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Abstract

A Cournot oligopoly in which firms face incomplete information with respect to production capacities is studied. For the case where the firms' capacities are stochastically independent, the functional form of equilibrium strategies is derived. If inverse demand is concave, a unique symmetric equilibrium exists, and if demand is linear, then every equilibrium is symmetric. In the case of duopoly, the impact on social welfare when firms commit ex-ante on exchanging information is analyzed. Sharing information increases expected output and social welfare in a large class of models. If the demand intercept is sufficiently large, sharing information increases producer surplus and decreases consumer surplus (and vice versa).

Keywords: Oligopoly, Incomplete Information, Cournot, Capacity Constraints, Information Sharing

JEL classification: C72, D43, L13

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1 Introduction

Previously conducted research on Bayesian-Cournot oligopolies deals with incomplete information with respect to inverse demand or production costs or both. In the paper at hand, a model in which the firms' production capacities are private information to the firms is analyzed. Models of this kind are not yet included in research concerning Bayesian Cournot oligopolies. In the well-known case where costs are unknown, the cost function is typically assumed to be convex; therefore, the model frameworks developed to deal with this source of uncertainty can not be applied to the case of unknown capacities (via production costs approaching infinity as output approaches the capacity limit). Alternatively, capacity constraints can be modeled via a penalty payment embedded in the firms' payoff function, such that firms receive a negative payoff if capacities are exceeded. However, this might destroy the quasi-concavity of the expected payoff function and thus may lead to the non-existence of equilibria.

Instead, we model a firm's capacity restriction by curtailing a firm's strategy space, ensuring that the existence of an equilibrium is implied by Nash's theorem under standard assumptions on inverse demand and costs if the common prior is finite. For the case where capacities are stochastically independent and the common prior may be infinite, we characterize the functional form of equilibrium strategies: In every equilibrium firms fully utilize their capacities up to some threshold. If the capacity with which firms are endowed exceeds this threshold, then firms produce a constant quantity that equals the inner maximum of the expected payoff function. This implies that a firm's strategy space is essentially one-dimensional.

Under the additional assumption that demand is strictly concave and that the firms' capacities are identically distributed, we show that a unique symmetric equilibrium exists. The expected output of the industry is smaller compared to the output of the standard form Cournot oligopoly. If demand is linear, we find that every equilibrium must be symmetric. This is because production decisions are strategic substitutes and each firm's best response function only depends on the expected aggregate output of the other firms.

For the special case of two firms and a simple common prior belief, we analyze the impact of information sharing on producer surplus, consumer surplus and social welfare. This is done by comparing the unique symmetric equilibrium when information is incomplete, the *private information equilibrium*, with the equilibrium of the corresponding complete information game, the *shared information equilibrium*.

In order to calculate expected profits and outputs, an explicit characterization of equilibrium strategies is required. However, due to non-negativity and capacity constraints standard techniques do not apply to derive closed-form solutions of equilibrium strategies. Therefore, we access the impacts on surplus and welfare by using inequality arguments.

While consumers benefit from an increase of expected output, they suffer from a decrease of the variance of outputs. Since under complete information equilibrium outputs are negatively correlated, the variance of total industry output is potentially reduced. We find that the net effect, which determines whether sharing information is beneficial for consumers, is ambiguous: While consumers benefit from sharing information when the horizontal demand intercept a is small, they increasingly suffer from information sharing when a increases. Thus, the change in consumer surplus is positive for small values of a and negative for sufficiently large values. This effect is driven by a constellation in which both firms are endowed with a large amount of capacity, thus leading to an "overproduction" under incomplete information as the total industry output exceeds the Cournot output. Due to information sharing, firms reduce their output accordingly. In contrast, the change of producer surplus may be negative for small values of a and is positive if a is sufficiently large.

2 Literature Review

Regarding Cournot oligopolies with complete information, a number of authors analyze equilibrium existence and uniqueness when production capacities are bounded and asymmetric. For example, Bischi et al. (2010) and Okuguchi and Szidarovszky (1999) discuss a wide range of oligopoly models and provide results on existence and uniqueness of equilibria. As in the paper at hand, production capacity is modeled by curtailing the strategy spaces.

For the case of incomplete information, Einy et al. (2010) provide a general framework of Bayesian-Cournot games and provide results of existence and uniqueness of Bayes-Nash equilibria. They allow for incomplete information with respect to the demand function as well as with respect to the cost function. However, the case of unknown capacities is not covered, and the model framework can not be applied.

The work on information sharing in oligopoly was pioneered by Novshek and Sonnenschein (1982), Clarke (1983) and Vives (1984). Novshek and Sonnenschein (1982) and Vives (1984) discuss a duopoly with uncertain linear demand, whereas Clarke (1983) analyzes an oligopoly where both demand and costs may be unknown. Raith (1996) provides a general model that allows for Bertrand or Cournot competition and incomplete information with respect to costs or demand. If parameters are specified in an appropriate way, virtually all models on information sharing in oligopoly follow in special cases. As Clarke (1983), Raith (1996) applies a general result provided by Radner (1962): If the joint distribution of private values is normal, then equilibrium strategies are affine. Raith (1996) shows that if firms' signals are independent private values and if each firm perfectly learns its private value, meaning that no noise is added, then industry-wide information sharing is always profitable for firms.

However, equilibrium strategies are not affine in our model. In the case of uncertain demand, Maleug and Tsutsui (1998) find that standard results on information sharing can be reversed if equilibrium strategies are not affine.

Here, non-linearity stems from non-negativity constraints or capacity limits. In particular, if demand is uncertain, consumer surplus can decrease although firms have an incentive to share information – in contrast, Raith (1996) and the literature cited therein find that firms do not have an incentive to share information when demand is uncertain, but consumers would profit from sharing information. Moreover, Maleug and Tsutsui (1998) demonstrate that information sharing is profitable as long as the the variation of demand is sufficiently large. In the case of uncertain costs, Shapiro (1986) finds that firms have an incentive to share information, and that information sharing increases social welfare but decreases consumer surplus.

In the work at hand, we also provide a result complementary to Maleug and Tsutsui (1998). We find that firms might not have an incentive to share their information, but consumers would benefit from sharing. Moreover, if the variance of the firms' capacities is sufficiently large, then firms have an incentive to share their information.

The remainder of the paper is structured as follows: Section 3 presents the model framework. Section 4 presents the private information equilibrium and provides a detailed characterization of the symmetric equilibrium strategy. Results on the impact of information sharing on surplus and welfare are derived in Section 5. Lastly, Section 6 concludes.

3 The Model

We consider a set $N = \{1, 2, \dots, n\}$ of firms that may face uncertainty regarding the other firm's endowment with production capacity. Firms only differ with respect to their production capacities. In a Bayesian approach, a strategy of firm i is a decision rule that specifies a firm's output for every possible information set with which the firm might be endowed.

