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Abstract: We study the interdependency between two markets, where the first involves offering production capacity, while on the second actual production is sold. The key issue is that the expected product market outcome determines the opportunity cost for bidding at the capacity market while the capacity sold on the capacity market, since no longer available for spot market bidding, influences the product market outcome. We show that a competitive simultaneous equilibrium exists. This equilibrium is unique and efficient. It is characterized by a u-shaped bidding function in the capacity market with respect to the marginal cost of suppliers. The leading example is the electricity industry, where there is a capacity market clearing before the spot market.

Keywords: Capacity Market, Procurement Auction, Electricity Market, Competitive Equilibrium

JEL classification: D41, D44, L94, L11

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1. Introduction

Electricity markets are characterized by some properties that complicate a matching of demand and supply. First, electricity is virtually non-storable economically in large quantities. Second, demand for and supply of electricity are not perfectly predictable. Third, supply has to equal demand at any time, since otherwise the electricity grid would collapse. Moreover, due to technical restrictions end-consumers cannot respond to real-time electricity prices, so demand for electricity is basically inelastic in the short term (see, for example, Patrick and Wolak (2001)).

To ensure system stability, a network operator procures capacity to compensate for prediction errors and to fill the gap between demand and supply in the short term.[1] If demand exceeds supply, capacity is called. The procurement of capacity is usually organized on a separate market platform.[2] Demand for capacity is defined by the transmission system operator to ensure a well-defined safety level regarding grid stability. In most European countries a procurement auction is implemented in which the pricing mechanism can be uniform or pay-as-bid. The market for capacity clears before the spot market does.

The key issue here is that providing generation capacity is costly for two reasons: First, providing capacity decreases the potential spot market revenues. These opportunity costs from foregone spot market participation are decreasing in a firm's marginal cost. Second, keeping capacity ready for delivery on demand induces costs which are increasing in a firm's marginal costs because of a power plant's minimum production condition when being in a ready-to-operate mode. Thus, a firm with low marginal costs will generate electricity even when demand is low and will have relatively high opportunity costs from foregone spot market revenues when offering capacity. Conversely, a firm with high marginal costs will generate electricity only when demand is high, i.e. its opportunity costs from spot market sales are low, and the firm's cost of capacity provision are driven by being ready for delivery on demand. That is,

[1] In the electricity industry, capacity procured is called "inremental reserve", "reserve capacity" or sometimes "balancing power".

[2] As is the case in Germany, for example.

firms with intermediate cost levels will place the lowest bids on the market for capacity.

Although this form of interaction between a spot market and a capacity market is specific to the electricity industry from what I know, there are several markets where at least one of these cost components occurs. Costs of foregone revenues from production always arise when assets are rented to somebody; costs of capacity provision arise when keeping capacity ready to operate is costly. The electricity industry, while being an important example for the problem sketched in this paper, is not the only industry where both effects occur simultaneously. We will discuss another example in the discussion at the end of the paper.

In the following the analysis is conducted in terms of a spot electricity market and a market for capacity. We consider a continuum of firms having pair wise different marginal costs of electricity generation. Each one has a fixed production capacity, which he can split up so that a firm can sell quantities on both markets at the same time. The technical restriction is imposed that if a supplier wants to offer capacity he has to generate electricity at least at some minimum production level. This is because a plant providing capacity has to be running to ensure a short response time when capacity is called. In this event, its electricity generation can be increased quickly. We will see that the unique equilibrium consists of a u-shaped capacity market bidding function and that the set of suppliers selected to provide capacity constitutes an interval. Moreover, a welfare analysis will show that the equilibrium is efficient.

There is small but growing literature on capacity procurement in the electricity sector. One important line of research is motivated by the fact that capacity auctions can be seen as a multi-unit auction with interdependent private values.[3] The theory is applied to electricity markets in Hortacsu and Puller (2008), for example. Swider (2007) introduces a model in which the spot market is competitive and the capacity market is not. The prices on the

[3]The players' signals are interrelated since the opportunity cost consideration of every player depends on the stochastic spot market demand.

capacity market are modelled as random variables, which the bidders anticipate. Creti and Fabra (2006) model a short-term capacity market. Optimal bidding strategies for market participants are derived under consideration of opportunity costs that arise from foregone sales on a domestic and a foreign electricity market. It is assumed that all players have identical marginal costs. The authors derive equilibrium srategies for both a monopolistic and a competitive market structure.

Closely related to the present paper is the work of Just and Weber (2008) and Just (2011). These papers, in turn, rely partly on Chao and Wilson (2002), who investigate optimal scoring rules on multi-dimensional procurement auctions for power reserves. Just and Weber (2008) model the interdependencies between markets for secondary reserve capacity and spot electricity to derive the pricing of capacity under equilibrium conditions in a uniform pricing setting. Just (2011) applies this model set up and adresses questions on appropriate contract durations in the German markets for reserve capacity. Both articles investigate the model numerically. The present work provides analytical results for a specific form of the stylized model Just and Weber (2008) developed, i.e it is proved that a unique efficient competitive equilibrium exists.

The remainder of this paper is structured as follows. In section 2 the model is defined and the equilibrium concept is introduced. Some properties of the model are derived, which allows for narrowing down the model's strategy space. In Section 3 the existence and uniqueness of an equilibrium is proved. Section 4 provides a welfare analysis which will show that the equilibrium derived earlier is efficient. Finally, Section 5 summarizes the results.

2. The Model

The supply side is given by a continuum $X = [0, 1]$ of suppliers that have constant and pair wise different marginal costs. Suppliers are sorted by their marginal costs, so that the market's marginal cost curve $c : X \to \mathbf{R}_+$ is strictly increasing. For the sake of analytical convenience, let c be differentiable. Moreover, c is common knowledge. I will refer to c as the "merit order curve" throughout the paper.

The production capacity of every supplier $x \in X$ equals one. Each supplier bids quantities on both the spot and capacity market. Capacity market bidding takes place first. Moreover, the result of the capacity market auction is revealed before the spot market bidding takes place. In a second step the spot market will clear. Some of the overall generation capacity is then no longer available since it has been contracted on the capacity market.

