Distributed Generation in Unbundled Electricity Markets

AUTHOR
Johannes Wagner

EWI Working Paper, No 18/01

June 2018
Distributed Generation in Unbundled Electricity Markets

Johannes Wagner

\textsuperscript{a}Department of Economics and Institute of Energy Economics, University of Cologne, Vogelsanger Strasse 321a, 50827 Cologne, Germany

Abstract

Electricity systems are increasingly characterized by distributed generation technologies, e.g. rooftop photovoltaic systems, which are used by end consumers to directly produce electricity. Additionally, empirical evidence suggests that electricity retailers exercise market power in many unbundled electricity markets. Against this backdrop this article analyzes the impact of distributed generation on imperfect retail markets for electricity in a spatial competition framework. I find that distributed generation puts competitive pressure on retailers and induces lower retail prices. Therefore even consumers who do not use distributed generation benefit. Based on this effect regulators can shift welfare to consumers by subsidizing distributed generation in order to position it as a competitor to grid based electricity. However, if only a limited share of demand can be supplied with distributed generation, there is a point at which retailers disregard the substitutable share of demand and focus on the non-substitutable consumption in order to realize higher mark-ups. As a result, increased subsidies for distributed generation can increase retail prices and harm consumers. With optimal subsidies this strategy of retailers is prevented by limiting usage of distributed generation.

Keywords: Distributed generation, renewable energy, retail unbundling, spatial competition

JEL classification: D43, L13, L50, L94, Q48

\textsuperscript{a}Email address: johannes.wagner@evi.uni-koeln.de, +49 221 27729 302 (Johannes Wagner)
1. Introduction

Electricity markets are increasingly influenced by distributed generation technologies such as rooftop photovoltaic systems, small scale combined heat and power plants or wind turbines, which are used by end consumers to directly produce electricity.\(^1\) End consumers use distributed generation to substitute grid based electricity, which is produced in large scale power plants and transported to consumers via transmission and distribution infrastructure. This development is also refereed to under the term ”prosumage”, which indicates that households or businesses are at the same time consumers and producers of electricity. Conceptually the choice whether to consume grid based electricity or produce electricity from distributed generation can be compared to ”make-or-buy” or ”do-it-yourself” decisions which are present in many markets.\(^2\)

In most cases distributed generation is currently not competitive to centralized electricity production. However, especially distributed generation technologies based on renewable energy sources often receive financial support either via direct subsides such as feed-in tariffs or via indirect support mechanisms. Indirect subsidization is typically a result of exemption rules which exempt distributed generation from tax or grid fee payments, which both account for a significant share of the total cost of grid based electricity in practice.\(^3\) Consumers compare the subsidized cost of distributed generation to the price of grid based electricity when they decide on becoming a ”prosumer”. Therefore direct subsidy payments, exemption rules and the prices charged by retailers are key drivers for the adoption of distributed generation.

In the course of the liberalization and restructuring of electricity markets over the last decades, many retail markets for electricity in the United States and the European Union have been unbundled and organized competitively.\(^4\) In competitive retail markets, consumers can choose between different retailers depending on their individual preference. Despite this possibility, empirical evidence indicates that only a small share of customers switches retailers in many of the restructured markets and in particular local retailers can realize substantial margins.\(^5\) One possible explanation for these margins are strong consumer preferences towards specific suppliers as a result of risk aversion, imperfect information or advertising activities.\(^6\)

---

\(^1\) A general discussion of distributed generation in electricity markets is provided in Pepermans et al. (2005).
\(^2\) See for example Sappington (2005).
\(^3\) The average total household electricity price in the European Union consisted of 27% network charges, 25% taxes and 13% charges for renewable energy support. See Agency for the Cooperation of Energy Regulators and Council of European Energy Regulators (2016).
\(^4\) Retail competition is mandatory in the European Union. In the United States roughly half of the states introduced retail competition. See International Energy Agency (2016) for an overview.
\(^5\) See Agency for the Cooperation of Energy Regulators and Council of European Energy Regulators (2016) for an overview of retail mark-ups in European electricity markets. A similar analysis for Texas can be found in Puller and West (2013).
Against the described backdrop this paper analyzes the impact of distributed generation on retail markets for electricity with imperfect competition. Based on this analysis, optimal regulatory strategies with respect to subsidies for distributed generation and grid fees are evaluated. The analysis builds on a standard Hotelling spatial competition framework in order to capture market power of retailers as a consequence of heterogeneous consumer preferences. Consumers may choose distributed generation as an alternative to grid based electricity purchased from retailers. However, only a limited share of total demand can be supplied with distributed generation, which means that some electricity is always received from retailers. This assumption reflects that not every consumer is able to use distributed generation and full autarky from the grid is very costly or even impossible with available technologies.

The analysis shows, that the availability of distributed generation increases competition in the retail market. Hence, as soon as distributed generation is competitive to grid based electricity, retailers adjust prices and reduce mark-ups. The regulator can exploit this behaviour by subsidizing distributed generation in order to position it as a competitor to grid based electricity, which reduces market power of retailers and shifts producer rents to consumers. As retail prices are reduced for all consumption, this strategy benefits also consumers who are unable to use distributed generation. However, there is a point where retailers discard the share of electricity consumption which can be substituted with distributed generation and prefer to serve only non-substitutable demand with high mark-ups. As a result, increasing subsidies for distributed generation increases retail prices and therefore harms consumers if retailers discard the substitutable share of demand. Additionally it is shown that optimal subsidization can be realized with grid fee exemptions. However, optimal subsidies can only be implemented with a two-part tariff structure. Grid fee exemptions with volumetric tariffs are not applicable to implement the optimal regulatory strategy.

The paper is mainly related to two literature streams. The first relevant literature stream examines distributed generation technologies in electricity markets. The majority of papers within this stream focuses on numerical simulations or general discussions. Formal analyses of distributed generation are scarce. Brown and Sappington (2017b) build a theoretical model to assess optimal compensation for distributed generation. They find that the optimal policy varies depending on the available instruments and the type of distributed generation technology. However, capacity charges are crucial in order to induce efficient investment into distributed generation. In Brown and Sappington (2017a) this analysis is extended in a very similar model framework in order to analyze net metering policies for small scale solar power generation.

---

7This model class was first presented in Hotelling (1929).
8Simulation studies on the impact of distributed generation can be found for example in Eid et al. (2014), Darghouth et al. (2016) or Munoz-Alvarez et al. (2017).
They conclude that the optimal payment for distributed generation should reflect changes in conventional
generation, distribution and network management costs as well as external effects such as environmental
benefits. However, a net metering mandate is unlikely to meet these requirements. In both analyses the value
chain of electricity supply is assumed to be vertically integrated, which means that unbundling and imperfect
retail markets are not considered. Gautier et al. (2018) analyze interactions between distributed generation
and grid infrastructure in a theoretical framework. They find that support of distributed generation via
net metering overencourages investment into distributed generation and that consumers without access to
distributed generation technologies cross subsidize distributed generation investments. The retail market is
assumed to be perfectly competitive in their analysis.

The second relevant literature stream consists of applications of spatial competition models. On the one
hand the paper is related to models of spatial competition with outside goods, which were first conceptualized
in Salop (1979). This model class has been applied for example in Balasubramanian (1998) or Nakayama
(2009) to analyze the impact of mail order businesses on traditional retail shops. On the other hand the
paper is related to applications of spatial competition frameworks in an energy context. Tode (2016) assesses
energy efficiency measures in a model with imperfect competition and imperfect consumer information.
Retail markets for electricity with switching costs are analyzed in Ruiz et al. (2015). Distributed generation
is not part of the analysis.

In summary the contribution of the paper is threefold. First, distributed generation in unbundled
electricity markets is analyzed in a theoretical model with an explicit representation of imperfect competition
in the retail market. Second, optimal regulatory strategies and subsidy mechanisms are assessed within this
model framework. Third, the impact of distributed generation on recovery of grid costs is evaluated.

The remainder of the paper is structured as follows. Section 2 introduces the basic model setup. Section
3 analyzes the retail market problem. Building on that, section 4 analyzes optimal subsidies for distributed
generation. In section 5, grid fee exemption rules and the impact of the share of electricity demand that
can be substituted with distributed generation are discussed as model extensions.

2. Model setup

We consider an electricity market with two symmetric retailers \( R_1 \) and \( R_2 \), who sell electricity to con-
sumers. Two types of consumers are differentiated: a mass \( \alpha \) of consumers \( C_s \), who can substitute grid based
electricity consumption with distributed generation and a mass \( 2 - \alpha \) of consumers \( C_{ns} \), who are unable
to use distributed generation. This differentiation reflects two practical issues. First, some consumers are
unable to use distributed generation for example because of financial, legal or constructional restrictions.
Second, even consumers who use distributed generation, typically maintain a grid connection and use both
grid based and self generated electricity. This is especially the case for distributed generation based on
weather dependent renewable energy sources such as wind or solar, where grid based electricity is used as a
back-up when wind and solar generation is unavailable. Consequently \( \alpha \) can be interpreted as the share of
demand of consumers who are unable to use distributed generation as well as the share of electricity demand
that can not be substituted because of unavailability of distributed generation for example during the night.
In the basic model, \( \alpha = 1 \) is assumed. The basic model results are generalized in section 5.2.

