  being a maximum for the expected profit function.  $\square$

This means that all renewable producers together submit a quantity of

$$q_{r1}^* = \mu_q - \frac{1}{N+1} \frac{a_1}{a_2} \mu_q \quad (21)$$

in the first stage (with  $a_2 > a_1$ ).

From Equation (21) we can conclude the following: (1)  $q_{r1}^*$  increases if conventional producers are less flexible ( $a_2 \gg a_1$ ); (2)  $q_{r1}^*$  increases with an increasing number of renewable producers  $N$ . For a perfectly competitive market (with  $N \rightarrow \infty$ ) it is optimal for each player to trade its share of the total expected quantity in the first stage.

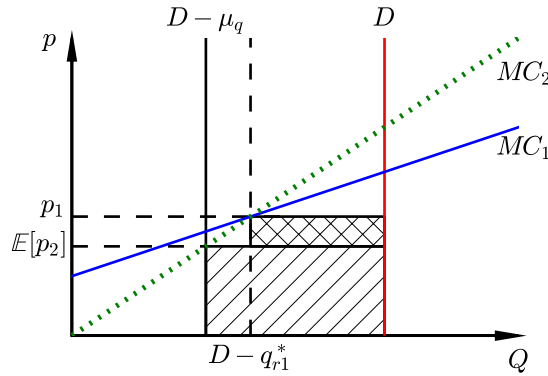


Figure 4: Profit of a renewable monopolist facing inflexible conventional producers

By looking at the example of a renewable monopolist in Figure 4, we can get a deeper understanding of the motives for a renewable producer who faces a market with inflexible conventional producers. As explained before, the marginal cost curve for the second stage rotates around the market clearing point of the first stage. The total production of the renewable producer that needs to be sold after both stages however does not change. Thus, the renewable producer has to decide what quantity to sell at a respective price in the first stage and sell the remaining quantity at a lower price in the second stage. The price is lower in the second stage due to the additional renewable quantities that are sold by the renewable producer. Basically, in Figure 4, the sum of the cross hatched area and the single hatched area needs to be maximized. The renewable producer is able to maximize both areas by a parallel shift of the marginal cost function for stage 2 (green dotted line). This means, the renewable producer has to optimize the quantity in the first stage in such a way that the profit from both stages is maximized. Summarizing, a more flexible power

plant fleet shifts the total optimal first stage bidding quantity of a renewable producer towards the expected production.

The described effects on the optimal quantity hold true for different numbers of renewable producers and different degrees of flexibility. This is shown exemplarily in Figure 5. Here, the optimal quantity converges more slowly to the expected production in the perfectly flexible case ( $\frac{a_2}{a_1} = 1$ ) compared to a highly inflexible conventional power plant mix ( $\frac{a_2}{a_1} = 4$ ). An increase in the number of renewable producers leads to a similar effect of a higher overall renewable quantity in the first stage.

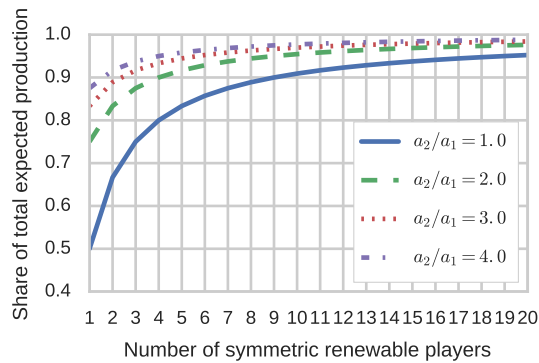


Figure 5: Optimal renewable quantity  $q_{r1}$  dependent on the number of players  $N$  and the ratio  $a_2/a_1$

## 5. Incentives of Renewable Producers to Withhold Production Competition in the First and Second Stage with Different Cost Functions

In this section we extend the analysis of strategic competition in the first stage by investigating the case in which renewable producers are allowed to withhold production in the second stage. Therefore, we relax the assumption that the renewable producer needs to sell all her realized production in the second stage. This means  $q_{r1} + q_{r2} \leq Q$  instead of  $q_{r1} + q_{r2} = Q$ . We still assume that renewable producers strictly avoid being short after stage 2, i.e. selling more production than they produce. The rationale is that high financial penalties need to be paid in case of an imbalance. All other model assumptions stay the same.

