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Abstract

A model of the global gas market is presented which in its basic version optimises the future development of production, transport and storage capacities as well as the actual gas flows around the world assuming perfect competition. Besides the transport of natural gas via pipelines also the global market for liquefied natural gas (LNG) is modeled using a hub-and-spoke approach. While in the basic version of the model an inelastic demand and a piecewise-linear supply function are used, both can be changed easily, e.g. to a Golombek style production function or a constant elasticity of substitution (CES) demand function. Due to the usage of mixed complementary programming (MCP) the model additionally allows for the simulation of strategic behaviour of different players in the gas market, e.g. the gas producers.

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JEL classification: C61, L72, Q34, Q41

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1. Introduction

The global gas market has undergone significant changes in the last decade and future development is highly uncertain as well. Gas market liberalisation in Europe, the rise of unconventional gas production in the US and the increase in LNG trade worldwide were major drivers for the development of the global gas market in recent years. The future trade relationships of gas exporting and importing countries might be affected by the increase of Chinese gas demand, major investments in LNG and pipeline infrastructure or the prospects of unconventional gas production just to mention a few possible influences.

Gas market models are helpful tools for simulating the complex interdependencies described above. The Institute for Energy Economics at the University of Cologne (EWI) has developed linear simulation models of European and global gas markets for many years to help decision makers in business and politics. These models usually minimise short-term or long-term costs of gas supply (i.e. production, storage and transport costs) subject to various constraints. Typical constraints are meeting inelastic, exogenous demand, achieving a certain mix of supply (national diversification targets with respect to supply), or accounting for existing long-term contracts.

The first EWI gas market model was EUGAS developed by Perner (2002). EUGAS is a long-term intertemporal optimisation model to analyse future European gas supply. Thereby, the model includes data of existing production and transport infrastructure and simulates investment decisions in future capacities as well. The model is implemented as a linear mixed-integer model since the investment decision is represented as a binary problem.

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In 2007, EWI developed the TIGER model [see Bothe and Lochner (2008) and Lochner (2011)] to analyse interdependencies within the European natural gas infrastructure. The TIGER-model is a dispatch model which optimizes the European gas supply under given infrastructure and demand assumptions. By minimising total costs of gas supply, the model enables an integrated evaluation of the infrastructure components, i.e. pipelines, storages and terminals and their interaction. The model is used for the evaluation of new investment projects and comprehensive analyses of physical market integration and security of supply.

The MAGELAN model by Seeliger (2006) extends the European gas market models to a global perspective in terms of geographical coverage. MAGELAN is a long-term gas supply model and optimises natural gas supply including investments in production and infrastructure capacity on a yearly basis up to 2035. It is an intertemporal and interregional cost-minimisation model. The model's objective function includes both capital and operating costs of global gas production and transport. Inefficiencies, which could arise due to strategic behaviour of market players, are therefore not accounted for by the model.

The COLUMBUS model which we present here allows for analysing such strategic behaviour. Opposed to MAGELAN, COLUMBUS has a monthly resolution and thus also endogenously calculates storage operation. COLUMBUS is an intertemporal global gas market model based on mixed complementary programming (MCP). The model uses a vertex/edge approach: the vertices represent production facilities (sources) or demand regions (sinks). The vertices are connected via edges, which represent either pipelines or LNG transport routes. Similar to e.g. Gabriel et al. (2005) or Egging et al. (2010) the model code is derived using the maximisation problems of the different players (see section 2) in the global gas market like gas producers, traders, regasifiers, etc. In its basic version, the model optimises the future development of production, transport and storage capacities as well as the dispatch of gas flows around the world assuming perfect competition. The model distinguishes physical gas flows from financial gas trades. Besides the transport of natural gas via pipelines also the global market of liquefied natural gas (LNG) is modeled using a hub-and-spoke approach. To reduce the model complexity, liquefaction and regasification terminals are not connected point-by-point but via a network of virtual LNG hubs, which are connected to each other. This approach reduces the number of LNG connections and therefore the number of variables by 60 % compared to the point-by-point connection approach. In oder to account for the intertemporal decisions of gas storage and resource extraction, the model incorporates a dynamic optimisation approach (using Hamiltonian functions). For representing the production and demand of natural gas, COLUMBUS uses an inelastic demand and a piecewise-linear supply function in the basic version of the model. Both can be changed easily though, e.g. to a Golombek style production function. As COLUMBUS is a MCP model, it can be extended to allow for the simulation of strategic behaviour of different players in the global gas market, e.g. gas producers. Market power on the demand side may be analysed as well.