Retailers maximize profits by buying electricity in a wholesale market at price \( w \) and selling it to con-
sumers at retail prices \( p_{R1} \) and \( p_{R2} \). Retailers are assumed to be price takers in the wholesale market.
Additionally retailers are horizontally differentiated and consumers have heterogeneous preferences towards
retailers. To model consumer preferences and horizontal differentiation a spatial competition framework is
applied, where parameter \( t \) represents the degree of differentiation. Retailers are not able to discriminate
prices. Therefore, they always charge the same retail price for both consumer groups \( C_s \) and \( C_{ns} \). The
cost of electricity production with distributed generation technologies is \( c_{DG} \).\(^9\) Additionally a subsidy \( \sigma \)
is in place that reduces the effective costs of distributed generation for end consumers. The subsidy is set
by a benevolent regulator. It is assumed that that \( c_{DG} - \sigma \geq w \), which means that the subsidized cost of
distributed generation exceeds the wholesale price for electricity.

The dynamic structure of the model consist of three stages. In the first stage, the regulator sets subsidies
for distributed generation \( \sigma \). In the second stage, the two retailers \( R_1 \) and \( R_2 \) set retail prices in order to
maximize profits. In the third stage, consumers choose between retailers and distributed generation. The
dynamic structure of the model is depicted graphically in figure 1. The model is solved by backward
induction. The retail market is considered first, followed by the regulator problem.

3. The retail market

3.1. Consumer problem

Consumers \( C_{ns} \) and \( C_s \) are assumed to be uniformly distributed along two separate Hotelling lines with
a normalized length of one.\(^10\) The two symmetric retailers \( R_1 \) and \( R_2 \) are located at the endpoints of both

\(^9\) The model considers only one period of electricity production and consumption. A differentiation between fixed and
variable costs is not required due to this simplification. Hence, \( w \) and \( c_{DG} \) can be interpreted as the total specific costs of
wholesale electricity and distributed generation over the model period.

\(^10\) The chosen model structure with two separate Hotelling lines that differentiate two groups of consumers is similar to the
model presented in Zégners and Kretschmer (2016).
Consumers choose between retailers and distributed generation. Regulator sets subsidy $\sigma$. Retailers $R_1$ and $R_2$ set $p_{R_1}$ and $p_{R_2}$. Consumers choose between retailers and distributed generation.

Figure 1: Dynamic model setting

The distance between the retailers represents horizontal differentiation and consumers are located at a location along the line according to their preference towards the retailers. The position of consumers $C_{ns}$ is denoted by $x_{ns} \in [0,1]$ and the position of consumers $C_s$ is denoted by $x_s \in [0,1]$. Every consumer receives a fixed utility $v$ from consuming one unit of electricity. It is assumed that $v$ is sufficiently large such that consumers always choose to consume electricity, which means that total electricity demand is perfectly inelastic. Because of $\alpha = 1$ demand of consumers $C_s$ and $C_{ns}$ is normalized to one in the basic model.

Depending on which retailer consumers choose, they pay a retail price $p_{R1}$ or $p_{R2}$ for electricity consumed from the grid. Additionally, consumers have costs $tx$ for consumption from retailer $R_1$ and $t(x - 1)$ for consumption from retailer $R_2$ depending on their position $0 \leq x \leq 1$. These costs can be interpreted as a disutility for consumers who cannot choose a retailer that perfectly matches their preferences. Consumers $C_s$ can substitute grid based electricity with distributed generation. The subsidized cost of distributed generation is $c_{DG} - \sigma$.

Formally the net utility consumers $C_{ns}$ and $C_s$ derive from grid based electricity consumption purchased via retailer $R_i$ can be described by equation (1a), where $x_i$ represents the position of retailer $i$.

$$U_{grid} = v - p_{R_i} - t|x_i - x|$$

$$U_{DG} = v - (c_{DG} - \sigma)$$

As consumers $C_{ns}$ are unable to use distributed generation, their net utility $U_{ns}$ of grid based electricity consumption is directly described by equation (1a). Because $v$ is sufficiently large by assumption, this utility is strictly positive and consumers $C_{ns}$ always consume grid based electricity. Consumers $C_s$ on the other hand compare net utility from grid based electricity to net utility from distributed generation. The net utility $U_s$ of grid based electricity consumption for consumers $C_s$ can therefore be determined by the

\[ \text{In the following } i \in \{1,2\} \text{ is used to symbolize retailers 1 and 2 in order to simplify notation. } -i \text{ stands for the corresponding other retailer.} \]
difference between equations (1a) and (1b). $U_s$ is only positive if the subsidized cost of distributed generation exceeds the sum of retail price and preference dependent disutility. Otherwise net utility from grid based consumption is negative and consumers $C_s$ use distributed generation to directly produce electricity. The formal expressions for $U_{ns}$ and $U_s$ are presented in equations (2a) and (2b):

$$U_{ns} = v - p_{Ri} - t|x_i - x_{ns}|$$ (2a)

$$U_s = c_{DG} - \sigma - p_{Ri} - t|x_i - x_s|$$ (2b)

Based on equations (2a) and (2b) the demand served by each retailer $i$ can be derived by solving for the indifferent consumer between purchasing from retailers $R_1$ or $R_2$ and for the indifferent consumer $C_s$ between using grid based electricity or distributed generation respectively. The following demand function can be derived:

$$q_{Ri} = \begin{cases} \frac{t + p_{Ri} - p_{Ri}}{l} & \text{if } p_{Ri} - t \leq p_{Ri} \leq 2(c_{DG} - \sigma) - p_{Ri} - t \\ \frac{t + p_{Ri} - p_{Ri}}{2l} + \frac{c_{DG} - \sigma - p_{Ri}}{l} & \text{if } 2(c_{DG} - \sigma) - p_{Ri} - t \leq p_{Ri} \leq c_{DG} - \sigma \\ \frac{t + p_{Ri} - p_{Ri}}{2l} & \text{if } p_{Ri} > c_{DG} - \sigma \end{cases}$$ (3)

Equation (3) shows that three cases can be distinguished for the demand function. In the first case, distributed generation is not competitive to grid based consumption for all consumers. As a result, demand from retailers depends only on retail prices and the preference dependent disutility for consumers. The subsidies for distributed generation are irrelevant as all consumption is grid based. In the second case, distributed generation is used by some consumers $C_s$. Consequently, retailers compete against distributed generation for the substitutable share of electricity demand. For this share, demand depends on the relationship between the subsidized cost of distributed generation $c_{DG} - \sigma$ and the retail price $p_{Ri}$. The non-substitutable consumption is still determined by competition between the retailers. In the third case, all substitutable demand is covered with distributed generation and retailers compete for consumers $C_{ns}$. The subsidized cost of distributed generation $c_{DG} - \sigma$ directly effects the demand function only in the second case. However, changes in $c_{DG} - \sigma$ shift the boundaries between the three cases of the demand functions. An increase in the subsidy for example shifts the boundaries to lower levels and enlarges the relative size of the second case of the demand function. The demand for distributed generation is determined by the residual $q_D = 2 - q_{R1} - q_{R2}$ in all three cases.
3.2. Retailer problem

The two retailers buy electricity in the wholesale market at an exogenous wholesale price $w$. They are located at the endpoints of the Hotelling lines and are assumed to maximize profits $\pi_{R1}$ and $\pi_{R2}$. Retailers set retail prices $p_{R1}$ and $p_{R2}$ according to problem (4). Quantities sold to consumers $q_{Ri}$ are determined by equation (3).

$$\max_{p_{Ri}} \pi_{Ri} = q_{Ri} \ast (p_{Ri} - w)$$  \hspace{1cm} (4)

Retailer profits depend on the different cases of the demand function, which means that profits differ if a retailer serves both consumer groups $C_{ns}$ and $C_s$ or if he focuses only on consumption that can not be substituted with distributed generation. Based on the demand function four different cases have to be distinguished in order to solve the retailer problem. These cases are illustrated graphically in figure 2.

Figure 2: Exemplary relations between retail price and cost of distributed generation

Figure 2 shows the total cost of grid based electricity consumption depending on the location $x_s$ of consumers $C_s$ in comparison to the subsidized cost of distributed generation $c_{DG} - \sigma$. In the first case, depicted in figure 2(a), the cost of distributed generation exceeds the sum of retail prices and preference dependent disutility for all $x_s$. As a result, all consumers use grid based electricity and choose the retailer which is closest to their preference. Demand is determined by the first case of equation (3). In figure 2(b) distributed generation has reached a cost level at which a marginal reduction would yield it competitive for consumers with the largest preference dependent disutility, which are located in the middle of the Hotelling line. Again all consumers use grid based electricity, however with a marginal cost reduction, some consumers would start to use it and demand would be determined by the second case of equation (3). In the third case
according to figure 2(c), distributed generation is the preferred option for some consumers. Consequently, consumers located between $x^D_1$ and $x^D_2$ avoid grid based electricity consumption by using distributed generation. Demand is described by the second part of equation (3). In the fourth case, depicted in figure 2(d), distributed generation is cheaper for all consumers and the substitutable electricity consumption is entirely supplied with distributed generation. Usage of grid based electricity is determined by the third case of equation (3).