More formally, we denote $T = [0, \hat{t}] \subseteq [0, \infty]$ as the set of possible capacity levels and $\Omega = \prod_{n \in N} T$ as the set of possible states of nature. The common prior belief μ is a probability measure on Ω (with respect to some appropriate σ-field). An element of Ω is denoted by $\omega = (\omega_1, \omega_2, \ldots, \omega_n)$. We assume that every firm is endowed with a production capacity exceeding zero with positive probability. The information with which a firm is endowed when making its output decision is described by a random variable $T_i : \Omega \to \Omega_i$, where Ω_i is chosen appropriately. Moreover, we assume that $E\left[|T_i|\right] < \infty$ for all i. The information sets of firm i are then the elements of the $\sigma-$algebra $\sigma(T_i)$ generated by T_i.[1] A strategy is a Borel measurable and integrable function $q_i : \Omega_i \mapsto \mathbb{R}_+$ satisfying $q_i(T_i(\omega)) \leq \omega_i$.[2] Lastly, the strategy space of firm i is denoted by S_i and the space containing all strategy profiles is given by $S = \prod_{i=1}^n S_i$.

As defined above, $q_i(T_i(\omega))$ denotes the output of firm i. We let $Q(\omega) := \sum_{i=1}^n q_i(T_i(\omega))$ denote the overall production. The inverse demand function and the cost function are denoted by P and C, respectively. The *state-dependent payoff function* u_i of firm i is given by

$$u_i(\omega, q_i, q_{-i}) = q_i(T_i(\omega))P(Q(\omega)) - C(q_i(T_i(\omega))). \tag{1}$$

The strategy profile $q \in S$ is a *Bayesian Cournot equilibrium* if for every i and $\tilde{q}_i \in S_i$ the *expected payoff function* is maximized,

$$E\left[u_i\left(\cdot, q_i, q_{-i}\right)\right] \geq E\left[u_i\left(\cdot, \tilde{q}_i, q_{-i}\right)\right], \tag{2}$$

meaning that in equilibrium, no firm has an incentive to unilaterally deviate from its strategy. Maximizing (2) is equivalent to maximizing the *conditional payoff expectation*, so that

$$E\left[u_i\left(\cdot, q_i, q_{-i}\right)\middle|\sigma\left(T_i\right)\right](\omega) \geq E\left[u_i\left(\cdot, \tilde{q}_i, q_{-i}\right)\middle|\sigma\left(T_i\right)\right](\omega) \tag{3}$$

[1] Following Einy et al. (2002), this is equivalent to the model by Harsanyi (1967-69) because each firm's σ-algebra is generated by a partition of Ω that is given by $\Pi_i = \{T_i^{-1}(\tilde{\omega}) | \tilde{\omega} \in \Omega_i\}$.

[2] Integrable means that $\int_\Omega |q_i(T_i(\omega))| d\mu < \infty$.

for all $i \in N$ and almost all $\omega \in \Omega$.[3]

Throughout the paper, we assume:

(A) The cost function C is convex, twice continuously differentiable and there are no fixed costs, meaning that $C(0) = 0$,

(B) Inverse demand P is nonincreasing and twice continuously differentiable,

(C) There exists $Z < \infty$ such that $qP(q) - C(q) \leq 0$ for all $q \geq Z$,

(D) The marginal revenue of firm i is strictly decreasing with the aggregate output of the other firms. This is equivalent to $P'(Q) + q_i P''(Q) < 0$ (the so-called *Novshek condition*, see Vives (1999)). Notice that (B) and (D) imply that P is strictly decreasing.

Remark 1. *If $\mu\left(T_i \geq Z\right) = 1$, the model reduces to a standard form Cournot oligopoly with complete information in which firms face the capacity constraint Z, which is never exceeded due to assumption (C). In this case, assumptions (A) and (D) ensure the existence of a unique equilibrium (Vives, 1999). Throughout the paper, we denote the corresponding standard form Cournot oligopoly equilibrium quantity by q^C and the corresponding best response function by r.[4] Under assumptions (A), (B) and (D), the best response r is twice continuously differentiable and $r' > -1$ (Vives, 1999), meaning that production decisions are strategic substitutes.*

Remark 2. *Assumptions (A), (B) and (D) ensure that the state-dependent payoff function (1) is concave in the output of firm i. Moreover, concavity is inherited by the expected payoff function (2) (Einy et al., 2010). If Ω is finite, then a firm's strategy space is compact and convex, and Nash's theorem implies the existence of an equilibrium.*

Notice that we allow for negative prices in the model, which is arguable from an economic point of view but which is helpful when it comes to proving existence of equilibria. If demand is truncated where it intercepts the horizontal axis in order to avoid negative prices, a firm's payoff function is no

[3] See Harsanyi (1967-69) and Einy et al. (2002).

[4] Thus, we implicitly assume $Z \leq \hat{t}$, which is not a limitation.

longer concave but only quasi-concave. This is not a problem in the complete information case, however the argument for equilibrium existence may collapse if we allow for incomplete information regarding the demand intercept. In this case, the quasi-concavity of the state-dependent payoff function does not necessarily translate into quasi-concavity of the expected payoff function (Einy et al., 2010). The same is true in the case where firms have incomplete information regarding the other firms' capacities and where demand is truncated. In contrast, allowing for negative prices ensures that the expected payoff function is concave.

4 Characterization of Equilibrium Strategies

First, we reconsider the case in which firms have asymmetric capacity constraints and share their information, meaning they are subject to complete information. The question of existence and uniqueness in this setting is treated extensively in the literature, as previously mentioned. In terms of the model formulation, we discuss the case where $T_i(\omega) = \omega$ for all i and all ω. If demand and costs are linear, then existence and uniqueness of an equilibrium are easily obtained.

In the remainder of the paper, we denote the shared information equilibrium strategy by q^S. For the duopoly case, the shared information equilibrium strategy q^S can be presented in a compact manner. As previously mentioned, r denotes the best response function of the unrestricted Cournot duopoly.

$$q^S(\omega_1, \omega_2) = \begin{cases} \min\left\{\omega_1, q^C\right\}, \text{ if } \omega_1 \leq \omega_2, \\ \min\left\{\omega_1, r\left(q^S(\omega_2, \omega_1)\right)\right\} \text{ otherwise.} \end{cases} \tag{4}$$

It is easily demonstrated that q^S is the unique equilibrium strategy. We use this representation of q^S in Section 5.

Notice that if in an equilibrium there is a firm with a binding capacity restriction, the total output of the industry is lower compared to the output of the standard form Cournot oligopoly. This property derives from the slope of the best response function r – if one firm decreases its output due to its capacity restriction, then the corresponding increase of the other firms is smaller.