2. The Model

In the paper at hand we focus on the model's basic mathematical structure – some details are left out, and the paper does not deal with any kind of data preparation at all.

The spatial structure model is formulated as a directed graph consisting of a set N of vertices and a set $A \subset N \times N$ of edges. The set of vertices can be subdivided into sources and sinks, where gas production facilities are modeled as sources and demand regions as sinks, for example. An overview of all sets, decision variables and parameters can be found in Table 1.

Table 1: Model sets, variables and parameters

Sets	
$n \in N$	all model nodes
$c \in C$	cost levels (steps of piecewise linear supply function)
$t \in T$	months
$y \in Y$	years
$p \in P \in N$	producer / production regions
$e \in E \in N$	exporter / trader
$d \in D \in N$	final customer / demand regions
$r \in R \in N$	regasifiers
$l \in L \in N$	liquefiers
$s \in S \in N$	storage operators
Primal Variables	
$pr_{p,c,t}$	produced gas volumes
$fl_{e,n,n1,t}$	physical gas flows
$tr_{e,d,t}$	traded gas volumes
$st_{s,t}$	gas stock in storage
$si_{s,t}$	injected gas volumes
$sd_{s,t}$	depleted gas volumes
$dr_{p,c,y}$	depleted resources
$ip_{p,c,y}$	annual investment into production capacity
$it_{n,n1,y}$	annual investment into pipeline transport capacity
$is_{s,y}$	annual investment into storage capacity
$ilng_y$	annual investment into LNG transport capacity
$ir_{r,y}$	annual investment into regasification capacity
$il_{l,y}$	annual investment into liquefaction capacity
Dual Variables	
$\lambda_{e,n,t}$	marginal costs of physical gas supply by exporter e to node n in time period t
$\sigma_{s,t}$	(intertemporal) marginal costs of storage injection
$\alpha_{p,c,y}$	marginal value of resources in node n at cost level c in year y
$\beta_{d,t}$	marginal costs / price in node n in time period t
$\mu_{p,c,t}$	marginal benefit of an additional unit of production capacity
$\phi_{n,n1,t}$	marginal benefit of an additional unit of pipeline capacity
$\epsilon_{s,t}$	marginal benefit of an additional unit of storage capacity
$\rho_{s,t}$	marginal benefit of an additional unit of storage injection capacity
$\theta_{s,t}$	marginal benefit of an additional unit of storage depletion capacity
ι_t	marginal benefit of an additional unit of LNG transport capacity
$\gamma_{r,t}$	marginal benefit of an additional unit of regasification capacity
$\zeta_{l,t}$	marginal benefit of an additional unit of liquefaction capacity

Parameter	
$dem_{d,t}$	final customer's demand for natural gas
$cap_{n,t/n,n1,t/n,c,t}$	monthly infrastructure capacity
$res_{n,c,y}$	maximum resources
$trc_{n,n1,t}$	transport costs
$prc_{n,c,t}$	production costs
$opc_{n,t}$	operating costs
$inc_{n,y/n,n1,y/n,c,y}$	investment costs
$dist_{n,n1}$	distance between node n and node $n1$ in km
$LNGcap$	initial LNG capacity
$speed$	speed of LNG tankers in km/h
cf_s	conversion factor used for storage inj. & depl. capacity
elt	economic life time of an asset

The model's time structure is represented by a set $T \subset \mathbb{N}$ of points in time (months). This time structure is flexible and the user can customize it, which means any year (y) until 2050 can be simulated with up to twelve month per year. We define Y_t as $Y_t := \left\{y \in Y \mid \left\lfloor \frac{t-1}{12} \right\rfloor + 1 > y\right\}$. Thus, the set Y_t comprises all years, i.e. elements of the set Y (modelled years), prior to the year associated with the actual month, i.e. element $t \in T$. A small example shall help to clarify this:

Example 1. Let us assume that we would like to model the years 2010 to 2015, i.e. $Y = \{1, 2, 3, 4, 5, 6\}$, with each year consisting of twelve month. Therefore, the set T would look like this $T = \{1, 2, ..., 12, 13, ...72\}$. Let us now assume that we would like to take a closer look at January 2012 ($t = 37$), we get $\left\lfloor \frac{37-1}{12} \right\rfloor + 1 = 4$ and thus $Y_{37} = \{1, 2, 3\}$.