The motivation for the relaxation of the second stage restriction to sell the whole production is threefold. First, we note that, in general, it is technically possible to reduce production for renewable producers. This happens for photovoltaic in critical grid situation if the voltage level extends a critical value (automatic shut down around 50.2 Hertz) or for wind turbines during storms. Second, a reduced production could be economically profitable in specific situation. Especially if prices are negative or, like in the investigated case, if market prices could be increased profitably by withholding production. Third, market manipulation by a



withhold of renewable production is not easy to prove by the regulator. It is hard to detect whether a wind turbine does not produce due to maintenance, local wind conditions or strategic production withholding.

We extend the model with cost functions by replacing the constraint  $q_{r1} + q_{r2} = Q$  with  $q_{r1} + q_{r2} \leq Q$ . Based on this model we obtain the following results.

**Proposition 5.** *If renewable producers are allowed to withhold production, they only withhold production after the second stage if the expected production of all producers is high compared to the demand  $D$ , i.e. if  $\mu_q > \frac{a_2 N(N+1)}{a_2(N+1)^2 - a_1 N} D + \frac{a_2(N+1)}{a_1(a_2(N+1)^2 - a_1 N)} b$ . This means the expected renewable production needs to be at least  $\frac{D}{2}$ . Otherwise, renewable producers sell the total realized production into the market (same result as of proposition 4).*

*Proof.* We use the same model as in section 4 (and corresponding proposition 4). The only difference is the relaxed constraint  $q_{r1} + q_{r2} = Q$  by  $q_{r1} + q_{r2} \leq Q$ . This allows the renewable producer to withhold production and to increase prices in the second stage. Since we adjusted an equality constraint by an inequality constraint, we face now a convex optimization problem with inequalities and can use the Karush-Kuhn-Tucker (KKT) conditions to solve it. The full proof can be found in the appendix.  $\square$

The main finding is that renewable producers have an incentive to withhold production after the second stage only if the (expected) production exceeds a threshold value which is at least  $\frac{D}{2}$  (but dependent on  $a_1, a_2, b$  and  $N$ ).<sup>11</sup> The exact threshold value is

$$Q_{threshold} := \frac{a_2(N+1)}{a_2(N+1)^2 - a_1 N} D + \frac{a_2(N+1)}{a_1(a_2(N+1)^2 - a_1 N)} b. \quad (22)$$

As long as the (expected) production is below this threshold, the renewable producers will sell their total realized production in the second stage. Nevertheless, the production is split between first and second stage to increase profits. By analyzing this threshold we find the following

- $Q_{threshold}$  is increasing in  $N$ : The more producers exist, the higher the threshold. Therefore, more competition between renewable producers limits the incentive for renewable producers to withhold quantities in the second stage.
- $Q_{threshold}$  is decreasing in  $a_2$  (with  $a_1$  fixed): The more inflexible the power plant fleet, the lower is the threshold. Therefore, renewable producers start to withhold production at a lower level of expected renewable production.
- $Q_{threshold}$  converges to  $\frac{N}{N+1} \left( D + \frac{b}{a_1} \right)$  for  $a_2 \rightarrow \infty$  but is strictly above  $\frac{D}{2}$ .

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<sup>11</sup>In stage 1, the expected production is the relevant quantity while in stage 2 the realized production is the relevant quantity. If both, expected and realized production, deviate from each other, it is possible that the renewable producers pursue a different strategy in each stage.

To sum up, renewable producers only have an incentive to withhold quantities in situations with very high renewable generation compared to the demand. Additional renewable producers as well as more flexible conventional producers increase the threshold ( $Q_{threshold}$ ) to withhold production quantities.

## 6. Prices, Welfare, Producer Surplus and Consumer Surplus

Trading in the day-ahead and intraday market has implications for overall welfare, producer surplus and consumer surplus. So far, we focused on the quantities of the renewable producers that maximize their respective profits. They determine the quantities that are traded by the conventional producers and thereby the prices in both stages. In order to disentangle the effects on overall welfare, producer and consumer surplus, we will first analyze the effects on prices in the two stages.