Based on the first order conditions derived from equation (4) the following reaction function can be obtained:\(^{(12)}\)

$$p_{R_i}(p_{R-i}) = \begin{cases} \frac{t + p_{R-i} + w}{2} & \text{if } p_{R-i} - t \leq p_{R_i} \leq 2(c_{DG} - \sigma) - p_{R-i} - t \\ \frac{t + p_{R-i} + 3w + 2(c_{DG} - \sigma)}{6} & \text{if } 2(c_{DG} - \sigma) - p_{R-i} - t \leq p_{R_i} \leq c_{DG} - \sigma \\ \frac{t + p_{R-i} + w}{2} & \text{if } p_{R_i} > c_{DG} - \sigma \end{cases}$$

Expressing the boundary conditions between the first and the second case of equation (5) in terms of $p_{R-i}$ yields the following equations:

$$p_{R-i} \leq \frac{4(c_{DG} - \sigma) - w - 3t}{3} := p'_{R-i}$$

$$p_{R-i} > \frac{10(c_{DG} - \sigma) - 3w - 7t}{7} := p''_{R-i}$$

Because $p'''_{R-i}$ is strictly larger than $p''_{R-i}$ for $c_{DG} - \sigma > w$ there is a region between $p''_{R-i}$ and $p'''_{R-i}$ where the best response is not defined by the three cases of equation (5). In this region $\frac{\partial \pi_{ri}}{\partial p_{ri}}$ is strictly positive for $p_{R_i} < 2(c_{DG} - \sigma) - p_{R-i} - t$ and strictly negative for $p_{R_i} > 2(c_{DG} - \sigma) - p_{R-i} - t$. As a result, the optimal reaction is $p_{R_i} = 2(c_{DG} - \sigma) - p_{R-i} - t$, which is exactly the boundary between cases 1 and 2 of equation (5).\(^{(13)}\)

Expressing the boundary conditions between the second and the third case of equation (5) in terms of $p_{R-i}$ yields the following equations:

$$p_{R-i} \leq \frac{4(c_{DG} - \sigma) - 3w - t}{3} := p''_{R-i}$$

$$p_{R-i} > \frac{2(c_{DG} - \sigma) - w - t}{7} := p'''_{R-i}$$

Because $p'''_{R-i}$ is strictly smaller than $p''_{R-i}$ for $c_{DG} - \sigma > w$ the best response can be given by both the second and the third case of equation (5) between $p''_{R-i}$ and $p'''_{R-i}$. Substituting both cases into the profit

---

\(^{(12)}\)The first order conditions are presented in equation (A.1) in Appendix A.

\(^{(13)}\)This case is discussed in detail in Mérel and Sexton (2010).
function and comparing the resulting profits yields $\hat{p}_{R-i} \equiv (1 + \sqrt{3})(c_{DG} - \sigma) - \sqrt{3}w - t$ as the boundary condition. Based on the described results, the reaction function is reformulated in equation (8).

$$p_{R}(p_{R-i}) = \begin{cases} 
\frac{t + p_{R-i} + w}{2} & \text{if } p_{R-i} \leq p'_{R-i} \\
2(c_{DG} - \sigma) - p_{R-i} - t & \text{if } p'_{R-i} < p_{R-i} \leq p''_{R-i} \\
t + p_{R-i} + 3w + 2(c_{DG} - \sigma) & \text{if } p''_{R-i} < p_{R-i} \leq \hat{p}_{R-i} \\
t + p_{R-i} + w & \text{if } p_{R-i} > \hat{p}_{R-i}
\end{cases} \quad (8)$$

The four cases of the reaction function correspond to the four cases depicted in figure 2. In the first case distributed generation is not used. In the second case distributed generation is at the margin to competitiveness. In the third case some consumers $C_s$ use distributed generation and in the fourth case all substitutable consumption is supplied with distributed generation.

Solving the reaction functions for the four possible equilibria and determining the parameter values under which they emerge gives the equilibrium solution of the retailer problem:

**Lemma 1.** There are four types of symmetric equilibria depending on the relationship between the subsidized costs of distributed generation $c_{DG} - \sigma$, wholesale price $w$ and the degree of horizontal differentiation $t$:

$$p_{R-i} = \begin{cases} 
w + t & \text{if } c_{DG} - \sigma \geq w + \frac{3}{2}t \\
c_{DG} - \sigma - \frac{t}{2} & \text{if } w + \frac{7}{6}t \leq c_{DG} - \sigma < w + \frac{3}{2}t \\
2(c_{DG} - \sigma) + 3w + t & \text{if } w + \frac{2\sqrt{3}}{5 + \sqrt{3}}t \leq c_{DG} - \sigma < w + \frac{7}{6}t \\
w + t & \text{if } c_{DG} - \sigma < w + \frac{2}{1 + \sqrt{3}}t
\end{cases} \quad (9)$$

**Proof.** See Appendix A.

The reaction functions are depicted graphically in figure 3. The decisive model parameter is the effective cost of distributed generation $c_{DG} - \sigma$ because it determines to which extent distributed generation interferes with the strategic interactions of the two retailers. The reaction function described in equation (8) consists of four parts of which the intermediate parts are directly affected by changes in $c_{DG} - \sigma$. Both are shifted downwards as $c_{DG} - \sigma$ decreases which explains the four possible equilibrium regions described in lemma 1.

If $c_{DG} - \sigma$ is very large, distributed generation is too expensive to be an alternative to grid based electricity for all consumers. Consequently, the standard result of spatial competition models applies. This case is depicted in figure 3(a).

As $c_{DG} - \sigma$ decreases, the first consumer is tempted to substitute grid based electricity with distributed
generation. The equilibrium answer of the retailers is to lower prices in order to render distributed generation just unattractive for consumers. As shown in figure 3(b) the reaction functions are downward sloping and overlap for this type of equilibrium. As a result there exist technically an infinite number of asymmetric equilibria. Restricting to symmetric equilibria yields the unique equilibrium described in lemma 1. The reaction functions are downward sloping because distributed generation is at the margin to competitiveness. If one of the retailers increases the price in this situation, consumers located in the middle of the Hotelling line start to use distributed generation. The best response of the corresponding other retailer is then to

---

As pointed out by Mérel and Sexton (2010), the focus on symmetric equilibria is not too restrictive because introducing even a slight elasticity into consumer demand establishes a unique symmetric equilibrium. Additionally the range of retail prices in the asymmetric equilibria is relatively small.
lower the price in order to gain market share and reestablish the situation in which distributed generation is just unattractive for the consumer with the largest preference dependent disutility.

If $c_{DG} - \sigma$ further decreases, it is no longer worthwhile for the retailers to fully compensate increased competitiveness of distributed generation with price reductions. Instead retailers give up on those customers least attracted to one of the two firms, which are located in the middle of the Hotelling line. Consequently, these consumers start to use distributed generation and avoid grid based electricity consumption. This equilibrium corresponds to the left intersection of the reaction functions in figure 3(c).

Finally, if distributed generation is very cheap, retailers give up on all substitutable electricity consumption. As a result retailers fully disregard consumers $C_s$ and focus on the non-substitutable share of electricity demand. As indicated by the right intersection of the reaction functions in figure 3(c), retailers return to the high equilibrium price of the first case. As shown in figure 3(c), the reaction functions can intersect twice, which means that serving consumers $C_s$ and $C_{ns}$ as well as as disregarding consumers $C_s$ are equilibrium solutions. From lemma 1 follows that this can only be the case for $w + \frac{2\sqrt{3}}{3+\sqrt{3}} t \leq c_{DG} - \sigma < w + \frac{2}{1+\sqrt{3}} t$.

Based on the described equilibria in the retail market proposition 1 is formulated.

**Proposition 1.** Increasing subsidies for distributed generation can increase the market price for grid based electricity.

**Proof.** See Appendix A.

Figure 4 depicts retail prices and the corresponding retailer profits as a function of the subsidized cost of distributed generation $c_{DG} - \sigma$ in order to clarify the intuition of proposition 1.\textsuperscript{15} Figures 4(a) and 4(b) distinguish five areas, which are discussed from right to left in the following.

In area I, the retail market is not affected by distributed generation and each retailer earns a profit of $t$ by charging a mark-up $t$ on wholesale prices, which corresponds to the first case of equation (9). In area II, retailers adjust retail prices to keep the market fully covered with grid based electricity as described in the second case of equation (9). Profits linearly decrease with $c_{DG} - \sigma$ because the quantity of sold electricity remains constant. In area III retailers further adjust prices but consumers in the middle of the Hotelling line start to use distributed generation. The slope of the price function in the third case is lower because there are price and quantity adjustments to changes in $c_{DG} - \sigma$. The profit function is quadratic for the same reason.

In area IV there are two possible equilibria which means that the reaction functions intersect in the third and in the fourth case of equation (8). As a result, price adjustments as in area III as well as disregarding...

\textsuperscript{15} The mathematical expressions of retailer profits are presented in the appendix.
consumers $C_s$ in order to serve only non-substitutable electricity consumption with higher mark-ups yield
stable symmetric equilibria. Retailer profits are strictly larger in the equilibrium where only consumers
$C_{ns}$ are served with grid based electricity in area IV. Finally in area V there is again only one symmetric
equilibrium, in which retailers discard consumers $C_s$ and all substitutable electricity consumption is met
with distributed generation.

![Graph](image)

(a) Retail prices in equilibrium
(b) Retailer profits in equilibrium

Figure 4: Solution of the retailer problem

With respect to the level of subsidization for distributed generation figure 4(a) shows that an increase
in subsidies lowers retail prices as long as both consumer groups $C_{ns}$ and $C_s$ are served by retailers because
distributed generation puts competitive pressure on retailers. However, if $c_{DG} - \sigma$ is already sufficiently low,
an increase in subsidization can shift the equilibrium from a situation in which both consumer groups $C_{ns}$
and $C_s$ are served to an equilibrium in which only consumers $C_{ns}$ are served by retailers. If this is the case,
the increased subsidization increases retail prices as stated in proposition 1.