Similarly, we define sets T_y and $\bar{T}_y$. Thereby, $T_y := \{t \in T \mid (y-1) * 12 < t \wedge (y - 1 + elt) * 12 \geq t\}$, where elt is defined as the economic lifetime. Thus, the set T_y comprises the months over the economic life time (e.g. 20 years) of an asset when the investment has been made in year y. $\bar{T}_y$ is defined as $\bar{T}_y := \{t \in T \mid (y-1) * 12 \geq t\}$. Hence, $\bar{T}_y$ comprises the months of the years previous to the actual year y.

Finally, we define sets $T(y)$ and $Y(t)$, where $Y(t) := \left\{y \in Y \mid \left\lfloor \frac{t-1}{12} \right\rfloor + 1 = y\right\}$ and $T(y) := \{t \in T \mid (y-1) * 12 < t \wedge y * 12 \geq t\}$. Hence, the set $Y(t)$ comprises the year which is associated with month t and the set $T(y)$ comprises the months t of year y. Again, a small example shall help to clarify this:

Example 2. Let us assume that we would like to model the years 2010 to 2020, i.e. $Y = \{1, 2, 3, .., 11\}$, with each year consisting of twelve month. Therefore, the set T would look like this $T = \{1, 2, ..., 12, 13, ...132\}$. Let us now assume that we would like to take a closer look at year 2018 ($y = 9$), we get $96 < t \leq 108$ and thus $T(9) = \{97, 98, ..., 108\}$.

The remainder of this section is organised as follows: We develop the optimisation problems of the different players modeled in COLUMBUS and the corresponding first-order optimality conditions for each of the players. The first-order conditions combined with the market clearing conditions form the partial equilibrium model.

The vector of variables in parentheses on the right-hand side of each constraint are the Lagrange multipliers used when developing the first-order conditions (Karush-Kuhn-Tucker (KKT) conditions). The complementary slackness condition is indicated by the perpendicular sign $\perp$, where $0 \leq x \perp y \geq 0 \Leftrightarrow x^t y = 0$ for vectors x and y.

2.1. The Exporter's Problem

The exporter $e \in E$ is here defined as a trading unit associated with one or more production regions $p \in P_e$, i.e. they are vertically integrated. Thus, the exporter buys gas from the different production regions and sells the gas ($tr_{e,d,t}$) on the wholesale markets of the demand nodes $d \in D$. Each exporter e maximises its profits, i.e. revenues minus costs of supply, over the modelled time period $t \in T$ and all demand regions d. Exporters may behave as price takers in the market, but can alternatively be modeled as if they were able to exercise market power.

The payoff function $\Pi_{eI}(tr_{e,d,t})$ is defined as[1]

$$\max_{tr_{e,d,t}} \Pi_{eI}(tr_{e,d,t}) = \sum_{t \in T} \sum_{d \in D} (\beta_{e,d,t} * tr_{e,d,t} - \lambda_{e,d,t} * tr_{e,d,t}) \tag{1}$$

where $tr_{e,d,t}$ is the corresponding decision vector of e and $\lambda_{e,d,t}$ corresponds to the exporter's costs of physical gas delivery to demand node d which is an endogenous variable (see equations 34 and 36). The feasible region of $tr_{e,d,t}$ is restricted by the non-negativity constraints $tr_{e,d,t} \geq 0$. The first-order condition of the exporter's optimisation problem is thus defined by the first partial derivative of the Lagrangian L_e with respect to the decision variable $tr_{e,d,t}$:

$$\frac{\partial L_{eI}}{\partial tr_{e,d,t}} = -\beta_{p,t} + \lambda_{e,d,t} \geq 0 \quad \perp \quad tr_{e,d,t} \geq 0 \qquad \forall e, d, t. \tag{2}$$

In reality, if an exporter sells natural gas on a wholesale market with physical delivery he also faces the decision of how to minimise transport costs by choosing the cost-minimal transport gas flows $fl_{e,n,n1,t}$. In COLUMBUS this is modeled by a separate optimisation problem of the following form:

$$\max_{fl_{e,n,n1,t}} \Pi_{eII}(fl_{e,n,n1,t}) = \sum_{t \in T} (\lambda_{e,n1,t} - \lambda_{e,n,t} - trc_{n,n1,t} - opc_{n,t}) * fl_{e,n,n1,t} \tag{3}$$

where $opc_{n,t}$ is defined as the operating costs at node n in month t and $trc_{n,n1,t}$ as the cost associated with transporting gas from node n to node $n1$. Therefore, if n is a regasification node $[r(n)]$, $opc_{n,t}$ would reflect the costs of regasifying a unit of natural gas. If $r(n)$ then $n1$ has to be a liquefaction node, thus $trc_{n,n1,t}$ would be the short-run marginal LNG transport costs from node n to node $n1$. The optimisation problem is subject to some physical transport constraints:

$$cap_{n,n1,t} + \sum_{y \in Y_t} it_{n,n1,y} - \sum_{e \in E} fl_{e,n,n1,t} \geq 0 \quad \forall n, n1, t \qquad (\phi_{n,n1,t}). \tag{4}$$