We assume every supplier x bids some price $a(x)$ on the capacity market and his marginal costs $c(x)$ on the spot market. The share every supplier offers on the capacity market is fixed and given by $\alpha > 0$.[4] Thus, spot market only strategies will be excluded. This is not a limitation since every supplier x may choose $a(x)$ arbitrarily high.

An independent system operator (ISO) ensures that demand is met cost efficiently, which means the ISO selects the lowest bids on both markets.

2.1. Strategy Space and Payoff Function

The ISO ensures that demand for capacity ist met cost-efficiently by selecting the lowest bids. This can be formalized by defining an *allocation* which is an integrable function

$$s : [0,1] \longrightarrow \{0,1\}$$

satisfying

$$\int_0^1 s(y)dy = D_c, \tag{1}$$

i.e. demand for capacity is met.

For any given strategy profile $a : [0,1] \mapsto \mathbb{R}$ we want to find an allocation s_a which is consistent with a in the sense that it ensures cost efficiency, which

[4]This share is determined by a power plant's minimum and maximum production level as well as the power plant's gradient (see Müsgens et al. (2011) for details). For simplicity let α be the same for every supplier. Typically, $\alpha \approx 0.1$ (see for example Stoft (2002), p. 307).

means that the following condition must hold:

$$\text{If } a(x) < a(y) \text{ and } s_a(y) = 1, \text{ then } s_a(x) = 1. \tag{2}$$

Suppliers selected to provide capacity are rewarded by the marginal bid, i.e. the capacity market auction is uniform pricing.[5] For a given a, let

$$a^* := \inf_{x \in X} \{a(x) | \ s_a(x) = 0\}$$

denote the marginal bid.

Thus, a strategy profile a and the corresponding allocation s_a ensuring cost efficiency are interdependent. In order to be able to solve the model I will provide some results for an arbitrarily chosen s which satisfies (1) but not necessarily (2). These results hold for all a and all s and thus, in particular, for a consistent pair (a, s_a). From now on, let s be arbitrarily chosen, but fixed.

The allocation s transformes the supply function on the spot market for two reasons: First, consider that demand for electricity on the spot market is high. Then the firms providing capacity may happen to be inframarginal but can only generate electricity at the level $1 - \alpha$. This leads to a higher price on the spot market compared to the case where no market for capacity is considered.

Second, we impose the technical restriction that a power plant needs to operate at a level of β in order to be able to provide capacity. This is because otherwise a power plant cannot respond fast enough when capacity is actually called. If a power plant is providing capacity, then the cost of generating β are sunk, which implies that the firm will bid the share β at a price of zero on the spot market. This leads to lower price on the spot market when demand is low (cp. Figure 1).

In order to cover demand D_c for capacity the accumulated must-run production amounts to $q_1 = \beta D_c / \alpha$, which means that q_1 is a technical lower bound

[5]Since the merit order curve c is common knowledge, a pay-as-bid auction mechanism would lead to the same market outcome (see Müsgens et al. (2011)). All results on existence, uniqueness and efficiency of equilibria translate to the pay-as-bid case.

for the overall electricity generation in our model. Accordingly, the maximum electricity production in the market is given by $q_2 = 1 - D_c$. Let $Q = [q_1, q_2]$ in the following. Let now D_e denote the random spot market demand, where the support of D_e equals Q.[6] Let D_e be distributed with respect to some probability measure $\mathbf{P}$, and let $\mathbf{E}$ denote the expectation operator with respect to $\mathbf{P}$. For a given s we now want to define the inverse demand function $p_s : Q \mapsto [0, \infty)$ which maps quantities to prices and which is consistent with the following assumptions:

- A supplier x who does not provide capacity, i.e. where $s(x) = 0$, bids all of his capacity at marginal costs into the spot market,
- a supplier providing capacity is comitted to bid his must-run share β at a price of zero into the spot market,
- he bids the remaining share $1-\alpha-\beta$, which is assumed to strictly exceed zero, according to his marginal costs in the spot market.

For a given allocation s let $m_s(x)$ denote the amount of electricity suppliers bid into the market at a price not exceeding $c(x)$. Then m_s models the merit order transformation resulting from s and the restrictions described above. The transformation m_s can be written as

$$\begin{aligned} m_s :& [0,1] \longrightarrow Q, \\ m_s(x) =& q_1 + \int_0^x 1 - s(y)(\alpha+\beta)dy. \end{aligned} \tag{3}$$

That is, the integrand equals 1 if and only if $s(y) = 0$, which means that supplier y will bid all of his capacity into the spot market. If $s(y) = 1$, he will bid only the share $1 - \alpha - \beta$ at marginal costs, whereas the share β is bid at a price of zero and incorporated in q_1. Since q_1 is bid into the market at a price of zero $m_s(0) = q_1$ holds.

[6]We may also allow for the support of D_e to be an interval which is a subset of Q. All results persist, but the proofs will sometimes become cumbersome.

Note that the merit order transformation m_s is continuous and that it is invertible as long as $\alpha + \beta < 1$. Now the inverse m_s^{-1} maps a given level of demand to the supplier who's marginal costs equal the spot market clearing price:

$$p_s(D_e) = c \circ m_s^{-1}(D_e). \tag{4}$$

Note that if $D_c = 0$, then m_s is the identity and

$$p_s(D_e) = c(D_e).$$

Figure 1 shows the transformation of the merit order c via m_s: Since the must-run capacity q_1 is bid into the spot market at a price of zero, electricity prices decrease when demand is low in comparison with the original merit order c. Accordingly prices increase when demand is high since capacity with low marginal costs is providing capacity instead of generating electricity.