### 3.3. Welfare effects

This section assesses the implications of the presented results on welfare. First the effect on consumer
surplus is discussed, followed by a discussion of total welfare effects.

#### 3.3.1. Consumer surplus

Consumer surplus consists of surplus of consumers $C_s$ and $C_{ns}$, which differs depending on the retail
market outcome. Both surplus functions can be determined by substituting the results of lemma 1 into the
utility functions and integrating over the consumer taste parameter $x$. The resulting total consumer surplus
function is presented in lemma 2.
Lemma 2. Consumer surplus in equilibrium is described by the following equation:

\[
CS = \begin{cases} 
2v - 2w - \frac{5}{2}t & \text{if } c_{DG} - \sigma \geq w + \frac{3}{2}t, \\
2v - 2(c_{DG} - \sigma) + \frac{t}{2} & \text{if } w + \frac{7}{6}t \leq c_{DG} - \sigma < w + \frac{3}{2}t, \\
2v - 2(c_{DG} - \sigma) + \frac{9(c_{DG} - \sigma - w + \frac{t}{2})^2}{25t} - \frac{t}{2} & \text{if } w + \frac{2\sqrt{3}}{5 + \sqrt{3}}t \leq c_{DG} - \sigma < w + \frac{7}{6}t, \\
2v - w - (c_{DG} - \sigma) - \frac{5}{4}t & \text{if } c_{DG} - \sigma < w + \frac{2}{1 + \sqrt{3}}t.
\end{cases}
\] (10)

Proof. See Appendix A.

The consumer surplus function consists of four parts, analogously to the four types of retail market equilibria. The subsidized cost of distributed generation \( c_{DG} - \sigma \) determine the retail market outcome and the subsequent level of consumer surplus. The main result with respect to the influence of subsidization of distributed generation on consumer surplus is described in proposition 2.

Proposition 2. Increasing subsidies for distributed generation can reduce consumer surplus even if consumers do not contribute to financing the subsidy payments.

Proof. See Appendix A.

To clarify the implications of proposition 2, figure 5 depicts the net effect of distributed generation on consumer surplus \( \Delta CS \) as a function of \( c_{DG} - \sigma \).\(^{16}\) Analogously to figure 4 five areas are distinguished. In area \( I \) the retail market is unaffected by distributed generation. In area \( II \), retailers adjust prices in order to keep the entire market covered with grid based electricity. As a result, consumer surplus increases as \( c_{DG} - \sigma \) decreases. Both consumer groups benefit from lower prices for distributed generation because prices are adjusted for all consumers. In area \( III \), consumers start to use distributed generation. Again, both consumer groups benefit from price adjustments as \( c_{DG} - \sigma \) decreases. Additionally consumer group \( C_s \) avoids costs due to taste mismatch by using distributed generation. Therefore, the surplus of consumers \( C_s \) in area \( III \) is strictly above surplus of consumers \( C_{ns} \) and the consumer surplus function is quadratic. In area \( IV \) there exist two equilibria, one in which both consumer groups \( C_{ns} \) and \( C_s \) are served and one in which consumers \( C_s \) are disregarded by retailers. In area \( V \), there is again a unique equilibrium in which only consumers \( C_{ns} \) are served by retailers. If only consumers \( C_{ns} \) are served by retailers, consumer surplus increases as \( c_{DG} - \sigma \) decreases because consumers \( C_s \) benefit from lower costs of distributed generation.

As shown in figure 5 there is a discontinuity in the consumer surplus function when consumers \( C_s \) are discarded by retailers. This discontinuity results of two effects. First, consumer group \( C_{ns} \) is charged a...\(^{16}\) Formally the net effect of distributed generation on consumer surplus is defined as \( \Delta CS = CS - (2v - 2w - \frac{5}{2}t) \).
higher retail price $p_{Ri} = w + t$. Because of the higher retail price surplus of consumers $C_{ns}$ is strictly below the surplus of consumers $C_s$ if consumer group $C_s$ is discarded by retailers. Second, all consumers $C_s$ are pushed into usage of distributed generation when retailers raise prices to $p_{Ri} = w + t$. A direct result from these two effects is that an increase in subsidy payments can decrease consumer surplus if the increased subsidy payments induce retailers to discard substitutable electricity demand in order to focus on the non-substitutable share of demand. This holds true even if the subsidy comes at no costs for consumers, which is assumed in this section.

3.3.2. Total surplus

Total welfare can be determined as the sum of retailer profits and consumer surplus. The aggregated welfare effects are described in lemma 3.

**Lemma 3.** Total surplus in equilibrium is described by the following equation:

$$TS = \begin{cases} 
2v - 2w - \frac{t}{2} & \text{if } c_{DG} - \sigma > w + \frac{7}{6}t \\
2v - 2(c_{DG} - \sigma) + \frac{84(c_{DG} - \sigma - w + \frac{t}{2})^2}{100t} - \frac{t}{2} & \text{if } w + \frac{2\sqrt{3}}{5 + \sqrt{3}t} \leq c_{DG} - \sigma < w + \frac{7}{6}t \\
2v - w - (c_{DG} - \sigma) - \frac{t}{4} & \text{if } c_{DG} - \sigma < w + \frac{2}{1 + \sqrt{3}t} 
\end{cases}$$

(11)

**Proof.** See Appendix A.

Because of the assumed inelastic electricity demand, total welfare changes are limited to two effects. First, consumers avoid costs due to taste mismatch when they use distributed generation. Second, distributed
generation is more costly than the wholesale price for electricity. Consequently consumers avoid paying rents to retailers by using an outside option that would not be competitive without the mark-ups charged by retailers. Based on the effect of subsidies for distributed generation on total surplus, proposition 3 is formulated.

**Proposition 3.** Usage of distributed generation increases total surplus if and only if $c_{DG} - \sigma < w + \frac{1}{4}t$.

*Proof.* See Appendix A.

To illustrate the intuition behind proposition 3, figure 6 depicts the net effect of distributed generation on total surplus $\Delta TS$.\(^{17}\) Again five areas are distinguished in figure 6. In area $I$, the retail market is unaffected by distributed generation. In area $II$, retailers adjust prices to keep the market fully covered with grid based electricity. However, total surplus remains unchanged because welfare is shifted from retailers to consumers without a net effect on total surplus. In area $III$ distributed generation enters the market and consumers avoid paying rents to retailers by directly producing electricity. However, distributed generation is still costly compared to the wholesale price of electricity when it enters the market because of the mark-up charged by retailers. As a result, the decrease in retailer profits outweighs the increase in consumer surplus and total surplus decreases as consumers start to adopt distributed generation.

\[ \Delta TS = TS - (2v - 2w - \frac{t}{4}) \]

In area $IV$ further price adjustments as well as discarding consumers $C_s$ are equilibrium solutions. A switch to an equilibrium, in which consumers $C_s$ are discarded by retailers always decreases total surplus in area $IV$ because all consumers $C_s$ are pushed into usage of distributed generation. In area $V$, only discarding...
consumers $C_s$ is an equilibrium solution. For low values of $c_{DG} - \sigma$ in area $V$ total surplus is higher compared to a situation without usage of distributed generation. This increase in total surplus emerges because all consumers $C_s$ use distributed generation and therefore avoid costs due to taste mismatch. Consequently, total surplus increases if the avoided costs due to taste mismatch exceed the difference between the subsidized costs of distributed generation $c_{DG} - \sigma$ and the wholesale price $w$.

4. Regulator problem

In the first stage of the model, the regulator decides on the subsidy for distributed generation. In this section, the optimal regulatory strategy is derived. In contrast to the welfare effects discussed in the previous section, the cost of the subsidy payments are accounted for. It is assumed that the regulator maximizes consumer welfare. Hence, a consumer surplus standard is applied in the model. Applying a consumer surplus standard instead of a total surplus standard in competition policy is controversial in economic literature. However it seems appropriate in the present context for two reasons. First, retail markets for electricity are still highly concentrated in many countries, which makes reducing market power of suppliers one of the main regulatory concerns in practice. Second, unbalanced powers between consumers and producers as a result of information asymmetries and lobbying activities, which is one of the main arguments in favor of a consumer surplus standard, seem to be an issue in the electricity industry.\footnote{For a general discussion of consumer surplus vs total surplus standard, see Motta (2004). A discussion of market concentration in retail markets for electricity in the United States and the European Union is provided in Morey and Kirsch (2016). Kang (2015) empirically analyzes lobby activity of the energy and electric utility industry in the United States.}

The regulator maximizes consumer surplus while taking into account the costs of the subsidy. Subsidy payments are assumed to be refinanced by end consumers on a per capita basis, which means that consumers can not avoid contributing to subsidy financing.\footnote{See section 5.1 for a discussion of a setting where consumers can avoid contributing to subsidy financing by using distributed generation.} The resulting maximization problem for the regulator is formulated in equation (12).

$$\max_{\sigma} CS - \sigma \cdot q_D$$

The regulator maximizes the difference between consumer surplus $CS$ and subsidy payments which are determined by the product of the level of subsidization $\sigma$ and the usage of distributed generation $q_D$. The regulator problem is solved by substituting the consumer surplus function formulated in lemma 2 into equation (12).