Thus, the sum over all transport flows (decided on by the traders) through the pipeline between node n and $n1$ has to be lower than the respective pipeline capacity $cap_{n,n1,t}$ and all past investments in additional capacity.

$$cap_{l,t} + \sum_{y \in Y_t} il_{l,y} - \sum_{e \in E} \sum_{n \in N} fl_{e,n,l,t} \geq 0 \quad \forall l, t \qquad (\zeta_{l,t}). \tag{5}$$

[1]In order to keep the formulae as simple as possible no discount factor is included in the following.

Along the lines of equation 4, this equation states that the sum over all transport flows (decided on by the traders) through the liquefaction terminal, i.e. all natural gas that is liquefied, has to be lower than the respective liquefaction capacity. The same holds true for the restriction of gas volumes which are regasified and then transported to a demand node d in month t

$$cap_{r,t} + \sum_{y \in Y_t} ir_{r,y} - \sum_{e \in E} \sum_{d \in D} fl_{e,r,d,t} \geq 0 \quad \forall r, t \qquad (\gamma_{r,t}). \tag{6}$$

Finally, we also have to account for a limitation of available LNG tankers. Hence, the sum of all gas volumes transported between a liquefaction terminal l and a regasification terminal r in month t is restricted by the available LNG transport capacity.

$$\left(LNGcap + \sum_{y \in Y_t} ilng_y \right) * 8760/12 * speed - \sum_{e \in E} \sum_{l \in L} \sum_{r \in R} 2 * (fl_{e,l,r,t} * dist_{n,n1}) \geq 0 \quad \forall t \qquad (\iota_t) \tag{7}$$

where *speed* is defined as the average speed of a LNG tanker (km/h), $dist_{n,n1}$ as the distance in km between node n and node $n1$ and $LNGcap$ as the number of existing LNG tankers times their average size in the initial model year. By using equation 7, we take into account that each LNG tanker which delivers gas to a regasification terminal has to drive back to a liquefaction terminal in order to load new LNG volumes. Thereby, we simplify by assuming that each imaginary LNG tanker drives back to the liquefaction terminal from where it started.

Taking the first partial derivative of the respective Lagrangian L_{eII} with respect to $fl_{e,n,n1,t}$ results in:

$$\begin{aligned} \frac{\partial L_{eII}}{\partial fl_{e,n,n1,t}} = & -\lambda_{e,n1,t} + \lambda_{e,n,t} + trc_{n,n1,t} + opc_{n,t} \\ & + \phi_{n,n1,t} + \zeta_{l,t} + \gamma_{r,t} \\ & + \iota_t * 2 * dist_{l,r} \geq 0 \quad \perp \quad fl_{e,n,n1,t} \geq 0 \qquad \forall e, n, n1, t. \end{aligned} \tag{8}$$

Thus, the optimisation problem defined by equations 3 to 7 may also be interpreted as a cost minimisation problem assuming a benevolent dictator. Therefore, the first-order condition above would be the same in a perfectly competitive transport market, since there will be gas flows between two nodes n and $n1$ until the absolute difference of the dual variables associated with the physical market clearing constraint (equation 34) of the two nodes ($\lambda_{e,n1,t} - \lambda_{e,n,t}$) equals the costs of transporting gas from node n to node $n1$. Hence, $\lambda_{e,n,t}$ can be interpreted as the exporter's marginal costs of supplying natural gas (including production costs $\lambda_{e,p,t}$) to node n as it has been done in equation 1.

2.2. The Producer's Problem

Each producer $p \in P$ is assumed to operate a single production region. The producers earn revenues from selling gas from its production region to the associated exporter. Each producer maximises its profits, i.e. revenues minus costs of production $prc_{p,c,t}$ and costs of investment $inc_{p,c,y}$ into additional production capacities. Producers behave as price takers in the market.