In the private information setting, every firm perfectly learns its own capacity but receives no information about the other firms' capacities. Speaking in terms of the model, we analyze the case $T_i(\omega) = \omega_i$. A strategy of firm i is now a function on T. In the following, we write $q_i(t)$ instead of $q_i(T_i(\omega))$.

Theorem 1 states that an equilibrium strategy q_i is completely determined by $q_i(\hat{t})$ if the firms' capacities are independent. That is, the relevant strategy space is one-dimensional.

Theorem 1. *If the firms' capacities are stochastically independent, then in every equilibrium $q = (q_1, q_2, \ldots, q_n)$ and for every firm i the strategy q_i is nondecreasing. More precisely, for every i there exists a threshold $s_i \in T$ such that $q_i(t) = t$ for all $t \leq s_i$ and $q_i(t) < t$ for all $t > s_i$.*

Proof. We assume that q is an equilibrium and choose $i \in N$ arbitrarily. If $q_i(t) = t$ for all $t \in T$, then the proposed statement follows. Therefore, we denote s_i as the infimum of the set $\{t \in T | q_i(t) < t\}$. If $t, u \in T$ so that $u > t > s_i$, we must have $q_i(u) = q_i(t) < t$ since $q_i(t)$ maximizes the conditional payoff expectation (3), which is concave, and because $q_i(t)$ lies in the inner of $[0, t]$, implying that $q_i(t)$ is the global maximum. Notice that either $q_i(t) = q_i(s) < s$ or $s = q_i(s) < q_i(t)$. □

The result of Theorem 1 is driven by the independence of $T_1, T_2, \ldots, T_n$ and does not generally hold, as shown in the following example. We consider a duopoly in which the inverse demand function is given by $P(q) = 2 - q$. The set of possible capacity levels equals $T = \{0, 1, 2\}$. We assume that μ is symmetric, meaning that for all ω_1, ω_2

$$\mu\left(T_1 = \omega_1, T_2 = \omega_2\right) = \mu\left(T_1 = \omega_2, T_2 = \omega_1\right).$$

Moreover, we assume that $\mu\left(T_1 = 0|T_2 = 1\right) = 1$ and $\mu\left(T_1 = 2|T_2 = 2\right) = 1$.[5] Then, the unique symmetric equilibrium is given by

$$\begin{aligned} q(0) &= 0, \\ q(1) &= 1, \\ q(2) &= 2/3. \end{aligned}$$

The equilibrium strategy is neither increasing nor decreasing. In fact, when allowing for an arbitrary common prior belief, then we can say nothing about the shape of the equilibria.

Recall that $\hat{t}$ denotes the maximal element in T. Theorem 1 states that a firm's equilibrium strategy q_i is completely determined by $q_i(\hat{t})$, since $q_i(t) = \min\{t, q_i(\hat{t})\}$. If we restrict the analysis to symmetric equilibria and assume that the firms' capacities are identically distributed, then the space of feasible strategy profiles becomes one-dimensional. Next, we show that there exists a unique symmetric equilibrium if the inverse demand function is concave. We use two arguments in the proof: A fixed point argument applied to the one-dimensional space of feasible strategy profiles described above and the existence of a unique Cournot equilibrium in the unrestricted, standard form Cournot oligopoly, characterized by a smooth best reply function (see Remark 1). In order to ease notation, we write $r(q)$ instead of $r((n-1)q)$ if q is a symmetric equilibrium strategy or quantity in the remainder of the paper.

Theorem 1 shows that a firm produces some constant output $q(\hat{t})$ if the capacity level the firm is endowed with exceeds a certain threshold. In Theorem 2 we show that this constant output exceeds the Cournot quantity q^C, but is smaller than the monopoly quantity q^M. We characterize $q(\hat{t})$ via

$$q^C < q(\hat{t}) = r(\lambda q^C) < q^M$$

[5]This specification of the conditional probabilities implies that firms have complete information. However, this is just for convenience. We obtain similar results if we allow for the conditional probabilities to be close to 1.

for an appropriate $0 < \lambda < 1$.

Clearly, $q(\hat{t})$ does not exceed the monopoly quantity, implying $\lambda > 0$. To encourage intuition why $q(\hat{t})$ exceeds the Cournot quantity, implying $\lambda < 1$, we consider a duopoly in which inverse demand is given by $P(q) = 1 - q$ and in which marginal costs are equal zero. Every firm's capacity may take values in $T = \{0, 1\}$, and each capacity level occurs with probability $p = 1/2$. If firm 1 is endowed with capacity 1, it maximizes

$$E\left[q_1(1 - q_2 - q_1)\right] = q_1(1 - E\left[q_2\right] - q_1)$$

subject to $q_1 \leq 1$. We let $\tilde{r}$ denote the best response function of firm 1. At equilibrium,

$$q_1(1) = \tilde{r}(q_2) = \frac{1 - E\left[q_2\right]}{2} = \frac{1 - \frac{1}{2}q_2(1)}{2},$$

because $q_2(0) = 0$. Since $q_1(1) = q_2(1)$ in a symmetric equilibrium, we obtain $q_1(1) = 2/5 > 1/3 = q^C$. The equilibrium strategy is then

$$q_1(t) = \min\{t, 2/5\},$$

which may be written as

$$q_1(t) = \min\left\{t, r\left(\frac{3}{5}q^C\right)\right\}.$$

In short, $q_1(1) \leq q^C$ implies $E\left[q_1\right] < q^C$, so that

$$r(q_2) = r(q_1) = r(E\left[q_1\right] > r(q^C) = q^C.$$

Theorem 2. *If capacities are i.i.d. and the inverse demand function is concave, there exists exactly one symmetric equilibrium and the equilibrium strategy* q^P *satisfies*

$$E\left[q^P\right] \leq q^C.$$

The inequality strictly holds if $\mu\left(T_i < q^C\right) > 0$.