Since we found in section 5 that renewable players only withhold production at very high production levels compared to demand  $D$ , we focus on the case in which renewable producers sell all their production after stage 2 (the case  $q_{r1} + q_{r2} = Q$ ).<sup>12</sup>

### 6.1. Prices and the Role of Arbitrageurs

By plugging in the optimal quantity from Equation (20) into the price equations for the case with flexibility constraints (Equation (3) and (4)) we obtain the following prices:

$$p_1 = Da_1 + b_1 - \frac{a_1}{a_2} \frac{\mu_q}{N+1} (a_2(N+1) - a_1) \quad (23)$$

$$\mathbb{E}[p_2] = Da_1 + b_1 - \frac{a_1}{a_2} \frac{\mu_q}{N+1} (a_2(N+2) - a_1). \quad (24)$$

From these two equations we can already see that the price in the first stage is higher than in the second stage. This becomes obvious by taking the difference between the two prices:

$$p_1 - \mathbb{E}[p_2] = \frac{a_1 \mu_q}{N+1}. \quad (25)$$

We can observe the following implications: First, the price difference between stage 1 and 2 is independent of the change in the slope of the marginal cost function ( $a_2$ ). The renewable producers choose their quantity dependent on the slope ( $a_2$ ). This has an effect on the absolute prices in the two stages but the price delta stays constant. Second, with a higher overall expected production from renewables ( $\mu_q$ ) also the price difference increases. The quantity that is withheld from trading in the first stage increases with the expected production and, thereby, the price difference increases. Third, the price difference decreases with

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<sup>12</sup>For a realistic number of renewable players  $N > 5$  and an arbitrary ratio of  $a_2$  to  $a_1$ , the threshold  $Q_{threshold}$  is at least  $0.85D$ .

an increasing number of renewable producers ( $N$ ). In a perfectly competitive market (with  $N \rightarrow \infty$ ), prices in both stages are equal. As we can observe in Figure 5, the quantity in the first stage tends towards the overall expected quantity and hereby prices in both stages converge.

Based on the price difference in both stages one could suspect arbitrageurs to be entering the market. By obtaining a short position in the day-ahead market and adjusting their position in the intraday market, they would be able to make a profit. The optimal strategy of an arbitrageur is therefore identical with the strategy of the renewable players. The only difference is that arbitrageurs do not necessarily own production assets. Each additional arbitrageur that would enter the market can nevertheless be regarded as an additional renewable player. This would in turn decrease the price difference between the day-ahead and intraday market (cf. Figure 6).

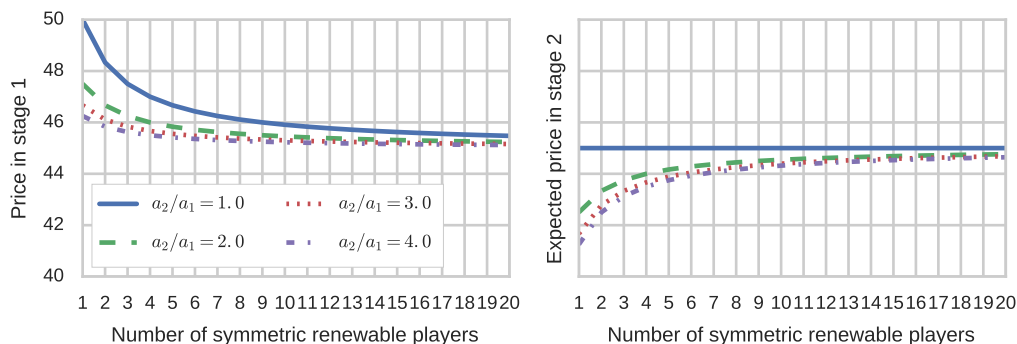


Figure 6: Prices in the two stages for an example with  $D = 70$ ,  $\mu_q = 20$ ,  $\sigma_q = 5$ ,  $b_1 = 20$  and  $a_1 = 0.5$

Still, electricity markets have some unique features that may prevent arbitrageurs from engaging in short-term electricity markets. First, the assets that are traded are not only financial but physical obligations to produce and deliver electricity. Therefore, some short-term market platforms restrict the participation to firms with physical production assets. This prevents for example banks from entering these markets. Second, there may be information asymmetries between renewable producers and arbitrageurs that may be hard to overcome. For example renewable producers can be assumed as having better knowledge about the expected production level of their assets. For the following discussions we will thus not focus on the case of additional arbitrageurs entering the market. Nevertheless, the implications of arbitrageurs entering the market can be observed implicitly by considering an increase in the number of renewable players ( $N$ ).

In order to gain a deeper understanding of the effects from changing cost functions and increased competition on prices, we plot this relationship in Figure 6 for an exemplary case. The direction of the effects

will stay the same for arbitrary  $a_1, a_2$  with  $a_2 \geq a_1$  and arbitrary  $D, Q$  and  $b$  with  $(D - Q)a_1 + b \geq 0$  ( $Q \sim \mathcal{N}(\mu_q, \sigma_q)$ ).