An important issue is that the regulator faces the possibility of multiple equilibria in the retail market, which means that the regulator can not anticipate with certainty the resulting equilibrium for some levels
of subsidization. Two different types of equilibria can emerge, in which retailers either choose to serve both consumer groups $C_s$ and $C_{ns}$ or choose to discard consumers $C_s$ and serve only consumers $C_{ns}$ in order to realize higher margins. The second type of equilibrium leads to strictly lower consumer surplus when multiple equilibria are possible. Because of this relation, it is assumed that the regulator does not risk the realization of the consumer harming equilibrium. This assumption can be interpreted as risk averse behavior of the regulator. Based on the described assumptions the optimal subsidy policy is summarized in lemma 4.

**Lemma 4.** Depending on the relationship between the cost of distributed generation and the wholesale price of electricity, the regulator chooses the following subsidies:

(i) For $c_{DG} - w > \frac{11}{6} t$, the regulator positions distributed generation as a competitor to grid based electricity with $\sigma = c_{DG} - w - \frac{2}{3} t$. There is no usage of distributed generation.

(ii) For $\frac{11}{6} t \geq c_{DG} - w \geq \frac{15 + \sqrt{3}}{5 + 5\sqrt{3}} t$, the regulator implements the optimal amount of distributed generation with $\sigma = \frac{1}{3}(2(c_{DG} - w) + t)$.

(iii) For $c_{DG} - w < \frac{15 + \sqrt{3}}{5 + 5\sqrt{3}} t$, the regulator avoids additional distributed generation in order to prevent retailers from charging the full mark-up while discarding consumers $C_s$ with $\sigma = c_{DG} - w - \frac{2}{3+\sqrt{3}} t$.

**Proof.** See Appendix A.

The implications of lemma 4 are best understood with the depiction of the consumer surplus function in figure 5. As discussed in section 3.3, consumers can benefit from distributed generation even if it is not used because retailers adjust prices in order keep the full market covered with grid based electricity. This can be exploited by the regulator to reduce market power of retailers and shift welfare from producers to consumers. Consequently, the regulator subsidizes distributed generation even if the usage is inefficient in order to position it as a competitor to grid based electricity which is described in the first part of lemma 4. This redistribution of welfare is without a cost because no distributed generation is used and no subsidy payments have to be made. In the second case of lemma 4, distributed generation is adopted by some consumers. The regulator chooses optimal subsidies in order to internalize the competitive effect of distributed generation into consumer decisions.

With increased adoption of distributed generation, retailers discard the substitutable share of electricity demand in order to charge higher mark-ups on the non-substitutable demand, which leads to a decrease in consumer surplus. In the third case of lemma 4, the regulator avoids this pricing strategy by setting the subsidy to a level, which ensures that retailers always choose to serve both consumer groups. Hence,

---

$^{20}$See lemma 2.

$^{21}$See figure 5.
the regulator avoids additional distributed generation in order to prevent retailers from raising prices. The regulator therefore never chooses a subsidy level that leads to full substitution of demand of consumers $C_s$ with distributed generation. This result is independent of the assumed risk averseness of the regulator. Under a different assumption, the regulator would risk the realization of the equilibrium where consumers $C_{ns}$ are discarded. However the regulator would still strictly prefer the retail equilibrium in which both consumer groups are served. The results are summarized in proposition 4.

**Proposition 4.** If the cost of subsidy payments is accounted for, maximal usage of distributed generation is never welfare optimal for consumers.

*Proof. See Appendix A.*

To give additional intuition for proposition 4, figure 7 shows the solution of the regulator problem as a function of $c_{DG}$. The depiction additionally differentiates between the two consumer groups $C_s$ and $C_{ns}$. In area $I$, no distributed generation is used but the regulator sets subsidies in order to position distributed generation as a competitor to grid based electricity which induces positive welfare effects for both consumer groups. In area $II$, distributed generation enters the market. Both consumer groups benefit as retail prices are further reduced. Consumers $C_s$ additionally avoid costs caused by taste mismatch which leads to a level of surplus strictly above the surplus of consumers $C_{ns}$ for $c_{DG} < w + \frac{11}{6}t$. In area $III$ the amount of distributed generation used by consumers $C_s$ is constant because the regulator avoids additional usage in order to protect consumers from higher retail prices. Nevertheless surplus for both consumer groups further increases with decreasing costs of distributed generation because the required subsidy payments decrease if distributed generation becomes more competitive.

![Figure 7: Consumer surplus in the solution of the regulator problem](image)
5. Extensions

This section presents two extensions of the basic model framework. Section 5.1 analyzes interactions between distributed generation and grid fees. Section 5.2 discusses the impact of the share of electricity demand that can be substituted with distributed generation.

5.1. Distributed generation and grid fees

In practice distributed generation is often subsidized indirectly with exemption rules. In many countries distributed generation is exempted from grid fee payments. In order to assess this within the presented model framework it is assumed that the electricity purchased from retailers has to be transported to consumers via a grid infrastructure, which causes fixed costs $fixc > 0$. Grid costs have to be recovered by charging grid fees. It is assumed that a benevolent grid operator sets grid fees in order to maximize consumer welfare analogously to the regulator in section 4. Two model settings are considered. In the first setting, the grid operator sets a two-part tariff consisting of an avoidable variable component $p_G$ and a fixed component $f$. This configuration is comparable to a network tariff regime with a volumetric component charged based on consumption from the grid and a fixed component charged based on the capacity of the grid connection. In the second setting, the grid operator can only set an avoidable variable component $p_G$, which corresponds to volumetric tariff structures in practice.

In the first analyzed model setting consumers can avoid the variable grid fee component by using distributed generation, while the fixed component $f$ can not be avoided. The exemption from grid fee payments is modeled by setting $\sigma = p_G$. The resulting problem of the grid operator is formulated in equations (13a) and (13b). Consumer surplus $CS$ is determined by the surplus function presented in lemma 3 with $\sigma = p_G$. Additionally, grid fee payments are added to the surplus function and the cost recovery constraint in equation (13b) is introduced.

$$\max_{p_G,f} CS$$

s.t. $p_G * (q_{R1} + q_{R2}) + 2 * f \geq fixc$

As there are no direct subsidy costs in the case of grid fee exemptions, the objective function (13a) consists only of consumer surplus. The solution of problem (13) is presented in lemma 5.
Lemma 5. Depending on the relationship between the cost of distributed generation and the wholesale price of electricity the grid operator chooses the following tariff structures:

(i) For \( c_{DG} - w > \frac{11}{6} t \), the grid operator positions distributed generation as a competitor to grid based electricity with \( p_G = c_{DG} - w - \frac{7}{6} t \). There is no usage of distributed generation and \( f = \frac{fixc}{2} - p_G \) ensures recovery of grid costs.

(ii) For \( \frac{11}{6} t \geq c_{DG} - w \geq \frac{15 + \sqrt{3}}{5 + 5\sqrt{3}} t \), the grid operator implements the optimal amount of distributed generation with:

\[
\begin{align*}
    p_G &= \frac{1}{7} \left(2(c_{DG} - w) + t\right) \\
    f &= \frac{49}{2} - \frac{6}{49} \left(c_{DG} - w + \frac{t}{2}\right)^2
\end{align*}
\]

(iii) For \( c_{DG} - w < \frac{15 + \sqrt{3}}{5 + 5\sqrt{3}} t \), the grid operator avoids additional distributed generation in order to prevent retailers from charging the full mark-up while disregarding consumers \( C_s \). Grid fees are set to \( p_G = c_{DG} - w - \frac{2}{1 + \sqrt{3}} t \) and \( f = \frac{fixc}{2} - (qR_1 + qR_2)p_G \).

Proof. See Appendix A.

Lemma 5 shows, that the optimal subsidy policy can be implemented with grid fee exemption rules. However, the optimal strategy can only be realized with a two-part tariff structure. In that case the grid operator can use the variable grid fee to incentivize optimal usage of distributed generation and adjust the fixed tariff accordingly in order to ensure recovery of grid costs. The fixed fee \( f \) could even be negative if the required subsides for distributed generation are large. Because of the two-part tariff structure it is ensured that all consumers contribute to financing fixed grid costs. Consequently, costs are allocated in accordance with the cost causation principle as distributed generation typically does not change fixed network costs in the short to medium term, especially if consumers keep a grid connection.\(^{22}\)

In practice, grid fees often consist only of volumetric tariffs charged based on the amount of electrical energy withdrawn from the grid. The main difference in a system with volumetric tariffs compared to a two-part tariff structure is that fixed grid costs have to be recovered with variable grid fees. This causes additional incentives to use distributed generation if decentralized production is exempted from grid fee payments because consumers can avoid contributing to fixed cost financing by using distributed generation.

Within the presented model framework this leads to the following reformulation of problem (13):

\[
\begin{align*}
    \max_{p_G} & \quad CS \\
    \text{s.t.} & \quad p_G (qR_1 + qR_2) \geq fixc
\end{align*}
\]

\(^{22}\)The issue of fixed cost recovery in the electricity system is discussed in detail in Borenstein (2016).
In the adjusted grid operator problem, there is only one decision variable \( p_G \). A direct result of this limitation is that the regulator is unable to position distributed generation as a competitor to grid based electricity because high variable grid fees directly reduce consumer surplus and a compensation via the fixed fee is not possible. Additionally, as distributed generation is adopted and consumers start to avoid grid fees by using distributed generation, the fixed grid costs have to be burdened on a smaller consumer base, which incentivizes additional usage of distributed generation. Because of this effect a stable solution where only a share of substitutable electricity demand is supplied with distributed generation exists only under strict conditions. If fixed grid costs are high compared to the other cost components of the electricity systems a spiral effect is induced and all substitutable demand is met with distributed generation as soon as it is the cheaper option for the first consumer.\(^{23}\) Consequently, a volumetric grid fee structure leads to inefficient levels of distributed generation within the presented model. The results are summarized in proposition 5. The detailed solution of problem (15) is presented in Appendix A.