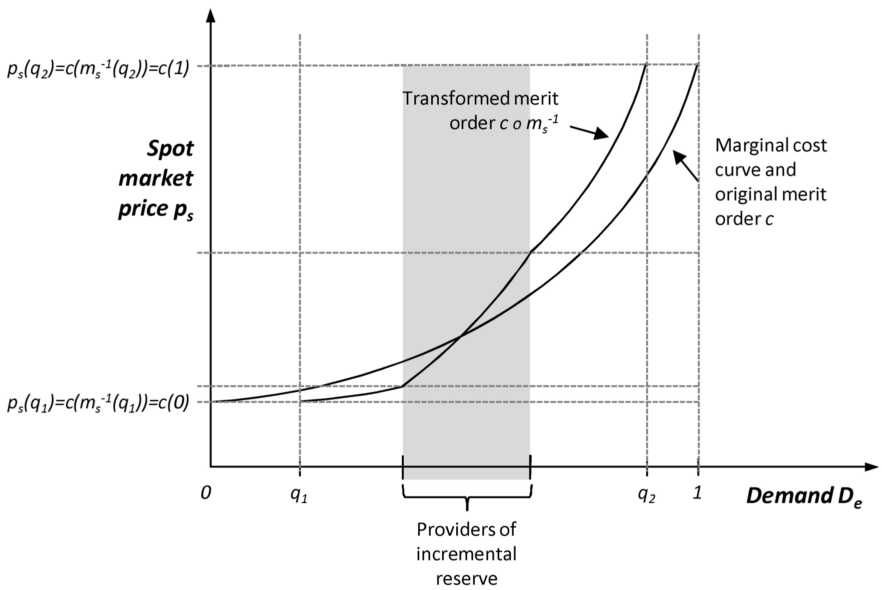

Figure 1: The transformation of the original merit order c via m_s.

After having defined the inverse demand function we can express the payoff function. The must-run costs of a supplier x are given by the expected difference of marginal costs and spot market price, multiplied by the minimum load factor β:

$$\beta\mathbf{E}\left[\left(c(x)-p_s(D_e)\right)\mathbb{1}_{\{c(x)\geq p_s(D_e)\}}\right]=\beta\mathbf{E}\left[\left(c(x)-p_s(D_e)\right)^+\right]. \tag{5}$$

Correspondingly, the spot market revenues are given by the expected difference between spot market price and marginal costs, multiplied with the remaining share $1-\alpha$ that is not contracted on the capacity market:

$$(1-\alpha)\mathbf{E}\left[\left(p_s(D_e)-c(x)\right)\mathbb{1}_{\{p_s(D_e)\geq c(x)\}}\right]=(1-\alpha)\mathbf{E}\left[\left(p_s(D_e)-c(x)\right)^+\right]. \tag{6}$$

The function Π describes a (risk neutral) supplier's profits for a fixed a and s and equals the sum of expected revenues on both markets minus the expected costs of keeping the plant running:

$$\Pi(x,a,s):=\begin{cases}\alpha a^*+(1-\alpha)\mathbf{E}\left[\left(p_s(D_e)-c(x)\right)^+\right]-\beta\mathbf{E}\left[\left(c(x)-p_s(D_e)\right)^+\right], & \text{if } a(x)\leq a^*,\\ \mathbf{E}\left[\left(p_s(D_e)-c(x)\right)^+\right] & \text{otherwise.}\end{cases} \tag{7}$$

Since the allocation s is arbitrarily chosen and does not ensure that demand for capacity D_c is met cost-efficiently, Π is not the payoff function but rather a helping function. The payoff function $\tilde{\Pi}$ is then given by

$$\tilde{\Pi}(x,a):=\Pi(x,a,s_a). \tag{8}$$

An equilibrium is a strategy profile a if for any x and any $\tilde{a}$ satisfying $\tilde{a}(y)=a(y)$ as long as $x\neq y$ it holds true that $\tilde{\Pi}(x,a)\geq\tilde{\Pi}(x,\tilde{a})$.

2.2. Firms Bid Opportunity Costs

In this section we will see that we can restrict the analysis to an opportunity cost function b that arises from expected gains and losses from spot market bidding. The basic argument is that given complete information every supplier bids his costs.

We will see that b is u-shaped along the merit order, which gives that those suppliers providing capacity constitute an interval in X in every equilibrium. This allows us to solve the interdependency of b and s_b.

We now define b in a way ensuring that the marginal bidder will exactly compensate his foregone sales on the spot market:

$$b(x,s) := \mathbf{E}\left[(p_s(D_e) - c(x))^+\right] + \frac{\beta}{\alpha}\mathbf{E}\left[(c(x) - p_s(D_e))^+\right]. \tag{9}$$

Note that $b(x,s)$ does not depend on the other supplier's bids. This is because s is fixed. Finding an equilibrium now reduces to finding the consistent allocation s, which means that s must be the cost-efficient procurement of D_c if suppliers bid according to $b(\cdot,s)$.[7]

If x places the highest accepted bid, if follows that $b(x,s) = b^*$ and thus

$$\Pi(x,a,s) = \mathbf{E}\left[(p_s(D_e) - c(x))^+\right], \tag{10}$$

i.e. the marginal supplier is indifferent between both markets. If supplier x places a bid that is not accepted, he will again generate revenues at the amount of

$$\Pi(x,a,s) = \mathbf{E}\left[(p_s(D_e) - c(x))^+\right]. \tag{11}$$

Any other supplier will place a bid that is lower than b^* and thus will generate higher revenues. Thus, the function $b(\cdot,s)$ can be seen as a weakly best response function. Moreover, every equilibrium a can be represented by a function of the form (9). The following proposition gives the formal statement.

Proposition 1. *If a is an equilibrium strategy profile, then $b(\cdot,s_a)$ is an equilibrium, too, and $s_a = s_b$ as well as $a^* = b^*$.*

Proof. See Appendix B. □

[7]Equivalently speaking, finding an equilibrium reduces to finding a fixed point of the mapping $s \mapsto s_{b(\cdot,s)}$.

That is, the equilibrium $b(\cdot, s_a)$ is equivalent to a in the sense that the market result does not change when moving from a to $b(\cdot, s_a)$. The basic intuition behind this result ist that firms bid their costs in a uniform pricing auction if the industrie's cost structure is common knowledge.

The first summand of b descibes the foregone spot market profits a firm faces when selling capacity. This cost component is decreasing in a firm's marginal costs. The second summand describes the costs of being in standby when offering capacity; these costs are increasing in a firm's marginal costs. Unsurprisingly the sum of both cost components is a convex function, as Theorem 1 implies:

Theorem 1. *The opportunity cost function $b(\cdot, s)$ is continuous and u-shaped.*

Proof. See Appendix A. □

U-shaped means that $b(\cdot, s)$ has a unique global minimum and strictly decreases beforehand and strictly increases afterwards. Theorem 1 states that suppliers at the boundary of X have high opportunity costs when bidding on the capacity market: For $x = 0$, expected losses from not bidding on the spot market are high, since the marginal costs are low. On the other hand, his must-run costs equal zero. Conversely, $x = 1$ has high must-run costs due to his high marginal costs, but his expected gains from spot market bidding are zero. If a supplier's marginal costs are close to the expected spot price he will place a relatively low bid.