In Figure 6, we chose the values such that one can easily find similarities to the German electricity market. A demand  $D$  of 70 GW can be observed during peak times, where also an expected renewable production  $\mu_q$  of 20 GW is quite common. Furthermore the parameters of the marginal cost function were chosen such that they represent common price levels.<sup>13</sup>

We can see that the prices in stage 1 and 2 converge to the same value with an increasing number of players. This benchmark is set by the perfectly flexible case ( $\frac{a_2}{a_1} = 1$ ), where the price in the second stage stays constant. In the next sections we will analyze the effects on producer surplus, consumer surplus and overall welfare.

## 6.2. Producer Surplus

The producer surplus is defined as the sum of the renewable producer surplus and the conventional producer surplus. For the case with a changing marginal cost function, the conventional producer surplus can be defined as

$$\mathbb{E}[\Pi_c(q_{r1})] = p_1(D - q_{r1}) + p_2 \int (q_{r1} - Q)f(Q)dQ - C_1(q_{r1}) - \int C_2(q_{r1})f(Q)dQ. \quad (26)$$

It is the difference between the income from sold quantities in stage 1 and 2 and the associated costs with the production of electricity.

The first stage costs  $C_1$  in our model depend on the quantities offered by the renewable producers  $q_{r1}$ . We can thus obtain the costs in the first stage by integrating over the marginal cost function  $MC_1$

$$C_1(q_{r1}) = \frac{1}{2}a_1(D - q_{r1})^2 + b_1(D - q_{r1}). \quad (27)$$

The formulation is more complex for the costs that are associated with the second stage of production. First, it depends on the quantity that is traded in the first stage by the renewable producer  $q_{r1}$ . Second, it depends on the realization of the final renewable production  $Q$ . In the first stage, the conventional producers plan to produce a certain quantity  $D - q_{r1}$ . In the second stage, this quantity has to be adjusted to meet the total residual demand of  $D - Q$ . This means if the renewable production turns out to be higher than the traded quantity in the first stage, the conventional producers need to reduce their planned production and can buy back quantities at a lower price. Meanwhile the slope of the cost function has changed from  $a_1$

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<sup>13</sup>Of course a linear marginal cost function is a crude assumption in this case, but it allows us to show the overall effects.

to  $a_2$ . This leads us to the following expected cost function for the second stage:

$$\mathbb{E}[C_2(q_{r1})] = \int \int_{q_{r1}}^{D-Q} (a_2 q_{c2} + (a_1 - a_2)(D - q_{r1}) + b_1) dq_{c2} f(Q) dQ \quad (28)$$

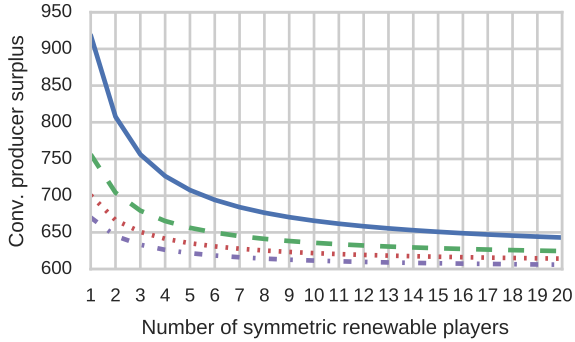
$$= (Da_1 - a_1 q_{r1} + b_1)(\mu_q - q_{r1}) + a_2 q_{r1}(\mu_q - \frac{q_{r1}}{2}) - \frac{a_2 n^2}{2} (\mu_{iq}^2 + \sigma_{iq}^2). \quad (29)$$

What is especially noticeable in this equation, is that for the first time in our analysis also the standard deviation ( $\sigma_{iq}$ ) of the expected renewable production plays a role. The reason for this lies in the non-linear cost function of the conventional producers. Here, deviations from the expected value are not multiplied by a linear curve and weighted equally but weighted by the non-linear function. This is why the standard deviation plays an important role. By inserting Equation (27) and (29) in (26), we obtain the total conventional producer surplus.

