On the functional form of short-term electricity demand response - insights from high-price years in Germany

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Abstract

Demand response is crucial for balancing supply and demand in the presence of intermittent electricity generation, particularly during scarcity situations with high prices. In 2021/2022, wholesale power prices in Germany have been dramatically higher than ever before, which offers the opportunity to investigate the demand response in high-price situations. This paper, thus, discusses the applicability of the two common functional forms of demand response under these circumstances, namely linear and log-log. Using a two-stage least squares approach, the short-term own-price elasticity of electricity demand in Germany is estimated for the period from 2015 to 2022, employing the two assumptions for the functional form of the demand function. The day-ahead forecast of wind power generation serves as an instrumental variable for the day-ahead price. The analysis shows that for low prices, the linear assumption tends to yield similar average elasticities to the constant elasticity obtained from the log-log specification. However, estimators based on linear functions exhibit significant variations depending on whether low or high prices are considered. This discrepancy arises from the observation that remaining demand at high prices tends to have limited flexibility, leading to distinct estimations. In contrast, the log-log approach provides smaller differences between estimates based on low and high prices. The exponential nature of the log-log function effectively captures the decrease in absolute demand response at high prices, resulting in more consistent estimations across the price range.

Keywords: Electricity demand, demand function, short-term elasticity, 2SLS

JEL classification: C36, D22, Q21, Q41, L94

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1. Introduction

In traditional, centralized power systems, demand is matched by supply from conventional and dispatchable electricity generation units. In this system, the potential of demand reacting flexibly to prices at short notice has played a subordinate role. With the increase in intermittent generation and the declining capacity of dispatchable power plants, understanding the short-term own-price elasticity of electricity demand becomes increasingly important in understanding market realities. Demand response becomes more relevant in the high-frequency matching of supply and demand as supply from intermittent generating units fluctuates.

Obtaining accurate estimates of short-term own-price elasticities of electricity demand is, therefore, important in planning the future electricity system. Assumptions on how electricity demand reacts to prices in peak load situations influence the extent of future capacity needs. Assumed short-term elasticities thus influence the design of capacity mechanisms such as capacity markets or payments. In addition, reliable information about the demand response in the electricity market can improve operational decisions, e.g. in grid operation or the dispatch of generation units.

Both in the case of medium and long-term planning decisions and the case of short-term operational decisions, elasticity information is incorporated into quantitative models. Various models, e.g. electricity market models, price forecast models, infrastructure models or general-equilibrium models, exist in academia and the industry that inform policy and business decisions. All these models require precise information on price elasticities.

The level of short-run own-price elasticities of electricity demand[1] is of particular interest in high-price situations. In the last two years, prices in Germany have been drastically higher and more volatile than historically. The corresponding availability of new observations on demand response offers the opportunity to update estimates of demand elasticity, with a specific focus on situations characterized by high prices. Furthermore, this provides a chance to explore the functional relationship between demand response and price levels. By incorporating these new observations, a more refined understanding of how demand responds to varying price levels can be attained. So

[1]Since the present paper deals with own-price elasticities, for simplicity, "price elasticity" or "elasticity" is used synonymously with "own-price elasticity of electricity demand" in the remainder of the paper.

far, there are no estimates of short-term elasticities that include data from these high-price years or any other information concerning comparably high prices.

Demand responses are commonly characterized using either linear or exponential price-demand relationships. The latter is modeled by applying the natural logarithm to both variables and implies a constant elasticity (log-log specification). While the disparities between these two approaches are likely small at lower price levels, the question arises as to their compatibility with the observed wide-ranging price dispersion. Since the shape of the demand curve is unknown ex-ante, this research aims to address the superiority of one assumption over the other, drawing upon theoretical expectations derived from the electricity market under consideration and the available data. In the course of this, this study will explore the extent to which estimates of demand elasticities, considering the newly available data on high prices, differ from existing estimates.