The next corollary gives more information about the minimum of $b(\cdot, s)$. This result is provided because our strategy to show existence and uniqueness of equilibria will rely on controlling the minimum's location.

Corollary 1. *The bidding function $b(\cdot, s)$ has a unique global minimum $\overline{x}$ and $\overline{x}$ is defined by*

$$\boldsymbol{P}\left(D_e \leq m_s(\overline{x})\right) = \frac{\alpha}{\alpha + \beta}. \tag{12}$$

Proof. The existence of the minimum follows from the shape of $b(\cdot, s)$. The characterization of the minimum's location follows from the proof of Theorem 1 (see Appendix A). □

Corollary 1 implies that if the set of suppliers providing capacity is an intervall, and if the interval moves to the right, then the minimum $\overline{x}$ moves to the left – this follows from the definition of m_s.

3. Existence and Uniqueness of an Equilibrium

The following Theorem 2 states that in every equilibrium the set of suppliers providing capacity is an interval in X, which is due to the shape of b. This result is the key to our solution procedure: It allows us to establish a one-to-one correspondance between X and the set of all allocations s that can eventually arise in an equilibrium.

Theorem 2. *In every equilibrium the set of suppliers providing capacity is an interval.*

Proof. The statement follows from the shape of $b(\cdot, s)$. □

From now on, define $h := D_c/\alpha$ to ease notation. That is, h is the length of the intervall of suppliers providing capacity. We define s_x by

$$s_x(y) = 1 \text{ if and only if } y \in [x, x+h]\,.$$

By a slight abuse of notation let m_x denote the corresponding merit order transformation.

Now, the strategy to prove the existence of equilibria relies on the observation that by restricting the shape of s we have established a mapping

$$x \mapsto b(\cdot, s_x)$$

which maps $[0, 1-h]$ bijective to the set of strategies in which every equilibrium must necessarily be located, which is due to Theorem 2. We will analyze the function g defined by

$$
\begin{aligned}
g :& [0, 1-h] \to \mathbf{R}, \\
& x \mapsto b(x+h, s_x) - b(x, s_x).
\end{aligned}
$$

Figure 2 provides the connection between g and equilibrium solutions. The horizontal axis shows the continuum of suppliers. The interval $[x, x+h]$ contains those suppliers that are selected by s_x to provide capacity.

Consider the case where g has a zero. This corresponds to Fig. 2 B, where $b(x, s_x) = b(x+h, s_x)$ holds true, and where every supplier providing capacity is located in the inner of X. An obvious condition for g to possess a zero would be that given the allocation s_0 the intervall $[0, h]$ of firms providing capacity is located on the left side of the minimum of $b(\cdot, s_0)$ and that given the allocation s_{1-h} the intervall is located on the right hand side of the minimum of $b(\cdot, s_{1-h})$. Since b is u-shaped, it would follow that $g(0) < 0$ and $g(1-h) > 0$, and since g is continuous, a zero would exist. The next proposition proves the existence of a zero under these two conditions just mentioned above.

Contrarily, in the cases (A) and (C) the underlying model parameters are specified in a way that there does not exist an equilibrium in which $b(x, s_x) = b(x+h, s_x)$ holds, which means g does not have a zero. In this case, it must be $g > 0$ or $g < 0$ everywhere, since g is continuous. If $g < 0$, define $x = 1-h$, which corresponds to Fig. 2 A, and if $g > 0$, define $x = 0$, which corresponds to Fig. 2 C. However, it is the case, but not apparent, that (A) and (C) constitute equilibria.

In the remainder I will sometimes refer to the equilibrium pictured in Fig. 2 B as an *inner equilibrium* or *inner solution*, since x lies in the inner of $[0, 1-h]$. The next proposition establishes some properties of g and provides sufficient conditions that g has a zero, which is intuitively the case when the ratio D_c/Q is sufficiently small and β is sufficiently large. Since this line-up is typically given in markets for capacity ($D_c/Q \leq 0.03, \beta \geq 0.3$), the inner equilibrium as pictured in Fig. 2 B can be seen as the typical equilibrium.

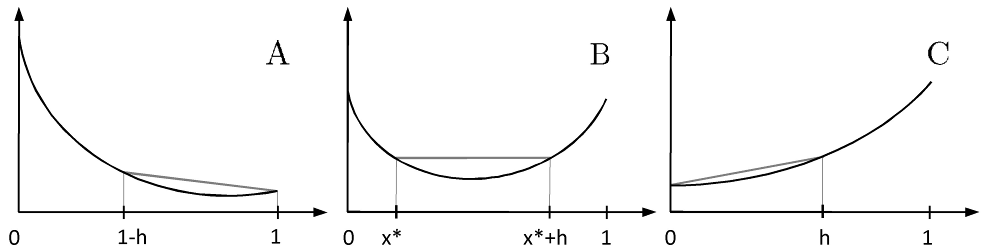

Figure 2: The three different possible types of equilibria.

Proposition 2. *The function g has at most one zero. If g has a zero x_0, then g is strictly increasing in a neighbourhood of x_0. A sufficient condition for g to have a zero is given by*

$$\boldsymbol{P}\left(D_e \leq D_c\left(\frac{1}{\alpha}-1\right)\right) \leq \frac{\alpha}{\alpha+\beta}$$

and

$$\boldsymbol{P}\left(D_e \leq 1-D_c\left(\frac{1-\beta}{\alpha}\right)\right) \geq \frac{\alpha}{\alpha+\beta}.$$

The intuition is that demand D_c for inremental reseve is relatively low compared to demand D_e for electricity (cp. Figure 2).

Proof. See Appendix C. □

The first inequality ensures that $g(0) < 0$, the second inequality leads to $g(1-h) > 0$. Since g is continuous, if follows that g has a zero.