Proof. We construct a symmetric equilibrium. Recall that firm i maximizes

$$E\left[u_i\left(\cdot, q_i, q_{-i}\right)\right] = E\left[q_i(T_i(\cdot))P(Q(\cdot)) - C(q_i(T_i(\cdot)))\right] \tag{5}$$

by choosing q_i. For every $t \in T$ and $\lambda \in \mathbb{R}$ we define the strategy q^λ by

$$q^\lambda(t) = \min\left\{t, r\left(\lambda q^C\right)\right\}. \tag{6}$$

Then, $q^\lambda(t)$ is continuous in λ since r is smooth (see Remark 1). We assume that the other firms $j \neq i$ apply q^λ for some $\lambda \in [0,1]$ and define

$$Q^\lambda_{-i}(\omega) := \sum_{j \neq i} q^\lambda(T_j(\omega)).$$

to be the corresponding, realized aggregate output, which is nonincreasing and continuous in λ. Consider the mapping $\phi : \mathbb{R}^2 \to \mathbb{R}$ defined by

$$\begin{aligned}\phi(\lambda, x) :=& E\left[u_i\left(\cdot, x, q^\lambda\right)\right] \\ =& xE\left[P\left(Q^\lambda_{-i}(\cdot) + x\right)\right] - C(x).\end{aligned}$$

Then ϕ is continuous in λ as well. Due to the assumptions placed on P and C, the integrand

$$xP\left(Q^\lambda_{-i}(\omega) + x\right) - C(x)$$

is strictly concave in x (see Remark 2) and this implies that ϕ is concave in x as well. We let $\gamma(\lambda)$ denote the global maximizer of $\phi(\lambda, \cdot)$. Then γ is strictly decreasing with λ because Q^{λ}_{-i} is strictly decreasing in λ,[6] and this implies that the maximizer γ must increase since we consider an oligopoly with strategic substitutes.

We prove indirectly that γ is continuous: Assume that γ has a discontinuity in μ. Then, since γ is nondecreasing, there exists an $\epsilon > 0$ and a sequence $\mu_n > \mu$ converging to μ such that

$$\gamma(\mu_n) - \gamma(\mu) > \epsilon \tag{7}$$

for all $n \in N$. Because $\gamma(\mu)$ maximizes $\phi(\mu, \cdot)$, which is strictly concave, we conclude

$$\phi(\mu, \gamma(\mu)) > \phi(\mu, \gamma(\mu) + \epsilon).$$

Because the sequence μ_n converges to μ and ϕ is continuous in its first argument, we can choose n^* large enough so that

$$\phi(\mu_{n^*}, \gamma(\mu)) > \phi(\mu_{n^*}, \gamma(\mu) + \epsilon).$$

This implies that $\gamma(\mu_{n^*}) < \gamma(\mu) + \epsilon$, since ϕ is strictly concave in its second argument (and thus continuous as well). But this yields

$$\gamma(\mu_{n^*}) - \gamma(\mu) < \epsilon,$$

contradicting (7).

Next, we demonstrate that there exists $\lambda > 0$ such that $\gamma(\lambda) = r(\lambda q^C)$ by applying the intermediate value theorem. If $\lambda = 0$, then $r(\lambda q^C) = r(0) = q^M$, where q^M is the monopoly output. Clearly, we must have $\gamma(0) < q^M$: $\gamma(0)$ is the maximizer of

$$xE\left[P\left(Q^0_{-i}(\cdot) + x\right)\right] - C(x)).$$

[6]More precisely, there exists a set $A \subset \Omega$ such that $Q^{\lambda}_{-i}(\omega)$ is strictly decreasing for almost all $\omega \in A$ and constant almost everywhere on A^c. In a non trivial setting, $\mu(A) > 0$, which is sufficient because γ does only depend on the expected value of Q_{-i}.

Since the inverse demand function is concave by assumption, we may apply Jensen's inequality and obtain

$$E\left[P\left(Q^0_{-i}(\cdot)+x\right)\right] \le P\left(E\left[Q^0_{-i}\right]+x\right) < P(x),$$

meaning that the expected price is smaller than the monopoly price for any x, implying that $\gamma(0) < q^M$.

Similarly, if $\lambda = 1$, then $r(\lambda q^C) = r(q^C) = q^C$, and $\gamma(1)$ exceeds q^C: The expected price satisfies

$$E\left[P\left(Q^0_{-i}(\cdot)+x\right)\right] \ge E\left[P\left((n-1)q^C+x\right)\right] = P\left((n-1)q^C+x\right),$$

implying that the expected price exceeds the price of the unrestricted Cournot oligopoly for any x and further that $\gamma(1)$ must exceed q^C. Since both r and γ are continuous, we conclude that there exists a λ as claimed. Notice that the inequality above strictly holds if $\mu\left(T_i < q^C\right) > 0$.

Lastly, we denote $\tilde{r}(t,\cdot)$ as the best response of the restricted oligopoly when $T_i = t$, meaning that $\tilde{r}$ maximizes $E\left[u_i\left(\cdot, x, q_{-i}\right)\right]$ subject to $x \le t$. When $q_j = q^\lambda$ for $j \ne i$, we obtain

$$\tilde{r}(t,q^\lambda) = \min\left\{t,\gamma(\lambda)\right\} = \min\left\{t, r(\lambda q^C)\right\} = q^\lambda(t).$$

This shows that q^λ is a fixed point of the best response function.

□

Ultimately, the result established in Theorem 2 stems from the fact that production decisions are strategic substitutes in the model setting (see Remark 1). If a firm's output is bounded with positive probability, then the remaining firms (state-wise) do not fully compensate this lack of production. It is easily verified that a similar result holds in the case complete information. Thus, Theorem 2 is a natural analog to the complete information case.

Remark 3. *Notice that if demand is linear, it follows* $\gamma(\lambda) = r\left(E\left[q^\lambda\right]\right)$*. This is because the expected payoff of firm* i *only depends on the expected aggregate output of the other firms. Since* $\gamma(\lambda) = r(\lambda q^C)$ *in the equilibrium, we conclude* $E\left[q^\lambda\right] = \lambda q^C$.

Since a firm's strategy is of the form $q_i(t) = \min\{t, q_i(\hat{t})\}$, the strategy is completely determined by its expected value, which is strictly increasing with $\hat{t}$. That is to say, a firm's decision variables are one-dimensional and the best response is of the form $\tilde{r}(t, Q_-) = \min\{t, r(Q_-)\}$ and thus depends only on the aggregate output. Under these conditions, only symmetric equilibria can exist if the slope of $\tilde{r}$ strictly exceeds -1.[7] In our case, $r' > -1$ (see Remark 1) and in fact, $r' > -1/2$ when demand is linear. Conversely, Theorem 3 may not hold if demand is not linear.

Theorem 3. *If capacities are i.i.d. and demand is linear, then every equilibrium is symmetric.*

Proof. First, we give a proof for the duopoly case. Second, we argue why the statement also holds true in an oligopoly. For an arbitrarily chosen equilibrium $q = (q_1, q_2)$ it is sufficient to show that $E[q_1] = E[q_2]$ due to Theorem 1. We define $x := E[q_1]$, $y := E[q_2]$ and

$$\phi_2(z) = E\left[u_2\left(\cdot, z, q_1\right)\right] = zP\left(E\left[q_1\right] + z\right) - C(z).$$

Clearly, ϕ_2 is maximized by $r\left(E\left[q_1\right]\right) = r(x)$ because the expected payoff function of firm 2 does only depend on the expected quantity of firm 1 as a result of the linearity of P.