Consequently, the present study firstly discusses the two distinct functional forms of demand-price response against the background of the German electricity market. Based on the insights gained from this examination, hypotheses are formulated for the empirical study. Subsequently, short-term own-price elasticities are estimated using data spanning the years 2015-2022 in the German wholesale electricity market, employing both functional forms. Recognizing the endogeneity issue between demand and prices, a two-stage least squares approach (2SLS) is employed, utilizing the day-ahead forecast of wind power generation as an instrumental variable for day-ahead electricity prices. Particular attention is paid to whether the short-term elasticity changed in 2021 and 2022 when consumers faced higher prices. To facilitate this examination, the analysis is divided into two distinct periods: the "low-price years" 2015-2020 and the "high-price years" 2021-2022.

Three hypotheses are put forward and then empirically confirmed: Firstly, in low-price years, the assumption of a linear relationship between demand and price yields similar average elasticities as the assumption of constant elasticities. This similarity, however, is not clear-cut and does not hold in high-price years. Secondly, when assuming a linear relationship between price and demand, the estimates for the demand response to a price increase of 1 EUR/MWh differ significantly between high- and low-price years (–34.3 MW/(EUR/MWh) vs −109.4 MW/(EUR/MWh)). The discrepancy arises, as the linear specification cannot account for decreasing absolute demand flexibility with increasing prices. Lastly, under the log-log specification, which assumes a constant elasticity, estimates are much closer together (–5.3 % vs –4.2 %). The exponential nature of the log-log function accounts for the decline in absolute demand response (per price change of 1 EUR/MWh) at high prices, resulting in more consistent estimations across the price range.

The paper is organized as follows. Section 2 provides an overview of the empirical literature on short-term (hourly) price elasticity estimation and the contribution of the paper at hand. Section 3 outlines basics of the German wholesale electricity market. Further, the Section introduces the two assumptions for the functional form of the demand function, discusses their properties and formulates hypotheses for the empirical study. Section 4 introduces the empirical strategy, drawing on the findings from Section 3, while Section 5 presents the data set utilized. The empirical results are shown and discussed in Section 6 before the paper concludes with Section 7.

2. Literature review and contribution

There is a wide range of literature on empirical estimation of price elasticities in the energy and electricity sectors. A general distinction is made between self-elasticity or own-price elasticity, i.e. the elasticity of demand to changes in the price of the good itself, and cross-price elasticities, i.e. the response of demand for one good to changes in the price of another good (in the case of electricity, for example, the price of electricity at other points in time (e.g. Filippini (2011)) or the price of natural gas (e.g. Woo et al. (2018) or Gautam and Paudel (2018)). Most papers, like the one at hand, deal with own-price elasticities. For readers who are interested in existing elasticity estimates, papers with comprehensive summaries of elasticity estimates and estimation methods have been published (Boogen et al., 2017; Andruszkiewicz et al., 2019; Ciarreta et al., 2023). In addition, explicit meta-studies have been conducted to review the state of research (Espey and Espey, 2004; Labandeira et al., 2017).

This paper seeks to estimate short-term price elasticities. Here, "short-term" refers to the hourly price reactions of consumers. The estimation and analysis of demand reactions in an hourly resolution (sometimes also called "real-time" elasticities (Lijesen, 2007)) constitutes a much smaller strand of literature. In this context, a distinction can be made between papers whose analyses are based on individual consumer consumption data and those that draw on aggregated data.

Taylor et al. (2005), Wolak (2011), Cosmo et al. (2014), and Fabra et al. (2021) use individual measurement data to examine the hourly demand response of individual consumers to various dynamic, time-of-use, or real-time pricing tariffs in different countries. These studies can be used, for example, to assess the impact and reactions of customers to the introduction of new price regulations. In contrast, there are studies that do not focus on individual consumers but on the overall demand response at the system level. As in the present paper, the goal is not to find out how pricing or taxation schemes affect individual consumers but rather to determine what demand reactions are caused by prices at the national level in the status quo. The first paper addressing this question for the Netherlands, based on data for the year 2003, is Lijesen (2007). He utilizes the lagged electricity price as instrumental variable to account for the simultaneity issue between price and demand. He estimates the price elasticity based on a linear specification of the demand curve to be -0.0013 and based on a logarithmic form to be -0.0043. These values are very low and imply an almost completely inelastic demand in the short run. Hirth et al. (2023), however, argue that the exogeneity assumption for the use of lagged prices as an instrumental variable, given the strong serial correlation and intertemporal interrelations, is unlikely to hold.