Note that if $\beta = 0$, no costs of keeping the plant running arise, so that the first equality always holds true. On the other hand, since $\alpha < 1$, the second inequality does not hold true for any configuration of the model parameters, as long as $\beta = 0$. This is consistent with the fact that b is strictly decreasing when $\beta = 0$, which is easy to see. The next theorem is an immediate consequence of the proposition above.

Theorem 3. *A unique equilibrium exists.*

Proof. We will split the existence proof in three parts.

Firstly, assume that g has a zero x_0. It is apparent that $b(\cdot, s_{x_0})$ is an equilibrium in this case.

Second, let $g > 0$ everywhere. The proof of Proposition 2 gives that

$$\mathbf{P}\left(D_e \leq m_0(h)\right) = \mathbf{P}\left(D_e \leq D_c\left(\frac{1}{\alpha} - 1\right)\right) > \frac{\alpha}{\alpha + \beta}. \tag{13}$$

The first equation is a calculation. Now, $b(\cdot, s_0)$ is an equilibrium (as pictured in Fig. 2 C): Combining expressions (12) and (13), we conclude that the minimum of $b(\cdot, s_0)$ is located in $[0, h]$, since the mapping $x \mapsto \mathbf{P}\left(D_e \leq m_0(x)\right)$ is strictly increasing in x. Since $b(\cdot, s_0)$ is strictly increasing on $[h, 1]$, condition (2) is satisfied.

Third, let $g < 0$ everywhere. Then $b(\cdot, s_{1-h})$ is an equilibrium as pictured in Fig. 2 A: We argue by similar considerations as in the second case that $\min_X b(\cdot, s_{1-h}) \in [1 - h, 1]$, which gives the statement.

As in the existence proof, we will examine three cases in order to prove uniqueness. First, let $g > 0$. Assume that there exists $x > 0$ and that $b(\cdot, s_x)$ is an equilibrium. Let $\overline{x}$ denote the minimum of $b(\cdot, s_x)$. It follows that $\overline{x} \in [x, x + h]$. Since $g > 0$, it holds true that $b(x, s_x) < b(x + h, s_x)$. Since $b(\cdot, s_x)$ is continuous and strictly decreasing on $[0, x]$, we may choose $y \in [0, x]$ so that $b(y, s_x) < b(x + h, s_x)$. On the other hand, it holds true that $s_x(x + h) = 1$, $s_x(y) = 0$, which is a contradiction to the cost-efficiency of s_x.

The second case in which $g < 0$ is similar to the first and is omitted.

Third, if there exists x_0 so that $g(x_0) = 0$, then x_0 is unique, which follows from Proposition 2.[8] We conclude that in this case there exists exactly one equilibrium of the form pictured in Fig. 2 B. Moreover, $b(\cdot, s_0)$ and $b(\cdot, s_{1-h})$ do not constitute equilibria either, since, according to Proposition 2, it holds

[8]If we allow for the support of D_e to be an interval which is a subset of Q, then g is not strictly increasing anymore but only nondecreasing. However, if $[x, x + h] \cap supp(D_e)$ is empty, then $g(x) \neq 0$, because then $b(x, s_x) - b(x + h, s_x)$ has a very simple form and will equal either $c(x + h) - c(x)$ or $c(x) - c(x + h)$. If $[x, x + h] \cap supp(D_e)$ is non-empty, then $g'(x) > 0$ (see appendix). That is, the null of g remains unique.

true that $\min_X b(\cdot, s_0) \notin [0, h]$ and $\min_X b(\cdot, s_{1-h}) \notin [1-h, 1]$. At last, for any $x \in X$ satisfying $x \neq 0, x \neq 1-h$ and $g(x) \neq 0$, $b(\cdot, s_x)$ does not constitute an equilibrium by the arguments of the first case. □

4. Welfare Analysis

In this section we will see that the equilibrium is efficient. Since demand for capacity is inelastic, an efficient supply allocation is sufficient for efficiency. A supply allocation, in turn, is efficient if it minimizes overall costs (Müsgens et al., 2011).

The result is not apparent for the following reason: The optimal bid on the market for capacity is determined by a firm's opportunity costs emerging from potential later spot market activities. Since the outcome of the market for capacity does transform the supply side of the spot market, one has to argue how a firm's opportunity costs transform into a consumption of ressources induced by electricity generation in order to prove efficiency.

To provide intuition we will discuss the issue in a heuristic manner. For the moment, let demand D_e for electricity on the spot market be constant and equal some value d. Apparently, any efficient allocation is given by an interval $[x, x+h]$ that contains the marginal supplier $m_x^{-1}(d)$: If the interval was located on the left, then there would exist a firm $y \geq x+h$ which is inframarginal, i.e. $c(y) < p_x(d)$. Clearly, one could reduce costs by shifting the interval to the right so that y provides capacity and a cheaper firm solely produces electricity. On the other hand, $x \leq d$ is an apparent necessary condition for cost efficiency.

Now, let $m_x^{-1}(d) \in [x, x+h]$. We will discuss the marginal effect of shifting the intervall $[x, x+h]$ to the right.

Note first that shifting to the right implies that x will not provide capacity anymore and can increase its production by the share α. At the margin, this leads to negative additional costs of electricity generation that equal

$$\alpha[c(x) - c(m_x^{-1}(d))].$$

On the other hand, additional must-run costs emerge that equal

$$\beta c(x+h).$$

This reduces costs by

$$\beta c(m_x^{-1}(d)).$$

Shifting to the right thus changes costs according to

$$\begin{aligned}\gamma'(x) :=& \beta c(x+h) - \beta c(m_x^{-1}(d)) + \alpha[c(x) - c(m_x^{-1}(d))] \\ =& \beta[c(x+h) - p_x(d)] - \alpha[p_x(d) - c(x)] \\ =& \alpha g(x).\end{aligned}$$

Now, if s_x is the allocation of an inner equilibrium, then $g(x) = \gamma'(x) = 0$. In fact, x minimizes γ, since g is strictly increasing in a neighbourhood of x .