**Proposition 5.** Optimal subsidization of distributed generation can be implemented with grid fee exemptions only with a two-part tariff structure.

**Proof.** See Appendix A.

5.2. The share of substitutable electricity demand

In the basic model \( \alpha = 1 \) is assumed. Consequently, the electricity demand that can be substituted with distributed generation equals the non-substitutable electricity demand. In reality the substitutable share of demand varies depending on a variety of factors such as technological constraints, geographical conditions, weather conditions or consumer characteristics. To analyze the impact of the share of substitutable electricity demand, this section generalizes the presented model by varying parameter \( \alpha \), while total electricity demand is kept unchanged. Hence, a share \( \alpha \) of total demand can be substituted with distributed generation while the remaining \( 2 - \alpha \) can only be supplied with grid based electricity.

The solution of the generalized model follows the same logic as the presented solution of the basic model. The detailed derivation is presented in Appendix B. Interestingly, varying the share of substitutable demand shifts the solution space but the main implications of the model remain. To illustrate the generalized model results, figure 8 depicts retail prices, retailer profits, consumer surplus and total surplus for different shares of substitutable electricity demand. As additional references, model results for \( \alpha = 0 \), which means that no distributed generation is available, and for \( \alpha = 2 \), which means that that electricity demand can be entirely supplied with distributed generation, are depicted in figure 8.

\(^{23}\)This effect is sometimes refereed to as the death spiral of public utilities, see Castaneda et al. (2017) for a discussion.
Figure 8(a) depicts the mark-up charged by retailers. It shows that the basic intuition described in lemma 1 and proposition 1 is independent of the value of $\alpha$. However, the higher the share of substitutable demand, the more retailers are willing to reduce the mark-up in order to compete against distributed generation. The reason is that the remaining demand they can cover if the substitutable share of demand is discarded, decreases as $\alpha$ increases. The corresponding effects on retailer profits are depicted in figure 8(b). The decrease in retailer profits is more pronounced the higher the share of substitutable demand. If only a small share of demand can be substituted with distributed generation, retailers choose earlier to supply only the non-substitutable share which stabilizes profits on a higher level.

The described dependency of retailer mark-ups on the level of $\alpha$ also shift the consumer surplus function.
as shown in figure 8(c). Again, the basic shape of the function described in lemma 2 remains. However, the potential gains in consumer surplus are higher, if a large share of demand can be supplied with distributed generation. Additionally, the drop in consumer surplus when retailers discard the substitutable share of demand, is smaller for large and small values of $\alpha$ and has a maximum for medium values. The reason for this is that as $\alpha$ increases, a smaller share of the consumers is affected when retailers raise prices. As $\alpha$ decreases on the other hand, retailers are less willing to adjust prices to distributed generation and discard the substitutable share of demand earlier, which leads to a less pronounced discontinuity.

The discussed effects directly transfer to the shape of the total surplus function depicted in figure 8(d). It can be seen that total surplus always decreases as consumers start to adopt distributed generation. The reason is that consumers avoid paying rents to retailers, which those generate by exercising market power. However, distributed generation is still more costly compared to the wholesale price for electricity when the first consumers start to use it, which leads to the decrease in total surplus. The breakeven point for total surplus is at $c_{DG} - \sigma = w + \frac{t}{\alpha}$ for most possible values of $\alpha$, which is consistent with proposition 3. Only for high values of $\alpha$ above 1.6, there are potentially positive welfare effects when both consumer groups are served by retailers which leads to a break-even point at slightly higher levels of $c_{DG} - \sigma$.\(^{24}\)

The dependency of consumer surplus on $\alpha$ also shapes the optimal regulatory strategy for subsidizing distributed generation. Nevertheless, the key properties of proposition 4 remain. It is always beneficial for consumers if the regulator positions distributed generation as a competitor to grid based electricity. However for low values of $\alpha$, retailers are more reluctant to reduce prices as a response to the outside competition because the share of non-substitutable demand is high. Consequently, the potential gains in consumer surplus due to subsidization of distributed generation are lower for low values of $\alpha$.\(^{25}\) For the same reason, retailers discard the substitutable share of demand at higher levels of $c_{DG} - \sigma$. In the basic model it is never optimal for the regulator to allow usage of distributed generation for all consumers $C_s$ as shown in proposition 4. In the generalized model this result remains true for a wide range of $\alpha$. Only for high shares of substitutable electricity demand full substitution with distributed generation can become welfare optimal for consumers. This result is summarized in proposition 6. The full generalized solution of the regulator problem is presented in Appendix B.

**Proposition 6.** If the subsidy costs are accounted for, full substitution of substitutable electricity demand with distributed generation can be optimal for consumers if and only if $\alpha \gtrsim 1.7$.

**Proof.** See Appendix B.

\(^{24}\)The exact value is $c_{DG} - \sigma - w = 0.2679t$. The calculation is based on the surplus function provided in Appendix B.

\(^{25}\)see figure 8(c).
6. Conclusion

This article analyzes the impact of distributed generation technologies on retail markets for electricity. A spatial competition framework is applied in order to account for horizontal product differentiation and heterogeneous consumer preferences with regard to electricity retailers. I find that distributed generation puts competitive pressure on retailers and induces lower retail prices. Therefore even consumers who do not use distributed generation benefit. Regulators can subsidize distributed generation in order to exploit this competitive effect and increase consumer surplus. However, if the cost of distributed generation is low and only a limited share of demand can be substituted with distributed generation, there is point at which retailers disregard the substitutable share of demand and focus on the non-substitutable consumption in order to realize higher mark-ups. As a result, increased subsidies for distributed generation can increase retail prices and harm consumers. In the optimal regulatory strategy this behaviour of retailers is therefore prevented by limiting usage of distributed generation.

The results of the analysis show that subsidies for distributed generation can be a regulatory tool to increase competition in retail markets for electricity. Hence, policy makers should design subsidy mechanisms for distributed generation with awareness for the competitive effects. In addition the analysis shows that grid fee exemptions, which are widely used in practice, are only suitable to implement the optimal regulatory strategy if a two part-tariff structure is in place. Exemption rules with volumetric grid fees lead to inefficient levels of distributed generation.

The analysis is conducted for distributed generation in electricity markets. However, the results can be also applied for the heating sector. Consumers can avoid gas consumption for heating by using alternative heating technologies based on renewable energy, for example solar thermal technologies. If gas is delivered to end consumers via a grid infrastructure, the discussed effects on refinancing of grid costs also apply for operators of gas grids.

In further research the presented theoretical framework could be extended to more complex representations of retail competition, for example by integrating switching costs into consumer decisions. Additionally, the wholesale market could be modeled in more detail by accounting for feedback effects of distributed generation on wholesale prices. Finally, an empirical evaluation of the presented propositions would be an important contribution to the understanding of the economics of distributed generation.
Proof of lemma 1.

Based on equation (4) the following first order conditions can be derived:

\[
\frac{\partial \pi_{Ri}}{\partial p_{Ri}} = \begin{cases} 
\frac{p_{Ri} - p_{Ri} + w}{t} & \text{if } p_{R-i} - t \leq p_{Ri} \leq 2(c_{DG} - \sigma) - p_{R-i} - t \\
\frac{p_{Ri} - 6p_{Ri} + 3w + 2(c_{DG} - \sigma)}{2t} & \text{if } 2(c_{DG} - \sigma) - p_{R-i} - t \leq p_{Ri} \leq c_{DG} - \sigma \\
\frac{p_{Ri} - p_{Ri} + w}{2t} & \text{if } p_{Ri} > c_{DG} - \sigma
\end{cases}
\] (A.1)

Setting \(\frac{\partial \pi_{Ri}}{\partial p_{Ri}} = 0\) and some reformulation yields equation (5). The four symmetric equilibria follow from the reformulations of the reaction function discussed in section 3.2. Despite the discontinuity and the nonmonotonicity of the reaction function (see equation (8)), existence of symmetric pure strategy equilibria is guaranteed because the game is symmetric with a one dimensional strategy space and all jumps in the best reply function are upwards (See theorem 2.6 in Vives (2001)). For the second case of lemma 1, symmetry is assumed.

Proof of proposition 1.

Distributed generation is competitive to grid based electricity if \(c_{DG} - \sigma < w + \frac{7}{6}t\). For \(w + \frac{7}{6}t \leq c_{DG} - \sigma < w + \frac{3}{2}t\) increased subsidies decrease retail prices as \(\frac{\partial p_{Ri}}{\partial \sigma} = -1\). For \(w + \frac{2\sqrt{3}}{5}t \leq c_{DG} - \sigma < w + \frac{7}{6}t\), \(\frac{\partial p_{Ri}}{\partial \sigma} = -\frac{2}{5}\). Consequently, increased subsidies decrease retail prices as long as both consumer groups are served by retailers. If retailers discard consumers \(C_{S}\) the retail price is \(w + t\), which is strictly larger than \(\frac{2(c_{DG} - \sigma) + 3w + t}{5}\) for \(c_{DG} - \sigma < w + 2t\). Consequently increased subsidies increase retail prices if the solution is shifted from an equilibrium where both consumer groups \(C_{S}\) and \(C_{NS}\) are served by retailers and \(c_{DG} - \sigma < w + \frac{3}{2}t\) to a solution where retailers discard consumers \(C_{S}\).