For Germany, three papers estimate hourly price elasticities of electricity demand. The first is Bönte et al. (2015). The paper uses ask prices from the EPEX SPOT power exchange for 2010-2014 to estimate a short-term elasticity of -0.43. This approach fundamentally differs from the one used in the present paper, as I utilize day-ahead prices and data on the realized aggregated demand. In contrast to the realized aggregated demand, the demand curves of the power exchange do not include the entirety of demand. However, they do include power traded over several markets and financial stages (Hirth et al., 2023) as well as prior internal matching of supply and demand of the market participants (Knaut and Paulus, 2016) (see Section 3.1).

The two papers that are methodologically closest to the one at hand are Knaut and Paulus (2016) and Hirth et al. (2023). Both papers use wind power generation as instrumental variable to estimate the hourly price elasticity of electricity demand in Germany. Knaut and Paulus (2016) estimate the elasticity, based on data for 2015, to be between -0.02 and -0.13, depending on the time of day. They assume a linear demand curve. Hirth et al. (2023) use data from 2015-2019 to estimate the average elasticity at –0.051 , assuming a linear price-demand relationship. The results are thus consistent with those of Knaut and Paulus (2016). The paper further includes regression models assuming a log-linear relationship[2] as well as non-parametric models and various sensitivities and robustness checks.

The present work adds to the literature in two ways. First, no previous work includes data from 2021 and 2022, the years in which prices were significantly higher than historically. The observations from recent years provide an opportunity to gain a better understanding of the demand response at high prices. Therefore, the analysis focuses on the two most commonly assumed functional forms of the demand curve: linear and log-log. While Lijesen (2007) uses both variants but does not discuss the differences in estimates, Bönte et al. (2015) uses only the log-log variant, Knaut and Paulus (2016) only the linear approximation, and Hirth et al. (2023) the linear, a log-linear, and non-parametric forms, but not the usual log-log variant. Given the wide dispersion of prices in recent years the implicit differences between the approaches could become more pronounced. Therefore, a well-founded discussion of the differences, implications and validity of both assumptions under current circumstances is needed.

3. Background

To support the interpretation of the applied method and results, this Section defines the estimated elasticities and explains the basics of the German wholesale electricity market and the demand response to be measured. Subsequently, the two applied assumptions for the functional form of the demand function are introduced, their properties are discussed, and hypotheses are formulated for the empirical study.

3.1. Short-term elasticity and wholesale prices

Price elasticity can be defined for different time scales. Long-run elasticity usually refers to demand adjustments over several years and thus include adjustments of the capital stock. Short-run elasticity is often defined as adjustments within one year (Labandeira et al., 2017). In the present

[2]Log-linear here means that the dependent variable (demand) was logarithmized, but not the explanatory variables. Alternatively, this procedure is also referred to as semi-log or log-level, while confusingly log-linear can also refer to models in which the explanatory variables are also logarithmized (here refered to as log-log).

analysis, *short-term* refers to an even shorter time scale: the demand response on an hourly basis. Sometimes this is also referred to as *real-time* elasticity (Lijesen, 2007; Hirth et al., 2023). Own-price elasticity of demand, the kind of elasticity covered in this paper, denotes the relative change in demand for a relative change in price for the product. As elasticities are not necessarily constant for all price-demand combinations, the own-price elasticity of demand (ϵ) can be calculated based on the absolute changes at a specific combination of *price* and *demand* using Equation 1.

$$\epsilon = \frac{\%\Delta demand}{\%\Delta price} = \frac{\Delta demand}{\Delta price} \cdot \frac{price}{demand} \tag{1}$$