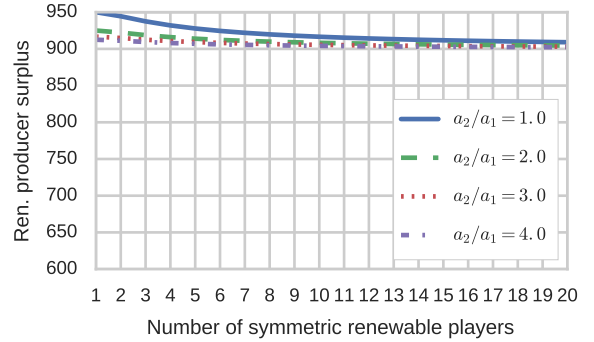
In the same way, we can also derive the producer surplus for the renewable producers.

$$\mathbb{E}[\Pi_r(q_{r1})] = p_1 q_{r1} + \int p_2(Q - q_{r1}) f(Q) dQ \quad (30)$$

By plugging in the results from Equation (20) it is possible to quantify the renewable and conventional producer surplus. We plot this for an exemplary cases in Figure 7.



(a) Expected conventional producer surplus



(b) Expected renewable producer surplus

Figure 7: Expected producer surplus for an example with  $D = 70$ ,  $\mu_q = 20$ ,  $\sigma_q = 5$ ,  $b_1 = 20$  and  $a_1 = 0.5$ .

As we could already see from Figure 6, prices in the first stage decrease with an increase in competition or a less flexible supply curve. At the same time prices in the second stage increase. This results in both, a dampening and an increasing effect on producer surplus. From Figure 7 we can observe that the decreasing effect of the first stage outweighs the increasing effect in the second stage. Overall, we see that the producer surplus decreases with the number of renewable producers  $N$  and with a less flexible power

plant mix. Especially the decrease in conventional producer surplus is noticeable. For renewable producers the decrease in surplus is not as prominent, since they are able to reduce the effects by adjusting their optimal quantity  $q_{r1}$ . For example the overall quantity traded by renewable producers ( $q_{r1}^*$ ) is increased when more renewable producers compete in the first stage. Also a less flexible power plant mix leads to a higher optimal quantity for renewable producers in the first stage (cf. Figure 5).

### 6.3. Consumer Surplus

In our model, consumers are represented as being completely inelastic in their demand behavior. In electricity markets it is common practice to assume consumers as completely price inelastic and consuming electricity up to the point when the price exceeds the value of lost load (VOLL). We therefore slightly adjust our assumptions by introducing the price  $p^{VOLL}$  which can be regarded as the upper limit for the willingness-to-pay for electricity consumption.

As consumers are assumed to be risk-averse, demand is already satisfied in the first stage at price  $p_1$ , as long as  $p_1 < p^{VOLL}$ . The consumer surplus can therefore be expressed as  $(p^{VOLL} - p_1)D$ . By plugging in the price formulation for the first stage from Equation (3), we get

$$CS = D \left( p^{VOLL} - Da_1 - b_1 + \frac{a_1}{a_2} \frac{\mu q}{N+1} (a_2(N+1) - a_1) \right). \quad (31)$$

We can now compare the consumer surplus for the different combinations of  $N$  and  $a_2/a_1$ . In order to circumvent an assumption for the upper price limit  $p^{VOLL}$ , we focus our analysis on changes in consumer surplus compared to a reference point. We therefore choose the reference point where consumer surplus is the lowest. This is the case for a renewable monopolist and perfectly flexible conventional producers ( $a_1 = a_2$ ).

As one could already expect from decrease in prices with an increasing number of players in Figure 6, the consumer surplus increases with the number of players. What may be counter intuitive is that consumers can profit from a less flexible power plant mix. The lower flexibility of conventional producers leads renewable producers to adjust their quantity, which has a price dampening effect for the first stage. Consumers can therefore profit from the lower prices in the first stage as it is shown exemplarily in figure 8(a).

### 6.4. Welfare

Combining the effects on producer and consumer surplus leads to changes in overall welfare. As we can only analyze differences in consumer surplus this also holds for the case of overall welfare. Again, we define the perfectly flexible case with a monopolistic renewable producer as a reference point for the analysis (cf. section 6.3). The difference in overall expected welfare to the monopolistic case can be defined as