Proof of lemma 2.

Consumer surplus is calculated by integrating over the utility function of consumers. Consumer surplus for consumers \(C_{NS}\) is determined by equation (A.2).

\[
CS_{ns} = 2 \ast \int_{0}^{\frac{t}{2}} (v - p_{Ri} - tx)dx
\] (A.2)

Consumer surplus for consumers \(C_{S}\) is determined by the sum of surplus resulting from grid based electricity and distributed generation, where \(q_{Ris}\) stands for electricity sold by retailer \(i\) to consumers \(C_{S}\) and \(q_{D}\) stands
for distributed generation:

\[ CS_s = 2 \int_0^{q_{R_s}} (v - p_{R_s} - tx)dx + \int_0^{q_{D}} (v - (c_{DG} - \sigma))dx \] (A.3)

Substituting the results of lemma 1 into equations (A.2) and (A.3) and summing \( CS = CS_{ns} + CS_s \) yields lemma 2 after some reformulation.

**Proof of proposition 2.**

For \( c_{DG} - \sigma < w + \frac{3t}{2} \), consumer surplus is strictly increasing in subsidies as long as both consumer groups \( C_s \) and \( C_{ns} \) are served by retailers. For \( w + \frac{3t}{2} \leq c_{DG} - \sigma < w + \frac{7t}{6} \), \( \frac{\partial CS}{\partial \sigma} = 2 \) and for \( w + \frac{2\sqrt{3}}{5 + \sqrt{3}} t \leq c_{DG} - \sigma < w + \frac{7t}{6} \),

\[ \frac{\partial CS}{\partial \sigma} = \frac{18(c_{DG} - w + \frac{\sigma}{100t}) + 31}{25t} \]

which is strictly positive for \( c_{DG} - \sigma < w \). If retailers discard consumers \( C_s \), consumer surplus is determined by \( CS' = 2v - w - (c_{DG} - \sigma) - \frac{5t}{4} \). If both consumer groups are served and \( c_{DG} - \sigma < w + \frac{7t}{6} \), \( CS'' = 2v - 2(c_{DG} - \sigma) + \frac{9(c_{DG} - w + \frac{t}{2})^2}{25t} \). Because of \( CS' < CS'' \) for \( t > 0 \), increased subsidies decrease consumer surplus if solution is shifted to an equilibrium where both consumer groups are served, to an equilibrium where consumer \( C_s \) are discarded by retailers.

**Proof of lemma 3.**

Total surplus is determined by \( TS = CS + 2 \pi_{R_s} \). \( \pi_{R_s} \) is determined by substituting the results of lemma 1 into equation (4). The following expression can be derived:

\[
\pi_{R_s} = \begin{cases} 
  t & \text{if } c_{DG} - w - \sigma > \frac{3t}{2} \\
  \frac{c_{DG} - w - \sigma - \frac{t}{2}}{2} & \text{if } \frac{7t}{6} \leq c_{DG} - w - \sigma \leq \frac{3t}{2} \\
  \frac{6(c_{DG} - w - \sigma + \frac{t}{2})^2}{25t} & \text{if } w + \frac{2\sqrt{3}}{5 + \sqrt{3}} t \leq c_{DG} - \sigma < w + \frac{7t}{6} \\
  \frac{t}{2} & \text{if } c_{DG} - \sigma < w + \frac{2}{1 + \sqrt{3}} t 
\end{cases}
\] (A.4)

With equation (A.4), the results of lemma 2 and \( TS = CS + 2 \pi_{R_s} \), lemma 3 follows after some reformulation.

**Proof of proposition 3.**

If distributed generation is not used, total surplus is determined by \( TS' = 2v - 2w - \frac{t}{2} \). If only a share of consumers \( C_s \) uses distributed generation, total surplus is determined by \( TS'' = 2v - 2(c_{DG} - \sigma) + \frac{84(c_{DG} - \sigma - w + \frac{t}{2})^2}{100t} - \frac{t}{2} \). Because \( TS'' < TS' \) for \( w + \frac{2\sqrt{3}}{5 + \sqrt{3}} t \leq c_{DG} - \sigma < w + \frac{7t}{6} \), total surplus is strictly
smaller in the second case. If all consumers $C_s$ use distributed generation, total surplus is determined by $TS'' = 2v - w - (c_{DG} - \sigma) - \frac{1}{4}t$. Because of $TS'' > TS'$ for $c_{DG} - \sigma < w + \frac{1}{4}t$, proposition 3 follows.

**Proof of lemma 4.**

Equation (12) is strictly increasing in $\sigma$ for $c_{DG} - \sigma \geq w + \frac{7}{6}t$ because $q_D = 0$.

For $c_{DG} - \sigma > w + \frac{7}{6}t$ distributed generation is used by consumers. The first order condition of equation (12) with respect to subsidy $\sigma$ is:

$$\frac{6(2(c_{DG} - w) + t - 7\sigma)}{25t}$$

(A.5)

Based on the first order condition $\sigma = \frac{1}{7}(2(c_{DG} - w) + t)$ can be derived. The second order condition $-\frac{42}{25t}$ is strictly negative for $t > 0$, which proves a maximum. The solution is however only valid as long as the optimal subsidy level guarantees an equilibrium where both consumer groups $C_s$ and $C_{ns}$ are served by retailers. The threshold value can be determined with lemma 1:

$$\frac{1}{7}(2(c_{DG} - w) + t) \leq c_{DG} - w - \frac{2}{1+\sqrt{3}}t$$

(A.6)

Reformulating equation (A.6) yields $c_{DG} - w \geq \frac{15+\sqrt{3}}{5+5\sqrt{3}}$. If this condition is not true, the optimal subsidy can lead to an equilibrium where retailers discard consumers $C_s$ and raise prices. The regulator avoids this by setting the subsidy at the boundary of lemma 1 $\sigma = c_{DG} - w - \frac{2}{1+\sqrt{3}}t$. The last step is to check, if there is a value of $c_{DG} - w$, where an equilibrium with maximum possible usage of distributed generation is welfare optimal. This can be verified by substituting the corresponding solutions for $\sigma$ into the objective function and comparing the results. $\sigma = c_{DG} - w - \frac{2}{1+\sqrt{3}}t$ yields the solution:

$$CS_{Reg1} = 2v - 2c_{DG} + (6\sqrt{3} - 3)(c_{DG} - w) - (353 - 144\sqrt{3})t$$

(A.7)

The regulator objective function with maximum distributed generation $q_D = 1$ yields:

$$CS_{Reg2} = 2v - c_{DG} - w - \frac{5}{4}t$$

(A.8)

Comparing equation (A.7) with (A.8) yields $CS_{Reg1} > CS_{Reg2}$ for $c_{DG} > 0, w > 0, c_{DG} > w, t > 0$ which is true by assumption. As a result maximum usage of distributed generation is never optimal and lemma 4 follows.

**Proof of proposition 4.**
Proposition 4 follows directly from lemma 4.

Proof of lemma 5.

Grid fees are integrated into consumer utility by changing equations (1a) and (1b) to:

\[ U_{grid} = v - f - p_G - p_{Ri} - t|x_i - x| \quad \text{(A.9a)} \]

\[ U_{DG} = v - f - p_G - (c_{DG} - \sigma) \quad \text{(A.9b)} \]

Setting \( \sigma = p_G \) exempts distributed generation from variable grid fee payments. The consumer surplus function changes accordingly. Based on problem (13) the following lagrangian function is derived, with \( \lambda \) as the dual variable of the cost recovery constraint:

\[ L = CS + \lambda \left( 2f - fixc + p_G \frac{3(2(c_{DG} - p_G - w) + t)}{5t} \right) \quad \text{(A.10)} \]

\( \frac{\partial L}{\partial p_G} = 0, \frac{\partial L}{\partial f} = 0 \) and \( \frac{\partial L}{\partial \lambda} = 0 \) yields:

\[ p_G = \frac{1}{t} \left( 2(c_{DG} - w) + t \right) \quad \text{(A.11a)} \]

\[ f = \frac{fixc}{2} - \frac{6}{49t} \left( c_{DG} - w + \frac{t}{2} \right)^2 \quad \text{(A.11b)} \]

\[ \lambda = 1 \quad \text{(A.11c)} \]

The remainder follows exactly the same logic as the proof of proposition 4 and is thus omitted.

Proof of proposition 5.

The first part of proposition 5 follows from problem (15) because increased grid fees directly reduce consumer surplus if compensation via the fixed component is not possible. Positioning distributed generation as a competitor to grid based electricity for \( c_{DG} - w + inc > \frac{11}{6} t \) is therefore not possible.