In order to estimate the elasticity according to Equation 1, one needs information on price and demand. For the price, I rely on the hourly day-ahead prices from the German wholesale electricity market (EPEX SPOT). Via an auction, supply and demand are traded daily at noon for each hour of the following day. Not all electricity volumes are procured on the power exchange. Alternatively, there is the option of bilateral trading or own-generation of required electricity quantities. Nevertheless, the day-ahead prices can be seen as relevant benchmarks for the other procurement options, as they form the opportunity costs for consumers and producers. Two aspects have to be considered further to interpret the estimated short-term price elasticities when using wholesale day-ahead prices. First, not all consumers are exposed to the hourly variation of electricity exchange prices. Residential electricity customers, for example, usually purchase their electricity at a fixed price per kWh from their electricity supplier. Other consumers acquire their electricity demand directly, such as energy-intensive industries. These are exposed to electricity price variations. Accordingly, the estimated demand response measures the demand response of these electricity consumers only. Second, electricity consumers are confronted with other price components besides the wholesale electricity price, such as taxes and surcharges, as well as regulatory incentives. Accordingly, the elasticity estimates are dependent on the existing regulatory background.

For the demand I utilize data on the hourly realized aggregated electricity demand in Germany. As described, while not all of the realized demand is traded at the day-ahead market, the day-ahead price is still a valid benchmark for the procurement cost of the entire demand. However, it should be emphasized that after the day-ahead prices are set, there is still continuous intraday trading that can affect realized demand. The estimate cannot reflect corresponding dynamics, e.g. triggered by unexpected events such as power plant outages (Lijesen, 2007).

As an alternative to using the day-ahead price and realized demand, it is also conceivable to use the aggregated hourly demand curves based on ask bids at the power exchange, e.g. EPEX SPOT, to estimate the price elasticities. Knaut and Paulus (2016) argue, however, that this is not easily possible, as market participants already match parts of their demand and supply internally before submitting their bids, which distorts the observable demand curve in the market. Furthermore, the observable demand curve on the wholesale market does not correspond to the real demand curve because it only reflects the volumes traded on the wholesale market and includes electricity that is traded over several market and financial stages (Hirth et al., 2023).

3.2. Functional forms of demand-price response and hypotheses

The actual demand curve in the electricity market is not known ex ante. When estimating elasticities, different functional forms of the demand-price relationship can be assumed. Two central options are the linear price-demand relationship and the exponential relationship described via constant elasticities (log-log specification).

In the literature, the linear approximation of the demand function according to Equation 2 is applied for example in Lijesen (2007); Knaut and Paulus (2016); Hirth et al. (2023); Fabra et al. (2021).

$$demand = b \cdot price + a \tag{2}$$

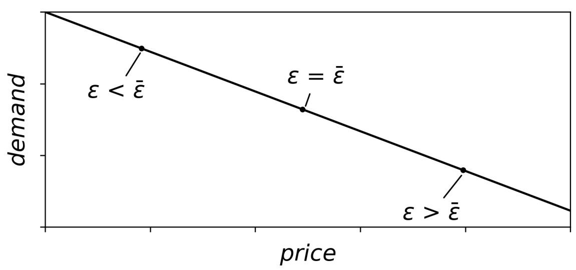
When estimating the demand response based on the linear formulation, one estimates the slope b of the demand-price curve. This slope is constant across all price levels. The elasticity corresponding to Equation 1 then varies depending on the point on the curve, since the absolute demand response $\frac{\Delta demand}{\Delta price}$ is constant, but the price-demand combination $\frac{price}{demand}$ is different at each point of the curve. The elasticity is then lower for low and higher for high prices. The results of this estimation are often communicated as average elasticity $\overline{\epsilon}$, calculated with Equation 3. This approach is convenient, but one must keep in mind that this average elasticity value is only valid for a single point on the linear demand-price curve (see Figure 1 (a)).

$$\overline{\epsilon} = b \cdot \frac{\overline{price}}{\overline{demand}} \tag{3}$$