The producer's payoff function $\Pi_p(pr_{p,c,t}, ip_{p,c,y})$ is defined as

$$\begin{aligned}\max_{\substack{pr_{p,c,t}\\ ip_{p,c,y}}} \Pi_p(pr_{p,c,t}, ip_{p,c,y}) = &\sum_{t\in T}\sum_{c\in C}(\lambda_{e,p,t} * pr_{p,c,t} - prc_{p,c,t} * pr_{p,c,t}) \\ &+ \sum_{y\in Y}\sum_{c\in C}(inc_{p,c,y} * ip_{p,c,y})\end{aligned} \tag{9}$$

where $pr_{p,c,t}$ and $ip_{p,c,y}$ are the corresponding decision vectors of p. The set of feasible solutions for $pr_{p,c,t}$ is restricted by the non-negativity constraints $pr_{p,c,t} \geq 0$ and by a constraint on maximum production capacities:

$$cap_{p,c,t} + \sum_{y\in Y_t} ip_{p,c,y} - pr_{p,c,t} \geq 0 \quad \forall p, c, t \qquad (\mu_{p,c,t}) \tag{10}$$

where $ip_{p,c,y}$ is the annual investment in additional monthly production capacities. Finally, $pr_{p,c,t}$ is restricted by a resource constraint:

$$res_{p,c,y} - dr_{p,c,y} \geq 0 \quad \forall p, c, y \qquad (\alpha_{p,c,y}) \tag{11}$$

where $dr_{p,c,y}$ represents the sum over the monthly natural gas production in the actual year y and the years prior to the actual year. Since $dr_{p,c,y-1} = \sum_{t\in \bar{T}_y} pr_{p,c,t}$ this can be rewritten as:

$$dr_{p,c,y} = dr_{p,c,y-1} + \sum_{t\in T(y)} pr_{p,c,t}. \tag{12}$$

Using equation 12 we can reformulate the resource constraint (equation 11):

$$res_{p,c,y} - dr_{p,c,y-1} - \sum_{t\in T(y)} pr_{p,c,t} \geq 0 \quad \forall p, c, y \qquad (\alpha_{p,c,y}). \tag{13}$$

Due to the recourse definition of the resource constraint we have to maximise a Hamiltonian function H_p with $pr_{p,c,t}$ being the control variable. Therefore, the first-order conditions of the producer's problem consists of constraints 10 and 13 as well as the following partial derivatives of the Hamiltonian function:

$$\frac{\partial H_p}{\partial pr_{p,c,t}} = -\lambda_{e,p,t} + prc_{p,c,t} + \sum_{y\in Y(t)} \alpha_{p,c,y} + \mu_{p,c,t} \geq 0 \quad \perp \quad pr_{p,c,t} \geq 0 \qquad \forall p, c, t \tag{14}$$

$$-\frac{\partial H_p}{\partial dr_{p,c,y}} = \dot{\alpha}_{p,c,y} = \alpha_{p,c,y+1} - \alpha_{p,c,y} \leq 0 \quad \perp \quad dr_{p,c,y} \geq 0 \qquad \forall p, c, y \tag{15}$$

$$\frac{\partial H_p}{\partial ip_{p,c,y}} = inc_{p,y} - \sum_{t\in T_y} \mu_{p,c,t} \geq 0 \quad \perp \quad ip_{p,c,y} \geq 0 \qquad \forall p, c, y. \tag{16}$$

Thereby, the first-partial derivative of H_p with respect to $ip_{p,c,y}$ gives us the optimality condition for investments in additional production infrastructure. Hence, the producers should invest into additional production capacities as long as the marginal value of additional production capacity ($\mu_{p,c,t}$) summed over the economic life time of production facilities is at least as large as the investment costs.

2.3. The Transmission System Operator's Problem

The Transmission System Operator (TSO) is modelled as a player in the natural gas market which is subject to regulation. Each pipeline $(n, n1) \in A$ is operated by one TSO. As also described in Gabriel et al. (2005) as well as Egging et al. (2010) the TSO allocates pipeline capacity to players like the exporter and physically reinforces the flows. Price regulation is assumed. Thus, revenues of the TSO are determined by the short-run marginal transport costs $trc_{n,n1,t}$ (exogenous component) and the congestion rent $\phi_{n,n1,t}$. The TSO may also invest into additional pipeline capacity $it_{n,n1,y}$. Hence, costs of the TSO are short-run marginal transport costs and investment costs. Since short-run marginal costs cancel out, the pay-off function of the TSOs looks the following:

$$\max_{it_{n,n1,y}} \Pi_{TSO}(it_{n,n1,y}) = \sum_{t \in T} \phi_{n,n1,t} - \sum_{y \in Y} (inc_{n,n1,y} * it_{n,n1,y}). \tag{17}$$