We let f denote a marginal probability density with respect to T_i, meaning that f is such that for all $c \in T$

$$\mu\left(T_i \leq c\right) = \int_0^c f(t)dt.$$

[7] See Vives (1999), p. 42–43, who discusses the complete information case.

We write

$$y = E\left[\min\left\{T_2, r(x)\right\}\right] \tag{8}$$

$$= \int_0^{r(x)} t f(t)dt + \int_{r(x)}^{\infty} r(x) f(t)dt \tag{9}$$

$$=: g(x). \tag{10}$$

Similarly, we conclude $x = g(y)$.

Next, we demonstrate that $g(x) = y$ and $g(y) = x$ implies $x = y$, which yields the given statement. The strategy is to show that $g' > -1$, implying that g can not intersect a linear function with derivative -1 twice; however, this is a necessary condition for the existence of $x \neq y$ satisfying $g(x) = y$ and $g(y) = x$. We calculate[8]

$$g'(x) = r'(x)r(x)f(r(x)) + r'(x)\int_{r(x)}^{\hat{t}} f(t)dt - r(x)r'(x)f(r(x))$$

$$= r'(x)\mu\left(T_i \geq r(x)\right) > -1. \tag{11}$$

We suppose that there exist $0 \leq x < y$ such that $g(x) = y$ and $g(y) = x$. We define the linear function h by $h(z) = x + y - z$. Then $h(x) = y$ and $h(y) = h(h(x)) = x$. On one hand, this implies that h intersects g at x and y, so that

$$g(x) - h(x) = g(y) - h(y) = 0. \tag{12}$$

On the other hand, $g' - h' > 0$, implying that $g - h$ is strictly increasing – a contradiction to (12).

It is important to notice that the proof does not rely on the demand function parameter a and b. This implies that for the oligopoly case we can define $\tilde{a} = a - b\sum_{j>2} E[q_j(T_j)]$ and apply the duopoly result to the residual demand

[8]If the common prior belief is discrete, then g is piecewise linear and thus differentiable almost everywhere.

function defined by $\tilde{a}$ (which is the same for both firm 1 and firm 2). Equivalently speaking, we proved that the firms' strategies are pair wise identical for any a, which is sufficient to prove the statement for the oligopoly case. □

The result is driven by by the linearity of the demand function: If demand is linear, then the best response function of a firm does only depend on the expected output of the other firms. Thus, the maximizer of a firm's payoff function inherits the slope of the Cournot best response r to some extent (see equation (11)).

Lastly, Theorem 3 implies the existence of a unique symmetric equilibrium in the linear case: g has exactly one fixed point, and the fixed points of g correspond to symmetric equilibria. Figure 1 shows the symmetric equilibrium of the oligopoly for the case in which demand is linear and the common prior belief is discrete and uniformly distributed.

5 Information Sharing

We discuss the effects of information sharing on producer surplus, consumer surplus and social welfare. We consider the two extreme cases in which information is not shared at all, the *private information equilibrium*, and where firms commit ex-ante to an industry-wide information sharing agreement, e.g. via some trade association, the *shared information equilibrium*. We find that even in simple examples, the impact on both consumer and producer surplus is ambiguous. This ambiguity is driven by the concavity of the firms' payoff function and by the covariance of firms' equilibrium outputs. For a large class of examples, social welfare increases. However, we provide an example where social welfare decreases.

In the following, we discuss a simple duopoly. The common prior belief is discrete and there are two possible capacity levels $t_L < t_H$ that may each occur with probability $p = 1/2$. The symmetry of the common prior belief is just for convenience – the results are not driven by this assumption. Without loss of

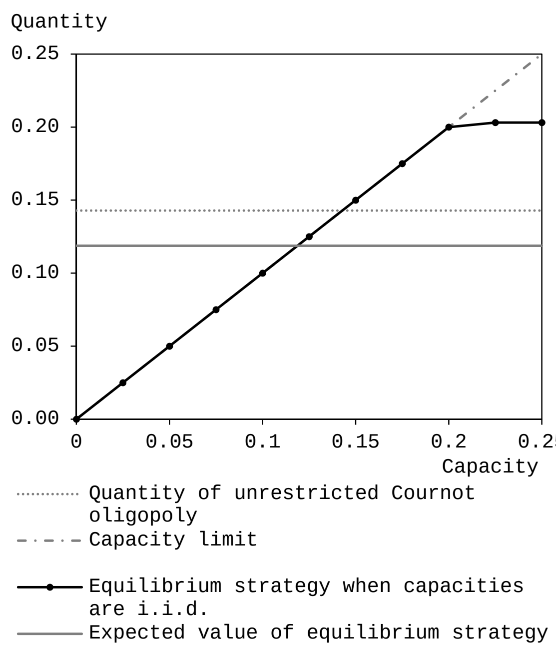

Figure 1: The unique symmetric equilibrium when capacities are stochastically independent and uniformly distributed ($a = 1$, $b = 1$, $c = 0$, $n = 6$, $\hat{t} = 0.25$, $|T| = 11$, $\lambda = 0.83$).

generality, we assume that costs equal zero and that inverse demand equals $p(q) = a - q$.[9] To avoid trivialities, we assume throughout the analysis that $t_L \leq q^C$, which is equivalent to $a \geq 3t_L$. In the limiting case $a = 3t_L$ the model reduces to a standard form Cournot duopoly in which both firms produce their Cournot quantity in both the private and shared information equilibrium.

Moreover, we assume that t_H is sufficiently large, meaning that t_H exceeds the monopoly output. This assumption simplifies the analysis, but we are still able to demonstrate the ambiguous effects on producer and consumer surplus. In contrast, under this assumption, social welfare increases when information is shared.

In this model specification we obtain (see Remark 3)

$$t_L < E\left[q^P\right] < r\left(E\left[q^P\right]\right).$$

This leads to

$$E\left[q^P\right] = p t_L + p r\left(E\left[q^P\right]\right).$$

Substituting $p = 1/2$ and solving for $E\left[q^P\right]$ yields

$$E\left[q^P\right] = \frac{2}{5}t_L + \frac{3}{5}q^C = \frac{2}{5}t_L + \frac{1}{5}a, \tag{13}$$

$$r\left(E\left[q^P\right]\right) = \frac{6}{5}q^C - \frac{1}{5}t_L = \frac{2}{5}a - \frac{1}{5}t_L. \tag{14}$$

We can see that the model becomes trivial if $a = 3t$: In this case, we obtain $q^C = t = E\left[q^P\right]$.

[9]If $c > 0$, we define $\tilde{a} = a - c$; if $b \neq 1$, we define $\overline{a} = \tilde{a}/b$. The payoff function of the case $b \neq 1$ is then a scaled version of the payoff function arising when demand equals $\overline{a} - q$. The same holds for consumer surplus.