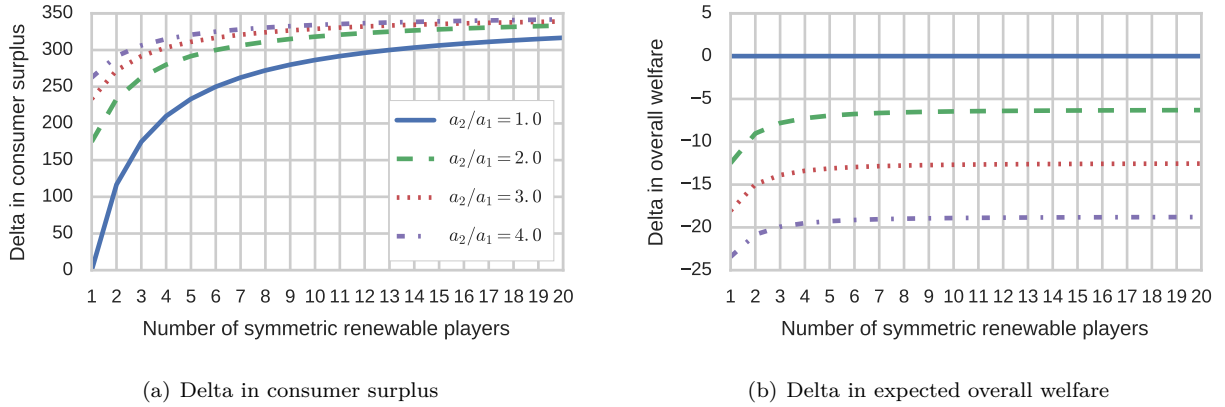


Figure 8: Delta in consumer surplus and expected overall welfare for an example with  $D = 70$ ,  $\mu_q = 20$ ,  $\sigma_q = 5$ ,  $b_1 = 20$  and  $a_1 = 0.5$

$$\Delta\mathbb{E}[\mathbb{W}(q_{r1})] = -\Delta\mathbb{E}[CE(q_{r1})] + \Delta\mathbb{E}[\Pi_p(q_{r1})]. \quad (32)$$

In Figure 8(b) we can observe these effects on overall welfare. The overall welfare stays constant for the case of a perfectly flexible power plant mix. In this case, the demand is always satisfied at the same costs which does not lead to a change in overall welfare. Negative effects on overall welfare occur only if the total production costs for electricity increase, i.e. if conventional power producers are less flexible. Especially if the power plant mix is highly inflexible, as in the case with  $\frac{a_2}{a_1} = 4$ , it will lead to a substantial decrease in overall welfare. Generally we can observe two effects. First, the effect on welfare has a smaller magnitude than the isolated effects on producer surplus or consumer surplus. The increase in consumer surplus and decrease in producer surplus counteract each other and lead only to a slightly reduced effect on overall welfare. Second, the welfare is generally decreased in a setting with less flexible power plants.

In a last step, we analyze the effects of uncertainty on overall welfare. So far, we assumed the production of the renewable producer in the final stage to be forecasted with a standard deviation of  $\sigma_q = 5$  in the numerical examples. Now, we assume that if forecasts are improved or trading time is delayed, the standard deviation decreases, as to Foley et al. (2012). A decrease in standard deviation could also be accomplished by delaying trading of the first stage (e.g. by trading in the evening of the day before physical delivery instead of at noon). We quantify the welfare effects by comparing them to the case with no uncertainty ( $\sigma_q = 0$ ) and a perfectly competitive market ( $q_{r1} = \mu_q$ ). From Figure 9 we can observe that a larger standard deviation results in welfare losses. From this we can conclude that it is desirable to increase the quality of forecasts or to change the timing of trading in order to increase overall welfare.

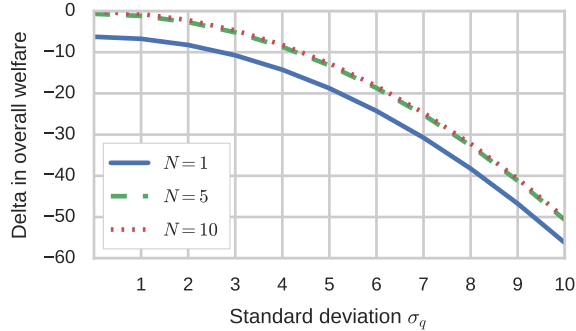


Figure 9: Delta in expected overall welfare for varying standard deviation of the forecast  $\sigma_q$  ( $D = 70$ ,  $\mu_q = 20$ ,  $b_1 = 20$ ,  $a_1 = 0.5$  and  $a_2 = 1$ )

## 7. Concluding Remarks

We derive the optimal quantities for renewable producers that are strategically selling their production in a two-stage game with uncertainty about production in stage 1 and knowledge about the realization of their production in stage 2. It is profit maximizing for renewable producers to bid less than their expected total quantity in the first stage, which we consider as the day-ahead market. Renewable producers are able to increase their profits by selling only part of their expected production in the first stage and thus raising the price in the first stage. The optimal quantity in the first stage tends towards the overall expected quantity with an increasing number of renewable producers. Conventional producers are considered as a competitive fringe that satisfies the residual demand in both markets. If conventional power producers are less flexible in their operation, renewable producers have a larger incentive to increase the traded quantity in the first stage. In general, prices in the first stage (day-ahead) are higher compared to the second stage (intraday), but with an increasing number of renewable producers or with arbitrageurs entering the market this difference decreases. In situations with very high production levels, that are at least able to serve half of the demand, renewable producers have an incentive to withhold production in the second stage. This effect is decreased by an increasing number of players but increases in a setting with low flexibility of conventional producers.