For the general case let us consider the expected costs of electricity production for a given s_x which equal

$$\gamma(x) := \mathbf{E}\left[\int_{q_1}^{D_e} p_x(q)dq\right] + \beta \int_x^{x+h} c(q)dq. \tag{14}$$

The first summand describes the the expected costs of generating electricity with respect to the transformed merit order when demand exceeds q_1, whereas the second summand describes the costs that arise from generating the amount q_1 due to the must-run condition.

Proposition 3. *The overall cost function γ satisfies $\gamma' = \alpha g$.*

Proof. See Appendix D. □

The factor α arises because the opportunity costs $b(\cdot, s_x)$ are per-unit costs, whereas γ describes the overall costs of production. Theorem 4 is an immediate consequence of Proposition 3.

Theorem 4. *Any equilibrium is efficient.*

Proof. This follows from the proposition above: If there exists an inner equilibrium and if s_x denotes the equilibrium allocation, then $\gamma'(x) = 0$. Since g is strictly increasing in a neighbourhood of x according to Proposition 2, x is a local minimum of γ. If g does not have a zero, then s_0 or s_{1-h} is the equilibrium allocation. Since $\gamma'(x) \neq 0$ for all x, γ is minimized by 0 or $1 - h$. Since the range of suppliers providing capacity must contain the bidding function's minimum in an efficient solution, γ is minimized by 0 if and only if s_0 is the equilibrium allocation. □

5. Discussion

We analyzed a stylized model that accounts for the main interdependencies between a spot electricity market and a capacity market. We have seen that the strategy space of the suppliers may be restricted to an opportunity cost function which is u-shaped: Opportunity costs arise from the alternative of spot market participation instead of providing capacity. These opportunity costs are decreasing in marginal costs. Additional costs of capacity provision arise from the technical requirement that power plants need to be running when they provide capacity, and these marginal costs are increasing with marginal costs.

An immediate consequence of this result is Theorem 2, which states that in every equilibrium the set of suppliers providing capacity is an interval. This gives that a unique equilibrium exists. Moreover, the equilibrium is efficient, since the opportunity costs a firm faces when placing a bid on the market for capacity turn into true costs when it comes to electricity generation on the spot market.

In the model suppliers differ only by their marginal costs. In reality, there is a large number of different power plants that exhibit very different technical and economical properties. For example, the share of capacity a power plant can offer on the capacity market depends on its specific technology, and some technologies do not even meet the technical requirements for providing capacity at all. Moreover, the minimum load condition varies extensively, and can even be zero (in the case of a pumped storage power station).

As mentioned earlier, the results developed might translate to other markets where there is demand for products as well as for production capacity. There are three essential characteristics the market must possess: (i) The firms differ with respect to their marginal profits per unit, (ii) the overall profit a firm generates is increasing in the product market demand and (iii) a firm has fixed costs of being ready-to-operate that are decreasing in the firm's marginal profit per unit. Beyond doubt, property (iii) is rare.

But consider, for example, two different restaurants A and B. Restaurant A has a reputation, whereas restaurant B has not. Every other restaurant in town is located in between of A and B regarding its reputation. All restaurants have an identical cost structure and provide service of equal quality. Due to restaurant A's reputation, prices in restaurant A are higher than in restaurant B, and the same translates to the profit per (customer). This gives property (i). Assume that the potential customers are equally distributed across those restaurants that are open on a specific day.[9] This gives property (ii).

Now, consider a small group of businessmen that wants to rent a dining hall in one of these two restaurants for a meeting. We will analyze the costs of renting the dining hall to the businessmen both restaurants face on a day with average demand.

Since restaurant A generates the highest profit per customer it will be the last restaurant in town to be closed when demand decreases. In particular, it will be open when demand is on average. Since the group of businessmen is sufficiently small, restaurant A will effectively loose customers when renting the dining hall to the businessmen. Thus, there arise opportunity costs from sending customers away.[10]

Conversely, due to the relatively low number of guests restaurant B will not be able to recover its labor costs that evening and thus it will be closed.[11]

[9] We just assume that this price structure and distribution of customers constitute a short term equilibrium.

[10] Here, we neglect the possibility that businessmen spend more money on average.

[11] On the basis of an ex-ante estimation, of course.

Now, if the group of businessmen is sufficiently small, the costs for restaurant B for renting a dining hall are driven by its labor costs which have to be recovered, i.e. by its costs of keeping capacity ready for delivery on demand. Now, if restaurant C has two dining rooms, it will operate the one that is not rented and thus will generate a contribution margin to cover its labor costs. That is, a restaurant with low marginal profits per unit has high costs of being ready-to-operate, which gives property (iii).

If prices decrease from A to B, then a restaurant with intermediate prices will offer the cheapest dining hall. Let C denote this restaurant. Notice that the allocation of the regular customers to the restaurants is transformed when C rents a dining hall to the businessmen: If C was open anyway, then renting the dining hall means decreasing supply on the product market. If C was originally meant to be closed, then renting the dining hall leads to an increase of supply on the product market, as long as C comes with at least two dining halls.

Appendix

A. Proof of Proposition 1

Let a be an equilibrium. We show that $b(x, s_a) \leq a^*$ if and only if $a(x) \leq a^*$: Choose $x \in X$ so that $s_a(x) = 1$ and $a(x) \neq b(x, s_a)$. Since a is an equilibrium, we must have $b(x, s_a) \leq a^*$, since $b(x, s_a) > a^*$ would imply $\Pi(x, a, s) < b(x, s_a)$, which is impossible since a is an equilibrium. This implies $s_a = s_b$ and the statement follows.