5.1 Producer Surplus

Since both the shared and the private information equilibria are symmetric, firms have an incentive to share their information if and only if sharing information increases producer surplus (PS). Producer surplus equals the expected profit of the industry. Thus, firms have an incentive to share their information if the difference

$$E[\Delta PS] := 2E\left[u_i\left(\cdot, q^S, q^S\right)\right] - 2E\left[u_i\left(\cdot, q^P, q^P\right)\right]$$

exceeds zero.

Information sharing may (ex-post) lead to losses for firm i if and only if its capacity restriction is binding and the capacity restriction of firm 2 is not binding in the private information equilibrium, allowing firm 2 to increase its output when learning that firm 1 produces little, and the other way round. If both firms' capacity restriction are not binding, then sharing information induces both firms to decrease outputs and thus increases profits. The net effect depends on the demand intercept a.

We derive the effects of information sharing on producer surplus by analyzing the possible states of nature separately. Ex-post, information sharing leads to losses for firm 1 if and only if $\omega_1 = t_L$ and $\omega_2 = t_H$. In this case, firm 2 produces $r\left(E\left[q^P\right]\right)$ in the private information equilibrium and $r(t_L) > r\left(E\left[q^P\right]\right)$ in the shared information equilibrium, whereas the output of firm 1 remains constant. If we combine the events (t_L, t_H) and (t_H, t_L) and multiply the expected difference of a firm's payoff by 2, the decrease in producer surplus arising from asymmetric capacities $PS^-(a)$ equals

$$PS^-(a) = 2t_L\left(a - r\left(E\left[q^P\right]\right) - t_L\right) - 2t_L\left(a - r(t_L) - t_L\right) = t_L(E\left[q^P\right] - t_L).$$

Via (13) and (14) we calculate

$$PS^{-}(a) = \frac{1}{5} a t_L - \frac{}{5} t_L^2$$

and

$$\frac{\partial}{\partial a} PS^{-}(a) = \frac{1}{10} t_L. \tag{15}$$

Thus, PS^{-} is linear and increasing. Clearly, if $a = 3t_L$, then $PS^{-}(a) = 0$.

Next, we examine two constellations that ex-post lead to an increase in producer surplus. The first is the counterpart of PS^{-}: If firm 1 is endowed with t_H and firm 2 is endowed with t_L, then firm 1 produces $r\left(E\left[q^P\right]\right)$ in the private information equilibrium and $r(t_L)$ in the shared information equilibrium. The output of firm 2 equals t_L in both equilibria. Again, we combine the events (t_L, t_H) and (t_H, t_L) and we denote the increase in producer surplus (when capacities are asymmetric) by $PS_1^{+}(a)$:

$$PS_1^{+}(a) = 2r(t_L)\left(a - t_L - r(t_L)\right) - 2r\left(E\left[q^P\right]\right)\left(a - t_L - r\left(E\left[q^P\right]\right)\right).$$

The expression PS_1^{+} has a zero at $a = 3t_L$. A straightforward calculation shows that

$$\frac{\partial}{\partial a} PS_1^{+}(a) = \frac{4}{25} a - \frac{12}{25} t_L. \tag{16}$$

This implies that PS_1^{+} is a parabola that has a local minimum at $a = 3t_L$.

Finally, both firms benefit (ex-post) from sharing information if $\omega_1 = \omega_2 = t_H$. In this case, both firms reduce their output to the Cournot quantity q^C when information is shared. The corresponding increase in producer surplus is denoted by PS_2^{+} and given by

$$PS_2^{+}(a) = 2q^C(a - 2q^C) - 2r\left(E\left[q^P\right]\right)(a - 2r\left(E\left[q^P\right]\right)).$$

Again, $PS_2^+(3t_L) = 0$. Moreover, a calculation shows that

$$\frac{\partial}{\partial a} PS_2^+(a) = \frac{140}{1125} a - \frac{6}{25} t_L. \tag{17}$$

Since

$$\frac{\partial}{\partial a} PS_2^+(3t_L) = \frac{2}{15} t_L > 0,$$

PS_2^+ is increasing as long as $a \geq 3t_L$.

All three events $(t_H, t_L), (t_L, t_H)$ and (t_H, t_H) occur with probability p^2, leading to

$$\begin{aligned} &E[\Delta PS](a) = \\ &p^2 \left(\frac{1}{2} PS_1^+(a) - \frac{1}{2} PS^-(a) \right) + p^2 \left(\frac{1}{2} PS_1^+(a) - \frac{1}{2} PS^-(a) \right) + p^2 PS_2^+(a) \\ =& p^2 \left(PS_1^+(a) + PS_2^+(a) - PS^-(a) \right). \end{aligned}$$

Using (15), (16) and (17), we obtain

$$\begin{aligned} \frac{\partial}{\partial a} E[\Delta PS](3t_L) =& p^2 \frac{\partial}{\partial a} \left(PS_1^+(3t_L) + PS_2^+(3t_L) - PS^-(3t_L) \right) \\ =& p^2 \left(0 + \frac{2}{15} t_L - \frac{2}{10} t_L \right) < 0. \end{aligned}$$

On one hand, since $E[\Delta PS](3t) = 0$ and $\partial / \partial a E[\Delta PS](3t) < 0$, we conclude that $E[\Delta PS](a) < 0$ if a is sufficiently small, meaning that firms do not have an incentive to share their information.

On the other hand, calculating the second derivative shows that

$$\partial / \partial^2 a E[\Delta PS] > 0.$$

This stems from the fact that PS^- is linear and implies $E[\Delta PS](a) > 0$ when a is sufficiently large. Thus, we have established:

Theorem 4. *If the demand intercept is sufficiently large, then firms have an incentive to exchange information.*

The result is driven by the concavity of the firms' payoff function. Consider that capacities are asymmetric and that $T_1 = t_L, T_2 = t_H$. Then firm 1 ex-post suffers from information sharing due to the price effect when firm 2 increases output, and theses losses are linear with respect to a. Conversely, firm 2 is subject to a price effect and a quantity effect. If a is large, then the quantity effect gains weight in a convex fashion, i.e. the marginal revenue of firm 2 is high (and vice versa).

5.2 Consumer Surplus

We let $Q(\omega, q^P)$ and $Q(\omega, q^S)$ denote the realized total output of the industry in the private information and shared information equilibria, respectively. We use $Q^2(\omega, q^P)/2$ and $Q^2(\omega, q^S)/2$ as a measure for consumer surplus. Sharing information leads to an increase in consumer surplus if and only if the expected difference

$$E\left[\Delta CS\right] = \frac{1}{2}E\left[Q^2(\cdot, q^S)\right] - \frac{1}{2}E\left[Q^2(\cdot, q^P)\right] \tag{18}$$

is positive.