A reduced forecast uncertainty leads to an increase in overall welfare. This leads us to two conclusions. First, overall welfare can be increased by delaying the trade in the day-ahead market closer to the time of physical delivery. For example by shifting the auction from noon to the evening. Second, an increase in forecast quality has a positive effect on overall welfare.

Based on the results it becomes obvious that in a future electricity system with high shares of renewables,



regulators need to pay attention to the possible abuse of market power by large renewable producers. In situations with low liquidity and the absence of arbitrageurs this could lead to significant distributional effects and even welfare losses.

In our whole analysis, we assumed the generation of all renewable producers to be perfectly correlated, as well as their forecast errors. This is not the case in reality and could be further investigated. Additionally, it would be possible to quantify welfare implications of improved forecast quality and alternative market designs at concrete examples.

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# Appendix

## AppendixA. Proof of Proposition 3

*Proof.* Because all players are symmetric we can denote the total traded renewable production of all players in stage 1 by  $q_{r1} = q_{ir1} + q_{-ir1}$  (where  $q_{-ir1}$  aggregates all players but not player  $i$ ), the realized production in stage 2 by  $Q = NQ_i$ , and the expected quantity by  $\mu_q = N\mu_{iq}$ . With these definitions, Equation (7) and (8) still hold for the oligopoly case.

The profit function of renewable producer  $i$  can be derived by plugging in those values into

$$\Pi_{ir}(q_{ir1}) = p_1(q_{r1})q_{ir1} + p_2(D - Q)q_{ir2} \quad (\text{A.1})$$

so that the profit function results in

$$\Pi_{ir}(q_{ir1}) = (a(D - q_{ir1} - q_{-ir1}) + b)q_{ir1} + (a(D - NQ_i) + b)(Q_{ir} - q_{ir1}). \quad (\text{A.2})$$

Remember that  $q_{ir2} = Q_i - q_{ir1}$  and that we assume  $Q_i$  to be uncertain. In order to derive the expected profit function we have to integrate for  $Q_i$  over the distribution  $f(Q_i)$ , where  $f(Q_i)$  is the probability density function for  $Q_i$ . After taking the first derivative, setting it equal to zero and replacing the expected values (analogous to Equations (11) and (12)), we get the necessary conditions

$$\frac{d}{dq_{ir1}} \mathbb{E}[\Pi_{ir}(q_{ir1})] = a(\mu_q - q_{ir1} - q_{-ir1}) \stackrel{!}{=} 0 \quad (\text{A.3})$$

and the corresponding solution is

$$q_{ir1}^* = \frac{1}{2}N\mu_{jq} - \frac{1}{2}q_{-ir1} \quad (\text{A.4})$$

for  $i = 1, \dots, N$ .

In an equilibrium of identical players we have identical solutions which results in  $q_{-ir1} = (N - 1)q_{ir1}$ . With this, we derive

$$q_{ir1}^* = \frac{1}{2}N\mu_{iq} - \frac{1}{2}(N - 1)q_{ir1}^* \quad (\text{A.5})$$

$$\Leftrightarrow q_{ir1}^* = \frac{1}{N + 1}\mu_q. \quad (\text{A.6})$$

Because the second derivative of Equation (A.2) is negative, we found the profit maximizing quantity  $q_{ir1}^*$   $\square$