Thereby, the congestion rent is determined by the pipeline capacity restriction (equation 4). Thus, the TSO's optimisation problem is defined by the partial derivative of the Lagrangian L_{TSO} with respect to $it_{n,n1,y}$:

$$\frac{\partial L_{TSO}}{\partial it_{n,n1,y}} = inc_{n,n1,y} - \sum_{t \in T_y} \phi_{n,n1,t} \geq 0 \quad \perp \quad it_{n,n1,y} \geq 0 \qquad \forall n, n1, y. \tag{18}$$

2.4. The Liquefier's problem

Liquefiers $l \in L$ receive natural gas from exporters e and liquefy it. The resulting LNG is then send downstream to the regasifiers $r \in R$ using LNG tankers. Liquefiers allocate liquefaction capacities to the traders. In return for liquefying natural gas they receive the sum of short-run variable liquefaction costs $opc_{l,t}$ and the congestion rent $\zeta_{n,t}$. Since, we assume the global LNG market, including the markets for liquefying and regasifying natural gas, to be perfectly competitive, this sum equals long-run marginal costs. Therefore, the liquefiers maximise the profit function $\Pi_l(il_{l,y})$[2]:

$$\max_{il_{l,y}} \Pi_l(il_{l,y}) = \sum_{t \in T} \zeta_{l,t} - \sum_{y \in Y} (inc_{l,y} * il_{l,y}). \tag{19}$$

Thereby, the congestion rent is determined by the liquefaction capacity restriction (equation 5). Thus, similar to the TSO the liquefier's optimisation problem is defined by the partial derivative of the Lagrangian L_l with respect to $il_{l,y}$:

$$\frac{\partial L_l}{\partial il_{l,y}} = inc_{l,y} - \sum_{t \in T_y} \zeta_{l,t} \geq 0 \quad \perp \quad il_{l,y} \geq 0 \qquad \forall l, y. \tag{20}$$

2.5. The Regasifier's Problem

Regasifiers r receive LNG transported by LNG tankers and regasify it. The natural gas is then transported to a demand node by the TSO of the respective pipeline. Hence, the optimisation problem is similar

[2] By analogy with the TSO's optimisation problem the short-run marginal costs, here $opc_{l,t}$, cancel out, thus are not included in equation 19.

to the liquifier's. Therefore, the profit function of the regasifier $\Pi_r(ir_{r,y})$ is defined as:

$$\max_{ir_{r,y}} \Pi_r(ir_{r,y}) = \sum_{t\in T} \gamma_{r,t} - \sum_{y\in Y} (inc_{r,y} * ir_{r,y}). \tag{21}$$

Thereby, the congestion rent $\gamma_{r,t}$ is determined by the regasification capacity constraint (equation 6). Hence, the regasifier's optimisation problem is defined by the partial derivative of the Lagrangian L_r with respect to $ir_{r,y}$:

$$\frac{\partial L_r}{\partial ir_{r,y}} = inc_{r,y} - \sum_{t\in T_y} \gamma_{r,t} \geq 0 \quad \perp \quad ir_{r,y} \geq 0 \qquad \forall r, y. \tag{22}$$

2.6. The LNG Problem

The approach to modelling the LNG market slightly deviates from the other optimisation problem, since no specific players are modelled. Instead, we assume one virtual investor who may invest in LNG transport capacities, i.e. LNG tankers. Similar to other investments in transport infrastructure the investor behaves perfectly competitive, thus investments into additional LNG tanker capacity take place until marginal investment costs equal marginal benefits. Therefore, the optimisation problem of investments in LNG transport capacities is defined as:

$$\max_{ilng_y} \Pi_{LNG}(ilng_y) = \sum_{t\in T} \iota_t - \sum_{y\in Y} (inc_y * ilng_y). \tag{23}$$

Thereby, the congestion rent ι_y is determined by the LNG capacity restriction (equation 7). Thus, the optimisation problem of the investor in LNG tanker capacity is defined by the partial derivative of L_{LNG} with respect to $ilng_y$:

$$\frac{\partial L_{LNG}}{\partial ilng_y} = inc_y - \sum_{t\in T_y} (\iota_t * 8760/12 * speed) \geq 0 \quad \perp \quad ilng_y \geq 0 \qquad \forall y. \tag{24}$$