Before we analyze the impact on consumer surplus, it is instructive to analyze the net quantity effect arising from information sharing. As performed in the last section, we can identify the states of nature that lead to a decrease or an increase of total output. A decrease can only occur if both firm 1 and firm 2 are endowed with t_H. We denote this quantity effect by Q^-. In this case, both firms produce $r\left(E\left[q^P\right]\right)$ in the private information equilibrium and $q^C < r\left(E\left[q^P\right]\right)$ in the shared information equilibrium. An increase, denoted by Q^+, occurs if both firms are endowed with different capacity levels: If firm 1 is endowed with t_L, then its outputs in both equilibria coincide. Firm

2 increases its output by $r(t_L) - r\left(E\left[q^P\right]\right)$. Notice that the same increase of output occurs if $\omega = (t_H, t_L)$.

The decrease in output (ex-post) amounts to

$$Q^- = 2r\left(E\left[q^P\right]\right) - 2q^C = q^C - E\left[q^P\right] = \frac{2}{15}a - \frac{2}{5}t_L.$$

The increase of output (ex-post), multiplied by 2, is given by

$$2Q^+ = 2\left(r(t_L) - r\left(E\left[q^P\right]\right)\right) = E\left[q^P\right] - t_L = \frac{1}{5}a - \frac{3}{5}t_L.$$

Both the increase and the decrease of output equal zero if $a = 3t_L$, as expected. Apparently, the expected difference of total output exceeds zero as long as $a > 3t_L$:

$$E\left[\Delta Q\right] = p^2Q^+ + p^2Q^+ - p^2Q^- = \frac{p^2}{15}a - \frac{p^2}{5}t_L.$$

That is to say, sharing information always leads to an increase in expected output. This stems from the fact that equilibrium strategies are concave. Shapiro (1986) finds that in the presence of uncertain costs and linear equilibrium strategies, a firm's output does not change when information is shared.

Moreover, the variance of a firm's output increases. By applying (13), we see that the increase in the output of firm 1 when firm 2 is endowed with t_L exceeds the decrease in output of firm 1 when firm 2 is endowed with t_H:

$$\begin{aligned}&\left(q^S(t_H, t_L) - q^P(t_H)\right) - \left(q^P(t_H) - q^S(t_H, t_H)\right)\\ =&r(t_L) - r\left(E\left[q^P\right]\right) - r\left(E\left[q^P\right]\right) + r\left(q^C\right)\\ =&\frac{1}{15}a - \frac{1}{5}t_L \geq 0 \end{aligned} \tag{19}$$

if and only if $a \geq 3t_L$. Since the output of firm 1 remains constant when endowed with t_L and since $q^S(t_H, t_L)$ exceeds $q^P(t_H)$, the variance of outputs of firm 1 increases due to information sharing. Because equation (19) increases with a, the increase of variance, in turn, increases with a.

In order to examine consumer surplus, we calculate the realized consumer surplus of the shared information equilibrium when both firm 1 and firm 2 are endowed with t_H:

$$CS^S(a, t_H, t_H) = \frac{\left(2q^C\right)^2}{2} = \frac{2}{9}a^2.$$

For the private information equilibrium, we find

$$CS^P(a, t_H, t_H) = \frac{\left(2r\left(E\left[q^P\right]\right)\right)^2}{2} = \frac{2}{25}\left(4a^2 - 4at_L + t_L^2\right).$$

The (ex-post) decrease in consumer surplus when both firms are endowed with t_H is then

$$CS^-(a) = CS^P(a, t_H, t_H) - CS^S(a, t_H, t_H) = \frac{2}{25}\left(\frac{11}{9}a^2 - 4at_L + t_L^2\right).$$

Similarly, if both firms have different capacity levels, we calculate the corresponding consumer surplus for both the shared and the private information equilibrium:

$$CS^S(a, t_L, t_H) = \frac{\left(t_L + r(t_L)\right)^2}{2} = \frac{1}{8}\left(a^2 + 2t_L a + t_L^2\right)$$

and

$$CS^P(a, t_L, t_H) = \frac{\left(t_L + r\left(E\left[q^P\right]\right)\right)^2}{2} = \frac{2}{25}\left(a^2 + 4at_L + 4t_L^2\right).$$

The (ex-post) increase in consumer surplus when firms have asymmetric capacities is then

$$CS^+(a) = CS^S(a, t_L, t_H) - CS^P(a, t_L, t_H) = \frac{1}{25}\left(\frac{9}{8}a^2 - \frac{7}{4}at_L - \frac{39}{8}t_L^2\right).$$

Since an increase in consumer surplus occurs in two states of nature, we may write

$$E\left[\Delta CS(a)\right] = 2p^2 CS^+(a) - p^2 CS^-(a).$$

Thus, it is sufficient to analyze the difference $2CS^+ - CS^-$ in order to determine the sign of $E\left[\Delta CS\right](a)$. Note first that both CS^+ and CS^- have a zero at $a = 3t$.

Differentiating with respect to a yields

$$2\frac{\partial}{\partial a} CS^+(a) = \frac{2}{25}\left(\frac{9}{4}a - \frac{7}{4}t_L\right)$$

and

$$\frac{\partial}{\partial a} CS^-(a) = \frac{2}{25}\left(\frac{22}{9}a - 4t_L\right).$$

Evaluating at $a = 3t$ shows

$$2\frac{\partial}{\partial a} CS^+(3t_L) = \frac{30}{75}t_L > \frac{20}{75}t_L = \frac{\partial}{\partial a} CS^-(a).$$

On one hand, this implies that $E\left[\Delta CS(a)\right]$ is positive when a is sufficiently small. On the other hand, calculating the second derivative yields

$$2\frac{\partial}{\partial^2 a} CS^+(a) = \frac{9}{100} < \frac{22}{225} = \frac{\partial}{\partial^2 a} CS^-(a).$$

This implies that $E\left[\Delta CS\right](a)$ is negative when a is sufficiently large. We have established:

Theorem 5. *If the demand intercept is sufficiently small, then information sharing increases consumer surplus.*

Ultimately, the result is due to the negative correlation of equilibrium outputs in the complete information case. This correlation effect decreases the variance of total industry output, which in turn lowers consumer surplus. When increasing a, the negative correlation of equilibrium outputs increases. If $q(T_1) + q(T_2)$ denotes the total industry output, we observe

$$
\begin{aligned}
& E\left[\left(q(T_1)+q(T_2)\right)^2\right] \\
=& VAR\left[q(T_1)+q(T_2)\right]+E\left[q(T_1)+q(T_2)\right]^2 && (20)\\
=& 2VAR\left[q(T_1)\right]+2COV\left[q(T_1),q(T_2)\right]+E\left[q(T_1)+q(T_2)\right]^2. && (21)
\end{aligned}
$$

As discussed on page 24, both expected output and variance of output of a *single* firm increase when information is shared. Theorem 5 and equation (20) imply that the variance of *total industry output* must decrease if a is sufficiently large. Lastly, equation (21) shows that the decrease of the variance of total industry output driven by a negative correlation of equilibrium outputs.