## AppendixB. Proof of Proposition 5

*Proof.* As before, we assume  $N$  identical (symmetric) renewable producers. Let us define our inequality constraint for producer  $i$  by

$$g(q_{ir1}, q_{ir2}) := q_{ir1} + q_{ir2} - Q_i \leq 0 \quad (\text{B.1})$$

Then the Lagrange function is

$$\begin{aligned} L(q_{ir1}, q_{ir2}, \lambda) := & q_{ir1} (a_1 (D - q_{ir1} - q_{jr1} (N - 1)) + b) + \\ & q_{ir2} (a_2 (D - q_{ir1} - q_{ir2} - q_{jr1} (N - 1) - q_{jr2} (N - 1))) + \\ & q_{ir2} (b + (a_1 - a_2) (D - q_{ir1} - q_{jr1} (N - 1))) \int f_i(Q_i) dQ_i + \\ & \lambda (Q_i - q_{ir1} - q_{ir2}), \end{aligned} \quad (\text{B.2})$$

which is the corresponding profit function of the first and second stage minus the function  $g$ . The conditions of the KKT which need to be fulfilled are

$$\text{Stationarity: } \frac{\partial L}{\partial q_{irk}} = 0, \quad k = \{1, 2\} \quad (\text{B.3})$$

$$\text{Primal feasibility: } q_{ir1} + q_{ir2} \leq Q_i \quad (\text{B.4})$$

$$\text{Dual feasibility: } \lambda \geq 0 \quad (\text{B.5})$$

$$\text{Complementary slackness: } \lambda(q_{ir1} + q_{ir2} - Q_i) = 0. \quad (\text{B.6})$$

We need to consider two cases:  $\lambda = 0$  or  $q_{ir1} + q_{ir2} = Q_i$  (binding capacity constraint).

To case 1 ( $\lambda = 0$ ):

From (B.3) we derive two equations which we can solve for  $q_{ir1}$  and  $q_{ir2}$ . Since we focus on symmetric probability distribution functions  $f_i$  for the renewable production, we can substitute  $\int f_i(Q_i) dQ_i = 1$ . Furthermore, due to symmetric renewable producers, we can plug in  $q_{ir1} = q_{jr1}$  and  $q_{ir2} = q_{jr2}$  for all renewable producers  $i$  and  $j$ . Therefore, the equilibrium solution aggregated for all identical renewable producers are

$$q_{r1}^* = \frac{a_2 N (N+1) - a_1 N}{a_2 (N+1)^2 - a_1 N} D + \frac{1}{a_1} \frac{a_2 N (N+1) - a_1 N}{a_2 (N+1)^2 - a_1 N} b \quad (\text{B.7})$$

$$q_{r2}^* = \frac{a_1 N}{a_2 (N+1)^2 - a_1 N} D + \frac{N}{a_2 (N+1)^2 - a_1 N} b. \quad (\text{B.8})$$

Note that the individual quantities are  $q_{irk} = q_{rk}/N$  for  $k = \{1, 2\}$ .

Now, we can plug the optimal quantities into the equation of the investigated case, i.e. into  $q_{r1} + q_{r1} < Q$ . This gives us the threshold value above which the renewable producers start to withhold production to increase prices. The threshold is

$$Q_{threshold} := \frac{a_2 N (N+1)}{a_2 (N+1)^2 - a_1 N} D + \frac{a_2 (N+1)}{a_1 (a_2 (N+1)^2 - a_1 N)} b. \quad (\text{B.9})$$

If the overall expected renewable production  $\mu_q$  exceeds this threshold, the renewable producers withhold production. Otherwise, the sold quantities are constraint and we are in case 2.

Note that the expected production has to reach a high level relative to the demand such that renewable producers withhold production.  $\mu_q$  has to be at least  $\frac{D}{2}$  (for the monopoly situation with a infinite inflexible power plant fleet) but increases with increasing number of players or more flexible power plant fleet (for a duopoly it is at least  $\frac{2D}{3}$ ).

To case 2 ( $q_{ir1} + q_{ir2} = Q_i$ ): This is the same case as shown in proposition 4. Therefore the optimal quantities for each individual renewable producer is

$$q_{ir1}^* = \frac{1}{N+1} \left( N+1 - \frac{a_1}{a_2} \right) \mu_{iq} \quad (\text{B.10})$$

$$q_{ir2}^* = \frac{1}{N+1} \frac{a_1}{a_2} \mu_{iq}. \quad (\text{B.11})$$

and for all renewable producers together are

$$q_{r1}^* = \mu_q - \frac{a_1}{a_2(N+1)} \mu_q \quad (\text{B.12})$$

$$q_{r2}^* = \frac{a_1}{a_2(N+1)} \mu_q \quad (\text{B.13})$$

if  $\mu_q \leq \frac{a_2(N+1)}{a_2(N+1)^2 - a_1 N} D + \frac{a_2(N+1)}{a_1(a_2(N+1)^2 - a_1 N)} b$ . Remember that  $N\mu_{qi} = \mu_q$ . This closes the proof.  $\square$