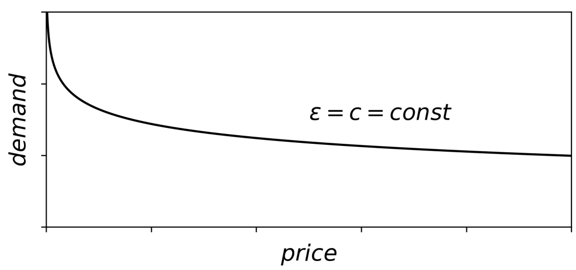
Alternatively, the demand function is often assumed to have a log-log functional form according to Equation 4 (e.g. Lijesen (2007); Bönte et al. (2015); Boogen et al. (2017)). This specification is based on the formation of the natural logarithm for both the dependent and the explanatory variables. The assumed relationship between demand and price is exponential. Confusingly, the term log-linear is also used to describe this specification. However, since this can lead to confusion with specifications in which only the dependent variable is logarithmized (also called semi-log or log-level), I use the term log-log specification in this paper, following Wooldridge (2013).

$$ln(demand) = c \cdot ln(price) + d \tag{4}$$

The parameter c, when estimated based on the assumption of a log-log demand function, can then be directly interpreted as a constant price elasticity of demand (see Figure 1 (a)). Using the approximation that $log(1 + x) \approx x$ for small x, the parameter c expresses the percentage change in demand for a one percent change in price, independent of the price level (Wooldridge, 2013). Thus, the log-log functional form assumes that the absolute effect of a price change on demand decreases with the price. Accordingly, the absolute demand response differs between linear and log-log specification, particularly at the edges of the curve, i.e. at very high prices.



(a)



(b)

Figure 1: Stylized demand-price functions: linear (a) and log-log (b)

As previously discussed, not all electricity market participants are confronted with the wholesale electricity price signal, resulting in a portion of demand that is completely unresponsive to short-term price changes. Figure 2 presents a highly stylized depiction of the composition of electricity demand during a specific hour. The total demand is divided into segments based on the maximum price that consumers are willing to pay before they curtail or shift their consumption. The shaded segment ($< d_1$) represents the perfectly inelastic portion of demand, such as the demand from household customers not exposed to price signals. Additionally, there are demand segments comprising customers who can adjust their consumption in response to wholesale market price signals (yellow areas). The extent to which these customers reduce or shift their consumption depends on their flexibility and opportunity costs. The figure illustrates this flexible demand using six equally sized demand shares. Starting from the bottom, the first segment represents industrial companies that only modify or shift their production at very high electricity prices (p_1). This is followed by increasingly flexible industrial and service applications, ultimately leading to flexible consumers such as storage units that curtail consumption even at relatively low prices. The size and number of these segments, the extent of demand reduction at different price levels, and, thus, the actual shape of the demand function are, in reality, unknown.

Function $D_1(p)$ serves as an illustrative linear demand function that captures the demand behavior in the low price range ($< p_1$). However, at high prices, the linear demand function deviates significantly from the assumed demand structure. Increasing the slope of the function may improve its representation of the low demand response to high prices, but this improvement comes at the cost of accurately depicting the demand response at low prices.

In contrast, Function $D_2(p)$ represents an exemplary log-log demand function. It maintains similarity to the linear description in the low price range, effectively capturing the demand structure. However, the model exhibits a good fit for the price range characterized by predominantly inelastic demand, thanks to its exponential characteristics that effectively capture the diminishing demand response as prices increase.

Certainly, there are other possible functional forms for the demand curve, such as piecewise linear functions, that can accurately represent these relationships. However, for the purpose of this paper, I focus on the two central options presented. In doing so, I aim to capture the fundamental characteristics of the demand response to price changes without delving into the complexities introduced by alternative functional forms.

Three hypotheses are derived from these observations, which will be substantiated in the following through empirical estimation:

Hypothesis 1: At low prices, estimates based on both assumptions yield similar results. The average elasticity based on the linear assumption will be similar to the constant elasticity derived from the log-log description.