Apparently, we can easily construct an example in which firms do not have an incentive to share information but consumers nevertheless profit from an information sharing agreement, shown by choosing a sufficiently small a. Similarly, an example in which firms do have an incentive to share information, but the sharing of information in turn decreases consumer surplus, is easily obtained by choosing a sufficiently large a.

The example presented in Table 1 shows that we can choose a such that both producer and consumer surpluses increase, a result that is not implied by the analysis conducted above. We choose $a = t_H = 5$ and $t_L = 1$, implying $r\left(E\left[q^P\right]\right) = 9/5$.

5.3 Social Welfare

Finally, we look at the expected change of social welfare, given by

$$
\begin{aligned}
E\left[\Delta W(a)\right] =& E\left[\Delta PS(a)\right]+E\left[\Delta CS(a)\right]\\
=& E\left[PS_1^+(a)+PS_2^+(a)-PS^-+2CS^+(a)-CS^-(a)\right].
\end{aligned}
$$

Using the results previously established and differentiating with respect to a show that $E\left[\Delta W(a)\right]$ is a quadratic function that has a zero at $a = 3t_L$ and is

Table 1: Equilibrium outputs for private (P) and shared (S) information equilibrium and effects on surplus and welfare ($a = 5, T = \{1, 5\}, \mu$ is uniformly distributed on T^2, implying $r\left(E\left[q^P\right]\right) = 9/5 = 1.8$).

ω	Output		P. surplus		C. surplus		Welfare	
	P	S	P	S	P	S	P	S
(1,1)	1.00	1.00	6.00	6.00	2.00	2.00	8.00	8.00
(1,5)	1.00	1.00	4.40	4.00	3.92	4.50	8.32	8.50
(5,1)	1.80	2.00	7.92	8.00	3.92	4.50	11.84	12.50
(5,5)	1.80	1.67	5.04	5.56	6.48	5.56	11.52	11.11
Expected Values	1.40	1.42	5.84	5.89	4.08	4.14	9.92	10.03
Variances	0.21	0.25	2.36	2.72	3.38	2.28	4.16	4.58

increasing as long as $a \geq 3t_L$. This implies that information sharing increases social welfare.

Lastly, we demonstrate that social welfare may decrease if t_H is sufficiently small. We discuss an example in which $t_H = r\left(E\left[q^P\right]\right)$. This implies that firms can never increase their outputs when moving from the private to the shared information equilibrium. The (ex-post) decrease in output that occurs when $\omega = (t_H, t_H)$ is not affected as long as $t_H \geq r\left(E\left[q^P\right]\right)$. Thus, both consumer surplus and social welfare decrease with t_H. Table 2 shows the equilibrium output and the corresponding surplus and welfare effects when we modify the example presented in Table 1 by defining $t_H = r\left(E\left[q^P\right]\right) = 9/5$.

Remark 4. *All results established in this section hold when we conduct the analysis in terms of t_L and keep the demand intercept constant. By lowering t_L, we increase the variance of μ. Theorem 4 implies that we can choose t_L small enough that firms have an incentive to share their information. This result is complementary to the results established by Maleug and Tsutsui (1998), who show that firms have an incentive to share their information if the variance of the common prior belief is sufficiently large.*

Table 2: Equilibrium outputs for private (P) and shared (S) information equilibrium when $t_H = r\left(E\left[q^P\right]\right) = 1.8$.

ω	Output		P. surplus		C. surplus		Welfare	
	P	S	P	S	P	S	P	S
(1,1)	1.00	1.00	6.00	6.00	2.00	2.00	8.00	8.00
(1,1.8)	1.00	1.00	4.40	4.40	3.92	3.92	8.32	8.32
(1.8,1)	1.80	1.80	7.92	7.92	3.92	3.92	11.84	11.84
(1.8,1.8)	1.80	1.67	5.04	5.56	6.48	5.56	11.52	11.11
Expected Values	1.40	1.37	5.84	5.97	4.08	3.85	9.92	9.82
Variances	0.21	0.18	2.36	2.15	3.38	2.11	4.16	3.77

6 Concluding Remarks

In the presence of uncertainty with respect to production capacities, equilibrium strategies are concave if capacities are stochastically independent. If firms are symmetric, a unique equilibrium exists. When inverse demand is linear, the best reply of a firm only depends on the expected output of the other firms, ensuring that every equilibrium is symmetric because output decisions are strategic substitutes.

Consistent with the literature, we find that capacity constraints can reverse standard results on information sharing. These results are established by discussing a Cournot duopoly in which the common prior belief is discrete and there exist two capacity levels $t_L < t_H$ such that t_H is sufficiently large. Due to the concavity of equilibrium strategies, information sharing leads to an increase in the expected aggregate output of the industry. Moreover, the variance of each firm's output increases with the horizontal demand intercept a when information is shared. However, the variance of total industry might decrease when information is shared, which is due to the negative correlation of the firms' equilibrium outputs. The net effect can lead to an increase as well as to a decrease in producer surplus. The same is true for consumer surplus, which can decrease when information is shared although total output increases. However, social welfare increases when information is shared due to the sufficiently large value of t_H. This effect can be reversed by choosing t_H small enough.

The question as to whether antitrust authorities should either encourage firms to share information or if they should prohibit information exchange can not be answered clearly for two reasons. First, we needed to specify the weights an authority assigns to producer surplus and consumer surplus. In case an authority relies on social welfare as the appropriate measure, sharing information is beneficial for a large class of markets. In case an authority emphasizes consumer surplus, the question as to whether information should be shared depends on the market parameters.

One can think of a number of possible applications of the model. Consider, for example, two markets A and B, where market prices P_A and P_B are common knowledge. If the markets are physically separated, firms who possess transport capacity may take advantage of arbitrage profits. If we assume that the price difference $P := P_A - P_B$ is positive and decreasing in the quantity q bought on market B and sold on market A, we can perceive the problem as a Cournot oligopoly with capacity constraints. These capacity constraints may be unknown: Consider that A and B are two market places for natural gas that are connected via Liquefied Natural Gas (LNG) carriers. Since firms do not know their rival's operation strategies, they do not know the amount of carriers that are available to serve the route between A and B.

The model is limited to the case of stochastically independent capacities. However, the assumption on independence might not be reasonable in markets where the uncertainty is driven by a common source of risk. In particular, local markets for agricultural products do not satisfy the assumption of independent signals, since the firms' harvest is determined by local weather conditions. However, independent capacities might be a suitable approximation.

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