2.7. The Storage Operator's Problem

Each storage facility is operated by one storage operator $s \in S$. The storage facilities are assumed to be located in the demand regions. The storage operator maximises its revenues by buying gas in months with low prices and reselling it in months with high prices. Furthermore, each year she may invest in additional monthly storage capacity ($is_{s,y}$). Hence, equivalent to the producer's problem each storage operator faces a dynamic optimisation problem of the following form:

$$\begin{aligned}\max_{\substack{si_{s,t}, sd_{s,t}\\ is_{s,y}}} \Pi_s(si_{s,t}, sd_{s,t}, is_{s,y}) = &\sum_{t\in T} \beta_{d,t}\,(sd_{s,t} - si_{s,t}) \\ &- \sum_{y\in Y} (inc_{s,y} * ip_{s,y}).\end{aligned} \tag{25}$$

Using injection $si_{s,t}$ as well as depletion $sd_{s,t}$ in month t, we can define the motion of gas stock ($st_{s,t}$), i.e. the change in stored gas volumes:

$$\dot{st}_{s,t} = st_{s,t+1} - st_{s,t} = si_{s,t} - sd_{s,t} \quad \forall s, t \qquad (\sigma_{s,t}) \tag{26}$$

Additionally, the maximisation problem of the storage operator is subject to some capacity constraints:

$$cap_{s,t} + \sum_{y \in Y_t} is_{s,y} - st_{s,t} \geq 0 \quad \forall s,t \qquad (\epsilon_{s,t}) \tag{27}$$

$$cf_s * (cap_{s,t} + \sum_{y \in Y_t} is_{s,y}) - si_{s,t} \geq 0 \quad \forall s,t \qquad (\rho_{s,t}) \tag{28}$$

$$cf_s * (cap_{s,t} + \sum_{y \in Y_t} is_{s,y}) - sd_{s,t} \geq 0 \quad \forall s,t \qquad (\theta_{s,t}). \tag{29}$$

Hence, we assume that storage capacity can be linearly transfered (by use of the parameter cf_s) into the restriction on maximum injection ($si_{s,t}$) and depletion ($sd_{s,t}$). The constraints stated above as well as the following derivatives of the Hamiltonian function H_s constitute the first-order conditions of the storage operator's optimisation problem:

$$\frac{\partial H_s}{\partial sd_{s,t}} = -\beta_{d,t} + \sigma_{s,t} + \theta_{s,t} \geq 0 \quad \perp \quad sd_{s,t} \geq 0 \qquad \forall s,t \tag{30}$$

$$\frac{\partial H_s}{\partial si_{s,t}} = -\sigma_{s,t} + \beta_{d,t} + \rho_{s,t} \geq 0 \quad \perp \quad si_{s,t} \geq 0 \qquad \forall s,t \tag{31}$$

$$-\frac{\partial H_s}{\partial st_{s,t}} = \epsilon_{s,t} = \dot{\sigma}_{s,t} = \sigma_{s,t+1} - \sigma_{s,t} \leq 0 \quad \perp \quad st_{s,t} \leq 0 \qquad \forall s,t \tag{32}$$

$$\frac{\partial H_s}{\partial is_{s,y}} = inc_{s,y} - \sum_{t \in T_y} [\epsilon_{s,t} + cf_{s,t} * (\rho_{s,t} + \theta_{s,t})] \geq 0 \quad \perp \quad is_{s,y} \geq 0 \qquad \forall s,y. \tag{33}$$

2.8. Market Clearing Conditions

The equilibrium problem comprises the first-order conditions derived from the different optimisation problems discussed previously. In addition, we have to include two market clearing conditions:

$$\sum_{c \in C} pr_{p,c,t} - tr_{e,d,t} + \sum_{n1 \in (n1,n) \in A} fl_{e,n1,n,t} - \sum_{n1 \in (n,n1) \in A} fl_{e,n,n1,t} = 0 \quad \perp \quad \lambda_{e,n,t} \text{ free} \qquad \forall e,n,t. \tag{34}$$

Equation 34 is a general market clearing condition which has to be fulfilled for each exporter $e \in E$ and every model node $n \in N_e$ at which the trader is active. For example, if we consider a production node $p(n)$, market clearing condition 34 collapses to:

$$\sum_{c \in C} pr_{p,c,t} - \sum_{n1 \in (p,n1) \in A} fl_{e,p,n1,t} = 0 \quad \perp \quad \lambda_{e,p,t} \text{ free} \qquad \forall e,p,t. \tag{35}$$