For the second part of proposition problem (15) is solved with the lagrangian:

\[ L = CS + \lambda \left( - fixc + p_G \frac{3(2(c_{DG} - p_G - w) + t)}{5t} \right) \quad \text{(A.12)} \]
\( \frac{\partial L}{\partial p} = 0 \) and \( \frac{\partial L}{\partial \lambda} = 0 \) yields:

\[
p_G = \frac{1}{2} (c_{DG} - w + \frac{t}{2}) - \frac{\sqrt{3}}{12} \sqrt{-4\text{fixc} \ast t + 3(2(c_{DG} - w) + t)^2} \tag{A.13a}
\]

\[
\lambda = \frac{3}{10} \left(1 + \frac{\sqrt{3}(2(c_{DG} - w) + t)}{\sqrt{-4\text{fixc} \ast t + 3(2(c_{DG} - w) + t)^2}}\right) \tag{A.13b}
\]

Based on equations A.13a and A.13b it follows that there exists a real solution only if:

\[
\text{fixc} > \frac{3(2(c_{DG} - w) + t)^2}{40t} \tag{A.14}
\]

Substituting the results into the objective function shows that welfare with volumetric tariffs is strictly lower compared to the two-part tariff case unless:

\[
\text{fixc} = \frac{3(2(c_{DG} - w) + t)^2}{49t} \tag{A.15}
\]

If condition (A.15) is true, the resulting welfare is the same in both cases.

**Appendix B. Substitutable share of demand**

Varying the share of substitutable electricity demand changes the demand function from the basic model to:

\[
q_{Ri} = \begin{cases} 
\frac{t + p_{R-i} - p_{Ri}}{t} & \text{if } p_{R-i} - t \leq p_{Ri} \leq 2c_{DG} - 2\sigma - p_{R-i} - t \\
\frac{(2 - \alpha) t + p_{R-i} - p_{Ri}}{2t} + \alpha \frac{c_{DG} - \sigma - p_{Ri}}{t} & \text{if } 2c_{DG} - 2\sigma - p_{R-i} - t \leq p_{Ri} \leq c_{DG} - \sigma \\
\frac{(2 - \alpha) t + p_{R-i} - p_{Ri}}{2t} & \text{if } p_{Ri} > c_{DG} - \sigma 
\end{cases} \tag{B.1}
\]

Following the same steps as described in section 3.2, the retailer problem can be solved to derive the following retail prices:

\[
p_{Ri} = \begin{cases} 
w + t & \text{if } c_{DG} - \sigma - w > \frac{3}{2} \\
c_{DG} - \sigma - \frac{t}{2} & \text{if } \frac{3}{2} t \geq c_{DG} - \sigma - w \geq \frac{6 + \alpha}{4 + 2\alpha} t \\
\frac{\alpha(2(c_{DG} - \sigma) - t + w) + 2(t + w)}{2 + 3\alpha} & \text{if } \frac{6 + \alpha}{4 + 2\alpha} t \geq c_{DG} - \sigma - w \geq \frac{(2 + 3\alpha)\sqrt{4 - \alpha^2} - 4 + \alpha^2}{\alpha(6 + 5\alpha)} t \\
w + t & \text{if } c_{DG} - \sigma - w < \frac{\sqrt{4 - \alpha^2} - 2 + \alpha}{\alpha} 
\end{cases} \tag{B.2}
\]
Summing retailer profits and consumer surplus yields total surplus with total electricity consumption. The resulting expressions for consumer surplus are:

Substituting retail prices into the profit equation yields:

\[
\pi_{Ri} = \begin{cases} 
  t & \text{if } c_{DG} - w - \sigma > \frac{3}{2}t \\
  c_{DG} - w - \sigma - \frac{t}{2} & \text{if } \frac{3}{2}t \geq c_{DG} - \sigma - w \geq \frac{6 + \alpha}{4 + 2\alpha}t \\
  (2 + \alpha)(2t + 2\alpha(c_{DG} - \sigma - w - \frac{t}{2}))^2 & \text{if } \frac{6 + \alpha}{4 + 2\alpha}t \geq c_{DG} - \sigma - w \geq \frac{(2 + 3\alpha)\sqrt{4 - \alpha^2} - 4 + \alpha^2}{\alpha(6 + 5\alpha)}t \\
  \frac{t}{2}(2 - \alpha) & \text{if } c_{DG} - w < \frac{\sqrt{4 - \alpha^2} - 2 + \alpha}{\alpha}
\end{cases}
\]

To determine consumer surplus, transport costs for consumers \(C_s\) and \(C_{ns}\) are normalized to the corresponding total electricity consumption. The resulting expressions for consumer surplus are:

\[
CS_{ns} = 2 \int_0^{\frac{t}{\alpha}} (v - p_{Ri} - \frac{t}{2 - \alpha}x) \, dx \quad (B.4a)
\]

\[
CS_s = 2 \int_0^{p_{Ri}} (v - p_{Ri} - \frac{t}{\alpha}x) \, dx + \int_0^{q_{Ri}} (v - (c_{DG} - \sigma)) \, dx \quad (B.4b)
\]

With retail prices from equation (B.2) and \(A = (2c_{DG} - 2\sigma - 2w - 5t)\), \(B = (2c_{DG} - 2\sigma - 2w - t)\) consumer surplus \(CS = CS_{ns} + CS_s\) can be reformulated to:

\[
CS = \begin{cases} 
  2v - 2w - \frac{5}{2}t & \text{if } c_{DG} - w - \sigma > \frac{3}{2}t \\
  2v - 2c_{DG} + 2\sigma + \frac{t}{2} & \text{if } \frac{3}{2}t \geq c_{DG} - \sigma - w \geq \frac{6 + \alpha}{4 + 2\alpha}t \\
  2v - 2w + \frac{1}{4(2 + 3\alpha)^2t} \big(4\alpha(A^2 - 30t^2) + 2\alpha^2(2A^2 - 41t^2) + \alpha^3B^2 - 40t^2) & \text{if } \frac{6 + \alpha}{4 + 2\alpha}t \geq c_{DG} - \sigma - w \geq \frac{(2 + 3\alpha)\sqrt{4 - \alpha^2} - 4 + \alpha^2}{\alpha(6 + 5\alpha)}t \\
  2v - 2w - \alpha(c_{DG} - \sigma - w - \frac{5}{4}t) - \frac{5}{2}t & \text{if } c_{DG} - \sigma - w < \frac{\sqrt{4 - \alpha^2} - 2 + \alpha}{\alpha}
\end{cases}
\]

Summing retailer profits and consumer surplus yields total surplus with \(B = (2c_{DG} - 2\sigma - 2w - t)\), \(C = (2c_{DG} - 2\sigma - 2w - \frac{5}{4}t)\):

\[
TS = \begin{cases} 
  2v - 2w - \frac{t}{2} & \text{if } c_{DG} - w - \frac{6 + \alpha}{4 + 2\alpha}t \\
  2v - 2w + \frac{1}{4(2 + 3\alpha)^2t} \big(4\alpha(B^2 - 10t^2) + 2\alpha^2(6C^2 - \frac{35}{3}t^2) + 5\alpha^3B^2 - 8t^2) & \text{if } \frac{6 + \alpha}{4 + 2\alpha}t \geq c_{DG} - \sigma - w \geq \frac{(2 + 3\alpha)\sqrt{4 - \alpha^2} - 4 + \alpha^2}{\alpha(6 + 5\alpha)}t \\
  2v - 2w - \alpha(c_{DG} - \sigma - w - \frac{t}{4}) - \frac{t}{2} & \text{if } c_{DG} - \sigma - w < \frac{\sqrt{4 - \alpha^2} - 2 + \alpha}{\alpha}
\end{cases}
\]
Following exactly the same logic as in the proof of lemma 4 the following optimal regulatory strategy can be determined for $\alpha \gtrsim 1.7$:

(i) For $c_{DG} - w > \frac{10+\alpha}{2(2+\alpha)} t$, $\sigma = c_{DG} - w - \frac{6+\alpha}{4+2\alpha} t$.

(ii) For $c_{DG} - w \geq \frac{1}{\alpha(2+3\alpha)} (2(\alpha - 2)(2\alpha + 1) + (2 + 5\alpha)\sqrt{4 - \alpha^2})$, $\sigma = \frac{2\alpha(c_{DG} - w) + (2-\alpha)t}{2+5\alpha}$.

(iii) For $c_{DG} - w < \frac{1}{\alpha(2+3\alpha)} (2(\alpha - 2)(2\alpha + 1) + (2 + 5\alpha)\sqrt{4 - \alpha^2})$, $\sigma = c_{DG} - w - \frac{\sqrt{4 - \alpha^2} - 2 + \alpha}{\alpha}$.

**Proof of proposition 6.**

If all substitutable electricity demand is supplied with distributed generation, the following solution for the regulator problem can be derived:

$$CS - \sigma * qD = 2v - 2w - \frac{5}{2}t + \alpha(w - c_{DG} + \frac{5}{4}t) \quad (B.7)$$

Comparing equation (B.7) with the result of the regulator problem for $\sigma = c_{DG} - w - \frac{\sqrt{4 - \alpha^2} - 2 + \alpha}{\alpha}$ yields that maximum usage of distributed generation can be welfare optimal for $\alpha \gtrsim 1.6985$.

Acknowledgments

I am grateful for helpful comments from Felix Höffler, Jakob Peter, Christian Tode and participants of the Research Colloquium in Energy Economics at the University of Cologne. The work was partly carried out within the ENSURE project and the research group on Energy Transition and Climate Change (ET-CC). ET-CC is an UoC Emerging Group funded by the DFG Zukunftskonzept (ZUK 81/1). ENSURE is funded by the German Federal Ministry of Education and Research (Support Code 03SFK1A). The financial support is gratefully acknowledged.

References


Borenstein, S., 2016. The economics of fixed cost recovery by utilities. The Electricity Journal 29 (7), 5–12.