B. Proof of Theorem 1

Since $b(\cdot, s)$ is an integral of a bounded function and since the merit order curve c is differentiable, $b(\cdot, s)$ is continuous everywhere and differentiable almost everywhere. Let $\tilde{X}$ denote the set of points where $b(\cdot, s)$ is not differentiable. Let $x \in X \backslash \tilde{X}$. We calculate:

$$\frac{d}{dx} b(x, s) = c'(x) \left[(1 + \beta/\alpha) \mathbf{P} \left(D_e \leq m_s(x) \right) - 1 \right].$$

Recall that $c' > 0$ by assumption. The term on the right-hand side is increasing in x and equals zero if and only if $\mathbf{P}(D_e \leq m(x)) = (1 + \beta/\alpha)^{-1}$. Let F denote the distribution function of D_e. Then F is invertible on $[0, 1]$ since $f(x) > 0$ for all $x \in Q$.. Define

$$\overline{x} := m_s^{-1} \left(F^{-1} \left(\frac{1}{1 + \beta/\alpha} \right) \right).$$

Since $b(\cdot, s)$ is not differentiable everywhere, it remains to show that $b(\cdot, s)$ is strictly decreasing on $[0, \overline{x}]$ and strictly increasing on $[\overline{x}, 1]$. We define

$$b'(x, s) := 0 \quad \forall x \in \tilde{X}.$$

As it is an antiderivative of a function that is integrable with respect to the Lebesgue measure, $b(\cdot, s)$ is absolutely continuous. Thus, we may express $b(\cdot, s)$

as

$$b(x, s) = b(0, s) + \int_0^x b'(t, s)dt.$$

If $x, y \in [0, \overline{x}]$ and $x < y$, we conclude

$$b(y, s) - b(x, s) = \int_x^y b'(t, s)dt < 0.$$

A similar argument shows that $b(\cdot, s)$ is strictly increasing on $[\overline{x}, 1]$.

C. Proof of Proposition 2

To see that g has a zero under the assumptions of Proposition 2, we will show that $g(0) < 0$ and that $g(1 - h) > 0$ holds true. Then the statement follows since g is continuous.

In order to prove $g(0) < 0$ we show that

$$h \leq \arg\min_{x \in X} b(x, s_0) =: \overline{x}.$$

This means that under the allocation s_0 the minimum of the corresponding bidding function $b(\cdot, s_0)$ is located on the right hand side of the interval $[0, h]$. This is sufficient, since according to Theorem 1 $b(\cdot, s_0)$ is strictly decreasing on $[0, \overline{x}]$.

A calculation shows that we have $m_0(h) = h - D_c$. Corollary 1 gives that $\mathbf{P}(D_e \leq m_0(\overline{x})) = (1 + \beta/\alpha)^{-1}$. Since the mapping $x \mapsto \mathbf{P}(D_e \leq m_0(x))$ is strictly increasing in x it is sufficient to show that

$$\mathbf{P}(D_e \leq m_0(h)) \leq \frac{1}{1 + \beta/\alpha},$$

which follows from the assumptions:

$$\mathbf{P}(D_e \leq m_0(h)) = \mathbf{P}\left(D_e \leq \frac{D_c}{\alpha} - D_c\right) \leq \frac{1}{1 + \beta/\alpha}$$

The proof that $g(1-h) > 0$ holds is similar. We have to show that

$$1 - h \geq \arg\min_{x \in X} b(x, s_{1-h}) := \overline{x},$$

which means that under the allocation s_{1-h} the minimum of the corresponding bidding function is located on the left hand side of the interval $[1-h, 1]$. It is sufficient to show that

$$\mathbf{P}\left(D_e \leq m_{1-h}(1-h)\right) \geq \frac{1}{1+\beta/\alpha},$$

which follows again from our assumptions:

$$\begin{aligned}\mathbf{P}\left(D_e \leq m_{1-h}(1-h)\right) &= \mathbf{P}\left(D_e \leq q_1 + 1 - h\right) \\ &= \mathbf{P}\left(D_e \leq 1 - D_c\left(\frac{1-\beta}{\alpha}\right)\right) \\ &\geq \frac{1}{1+\beta/\alpha}.\end{aligned}$$

We will now see that we can find values $x_1, x_2 \in [0, 1-h]$ so that $g(x) < 0$ if $x \leq x_1$, $g(x) > 0$ if $x \geq x_2$ and so that g is strictly increasing on $[x_1, x_2]$. This is sufficient to prove the proposition.

Note first that Corollary 1 implies that if the range $[x, x+h]$ of firms providing capacity moves to the right, then the minimum of $b(\cdot, s_x)$ moves to the left, because the mapping $x \mapsto m_x(\cdot)$ is increasing in x. This means that under the assumptions of the proposition there exists a value x_1 so that the right edge of the interval $[x_1, x_1+h]$ and the minimum of the corresponding bidding function $b(\cdot, s_{x_1})$ coincide, i.e. $x_1 + h$ minimizes $b(\cdot, s_{x_1})$. On the other hand, there exists x_2 so that x_2 minimizes $b(\cdot, s_{x_2})$.

The u-shape of the bidding function and the fact that $g(0) < 0$ imply that $g(x) < 0$ if $x \leq x_1$ and $g(1-h) > 0$ implies that $g(x) > 0$ if $x \geq x_2$. It remains to show that g is strictly increasing on $[x_1, x_2]$. Choose x and y satisfying

$x_1 < x < y < x_2$. We will show that $g(y) - g(x) > 0$. This can be written as

$$\begin{aligned} g(y) - g(x) &= b(y+h, s_y) - b(y, s_y) - (b(x+h, s_x) - b(x, s_x)) \\ &= \underbrace{b(y+h, s_y) - b(x+h, s_x)}_{(I)} + \underbrace{b(x, s_x) - b(y, s_y)}_{(II)}. \end{aligned}$$

Let's have a look at (II):

$$b(x, s_x) - b(y, s_y) = \underbrace{b(x, s_x) - b(x, s_y)}_{A} + \underbrace{b(x, s_y) - b(y, s_y)}_{B}$$