Thus, the gas volumes produced have to match the physically flows out of node $p(n)$. A similar example could be derived for every other subnode of N. For example, if we consider a demand node $d(n)$, market clearing condition 34 simplifies to

$$\sum_{n1 \in (n1,d) \in A} fl_{e,n1,d,t} - tr_{e,d,t} = 0 \quad \perp \quad \lambda_{e,d,t} \text{ free} \qquad \forall e,d,t. \tag{36}$$

Hence, equation 34 also assures that the gas volumes, which exporter e sold at the wholesale market of demand node d, are actually physically transported there.

$$\sum_{e \in E} tr_{e,d,t} + sd_{s,t} - si_{s,t} - dem_{d,t} = 0 \quad \perp \quad \beta_{d,t} \text{ free} \qquad \forall d,t. \tag{37}$$

The last market clearing condition (equation 37) states that final customer's demand for natural gas ($dem_{d,t}$) and gas volumes injected ($si_{s,t}$) into storage facility at node $s(d)$ is met by the sum over all gas volumes sold at the wholesale market by traders e and gas volumes depleted ($sd_{s,t}$) from storage facility s. Thus, the dual associated with equation 37 ($\beta_{d,t}$) represents the wholesale price in demand node d in month t.

3. Summary

The paper at hand presents a mixed complementary program for simulating dispatch and investment decisions in the global gas market. Earlier modelling approaches developed at EWI were consolidated and refined. Among other things new developments implemented in the COLUMBUS model now allow to include non-linear elements like elastic demand functions or Golombek-style production functions in the model and to account for non-competitive behaviour of players in the global gas market as well as the intertemporal optimisation of resource extraction or storage utilisation. Therefore, COLUMBUS allows for short-term as well as long-term analyses using different assumptions about the market structure and thus the market power of the various players in the global gas market.

References

Bothe, D., Lochner, S., 2008. Erdgas für Europa: Die ewiGAS2008 Prognose. Zeitschrift für Energiewirtschaft 32 (1), 22–29.

Egging, R., Holz, F., Gabriel, S. A., 2010. The world gas model: A multi-period mixed complementarity model for the global natural gas market. Energy 35 (10), 4016 – 4029.

Gabriel, S., Kiet, S., Zhuang, J., 2005. A mixed complementarity-based equilibrium model of natural gas markets. Operations Research 53 (5), 799.

Lochner, S., 2011. Identification of congestion and valuation of transport infrastructures in the European natural gas market. Energy 36 (5), 2483–2492.

Perner, J., 2002. Die langfristige Erdgasversorgung Europas – Analysen und Simulationen mit dem Angebotsmodell EUGAS. No. 60 in Schriften des Energiewirtschaftlichen Instituts. Oldenbourg Industrieverlag, München.

Seeliger, A., 2006. Entwicklung des weltweiten Erdgasangebots bis 2030. No. 61 in Schriften des Energiewirtschaftlichen Instituts. Oldenbourg Industrieverlag, München.

ABOUT EWI

EWI is a so called An-Institute annexed to the University of Cologne. The character of such an institute is determined by a complete freedom of research and teaching and it is solely bound to scientific principles. The EWI is supported by the University of Cologne as well as by a benefactors society whose members are of more than forty organizations, federations and companies. The EWI receives financial means and material support on the part of various sides, among others from the German Federal State North Rhine-Westphalia, from the University of Cologne as well as – with less than half of the budget – from the energy companies E.ON and RWE. These funds are granted to the institute EWI for the period from 2009 to 2013 without any further stipulations. Additional funds are generated through research projects and expert reports. The support by E.ON, RWE and the state of North Rhine-Westphalia, which for a start has been fixed for the period of five years, amounts to twelve Million Euros and was arranged on 11th September, 2008 in a framework agreement with the University of Cologne and the benefactors society. In this agreement, the secured independence and the scientific autonomy of the institute plays a crucial part. The agreement guarantees the primacy of the public authorities and in particular of the scientists active at the EWI, regarding the disposition of funds. This special promotion serves the purpose of increasing scientific quality as well as enhancing internationalization of the institute. The funding by the state of North Rhine-Westphalia, E.ON and RWE is being conducted in an entirely transparent manner.