Now, expression (B) strictly exceeds zero, because the function $b(\cdot, s_y)$ is u-shaped, and the function's minimum strictly exceeds y by construction of $[x_1, x_2]$. It remains to show that expression (A) is non-negative. To see this, choose $z < x < y$ and consider the difference $b(z, s_x) - b(z, s_y)$. We will show that this difference is non-negative, and since the difference is continuous in z, the limit $z \to x$ is non-negative. Key to this result is the observation that the must-run costs of firm z are equal for both the allocation s_x and s_y, because must-run costs only occur for z when a firm $\tilde{z} < z$ happens to be the marginal supplier on the spot market. Since $\tilde{z} < z < x < y$, the must-run costs of z are not affected when moving from s_x to s_y. On the other hand, the foregone spot market revenues for z decrease when the allocation moves from s_x to s_y, because the spot market price (weakly) decreases. More formally (note that $m_x \leq m_y$):

$$\begin{aligned} b(z, s_x) - b(z, s_y) &= \mathbf{E}\left[(p_x(D_e) - c(z))^+\right] - \mathbf{E}\left[(p_y(D_e) - c(z))^+\right] \\ &= \int_{m_x(z)}^{q_2} (c(m_x^{-1}(t)) - c(z)) f(t) dt - \int_{m_y(z)}^{q_2} (c(m_y^{-1}(t)) - c(z)) f(t) dt \\ &\geq \int_{m_x(z)}^{q_2} (c(m_x^{-1}(t)) - c(z)) f(t) dt - \int_{m_x(z)}^{q_2} (c(m_y^{-1}(t)) - c(z)) f(t) dt \\ &\geq \int_{m_x(z)}^{q_2} c(m_x^{-1}(t)) f(t) dt - \int_{m_x(z)}^{q_2} c(m_x^{-1}(t)) f(t) dt = 0. \end{aligned}$$

It remains to show that (I) is non-negative. The proof is similar to the proof that (II) exceeds zero: By construction, $x+h$ and $y+h$ are located on the right hand side of the minimum of $b(\cdot, s_x)$ so that we can take advantage of the u-shape of $b(\cdot, s_x)$. Moreover, the foregone spot market revenues of a firm $z > y+h$ are not affected when the allocation moves from s_x to s_y in analogy to the situation above. The details are omitted.

D. Proof of Proposition 3

Proof. By applying Fubini's theorem to the first summand of (14) and then transformation formula with transformation m_x we calculate (remember $m_x(0) = q_1$, $m_x(1) = q_2$ and the definition of m_x.):

$$\begin{aligned}\gamma(x) &= \int_{q_1}^{q_2} p_x(q)\mathbf{P}\left(q \leq D_e\right) dq + \beta \int_x^{x+h} c(y)dy \\ &= \int_0^1 c(y)\left(1 - s_x(y)(\alpha+\beta)\right)\mathbf{P}\left(m_x(y) \leq D_e\right) dy + \beta \int_x^{x+h} c(y)dy. \quad (15)\end{aligned}$$

In order to be able to calculate the derivative of γ we write (not that on the intervals $[0, x], [x, x+h], [x+h, 1]$ the function s_x is constant and equals 0 or 1):

$$\begin{aligned}\gamma(x) = &\int_0^x c(y)\mathbf{P}\left(m_x(y) \leq D_e\right) dy && (16)\\ &+ \int_x^{x+h} c(y)(1-\alpha-\beta)\mathbf{P}\left(m_x(y) \leq D_e\right) dy && (17)\\ &+ \int_{x+h}^1 c(y)(1-\alpha-\beta)\mathbf{P}\left(m_x(y) \leq D_e\right) dy && (18)\\ &+ \beta \int_x^{x+h} c(y)dy. && (19)\end{aligned}$$

Note now that on $[0, x]$ and $[x+h, 1]$, the function $m_x(\cdot)$ does not depend on x, which makes it easy to differentiate expressions (16) and (18) with respect to x. Expression (17) is differentiated by applying the multi-dimensional chain rule to the function $\tilde{g}(\phi(x))$, where

$$\begin{aligned}\tilde{g}(x,z) &:= \int_x^{x+h} c(y)(1-\alpha-\beta)\mathbf{P}\left(m_z(y) \leq D_e\right) dy \\ \phi(x) &:= (x,x).\end{aligned}$$

We calculate:

$$\begin{aligned}\gamma'(x) =& c(x)\mathbf{P}\left(m_x(x) \leq D_e\right) - c(x+h)\mathbf{P}\left(m_x(x+h) \leq D_e\right) \\ &+ (1-\alpha-\beta)\left[c(x+h)\mathbf{P}\left(m_x(x+h) \leq D_e\right) - c(x)\mathbf{P}\left(m_x(x) \leq +D_e\right)\right] \\ &+ \beta\left(c(x+h) - c(x)\right) - (\alpha+\beta)\int_{m_x(x)}^{m_x(x+h)} f(y)p_x(y)dy \\ =& c(x)\left[(\alpha+\beta)\mathbf{P}\left(m_x(x) \leq D_e\right) - \beta\right] \\ &- c(x+h)\left[(\alpha+\beta)\mathbf{P}\left(m_x(x+h) \leq D_e\right) - \beta\right] \qquad (20) \\ &- (\alpha+\beta)\int_{m_x(x)}^{m_x(x+h)} f(y)p_x(y)dy. \qquad (21)\end{aligned}$$

Second, we will derive $\alpha g(x)$:

$$
\begin{aligned}
\alpha g(x) =&\alpha \int_{m_x(x+h)}^{q_2} f(y)\left(p_x(y)-c(x+h)\right)dy\\
&+\beta \int_{q_1}^{m_x(x+h)} f(y)\left(c(x+h)-p_x(y)\right)dy\\
&-\alpha \int_{m_x(x)}^{q_2} f(y)\left(p_x(y)-c(x)\right)dy\\
&-\beta \int_{q_1}^{m_x(x)} f(y)\left(c(x)-p_x(y)\right)dy\\
=&c(x)\left[\alpha\mathbf{P}\left(m_x(x)\le D_e\right)-\beta\mathbf{P}\left(D_e\le m_x(x)\right)\right]\\
&-c(x+h)\left[\alpha\mathbf{P}\left(m_x(x+h)\le D_e\right)-\beta\mathbf{P}\left(D_e\le m_x(x+h)\right)\right]\\
&-(\alpha+\beta)\int_{m_x(x)}^{m_x(x+h)} f(y)p_x(y)dy\\
=&c(x)\left[(\alpha+\beta)\mathbf{P}\left(m_x(x)\le D_e\right)-\beta\right]\\
&-c(x+h)\left[(\alpha+\beta)\mathbf{P}\left(m_x(x+h)\le D_e\right)-\beta\right]\\
&-(\alpha+\beta)\int_{m_x(x)}^{m_x(x+h)} f(y)p_x(y)dy\\
=&\gamma'(x).
\end{aligned}
$$

$\square$

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