Hypothesis 2: The estimators based on linear functions will exhibit significant differences depending on whether low or high prices are considered. This discrepancy arises from the observation that demand at high prices tends to have limited flexibility, leading to distinct estimations.

Hypothesis 3: Estimators using the log-log approach will demonstrate smaller differences between estimates based on low and high prices. This is because the exponential nature of the log-log function takes into account the decrease in absolute demand response (per price change of 1 EUR/MWh) at high prices, leading to more consistent estimations across the price range.

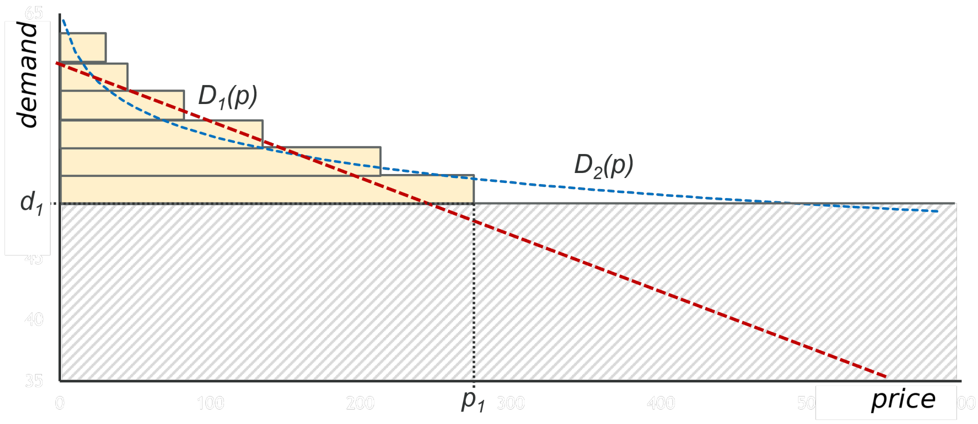


Figure 2: Stylized demand structure.

4. Empirical strategy

The objective of the analysis is to estimate the short-term (i.e. hourly) response of electricity demand in Germany to changes in electricity prices. Equation 5 shows the estimated relationship as a linear regression model.

$$Demand_t = \alpha_0 + \alpha_1 \cdot Price_t + \boldsymbol{\alpha_2} \cdot \boldsymbol{X_t} + \epsilon_t \tag{5}$$

The variable $Demand_t$ represents the hourly realized electricity demand in Germany, while the variable $Price_t$ represents the deflated (real) hourly day-ahead electricity price in the German electricity market. X_t denotes a vector of covariates.

The interaction of demand and supply endogenously determines price and realized demand. Due to this simultaneity, the explanatory variable $Price_t$ is endogenous: In a linear regression model (as shown in Equation 5), the $Price_t$ is correlated with the error term ϵ_t. An estimation of the response of demand to changes in price using a common linear regression model is therefore biased.[3]

To handle the simultaneity issue, I apply a two-stage least squares (2SLS) approach, using an instrumental variable (IV) for the endogenous explanatory variable of the electricity price. As IV, I utilize the grid operators' day-ahead wind power generation forecast. Figure 3 illustrates the relationship between the outcome $Demand_t$, the central explanatory variable, or treatment, $Price_t$, and the IV day-ahead forecast of wind power generation $Wind_t$.

The chosen IV meets the three criteria central to an appropriate instrument choice: The relevance and exogeneity criteria as well as the exclusion restriction (indicated in Figure 3). First, $Wind_t$ is relevant as an instrument for the treatment variable $Price_t$ as it is correlated with $Price_t$: electricity generation from wind has, ceteris paribus, a negative effect on the price of electricity because electricity from wind turbines is offered in the market at marginal costs close to zero. As a result, more expensive forms of generation, such as coal or gas-fired power plants, are priced out of the market equilibrium. In Section 6, it is shown quantitatively that $Wind_t$ has significant and high explanatory power for $Price_t$. Since day-ahead prices are determined by auction at noon

[3] In Appendix A, the main model results of this paper are compared to the results of an OLS estimation to show magnitudes of bias.

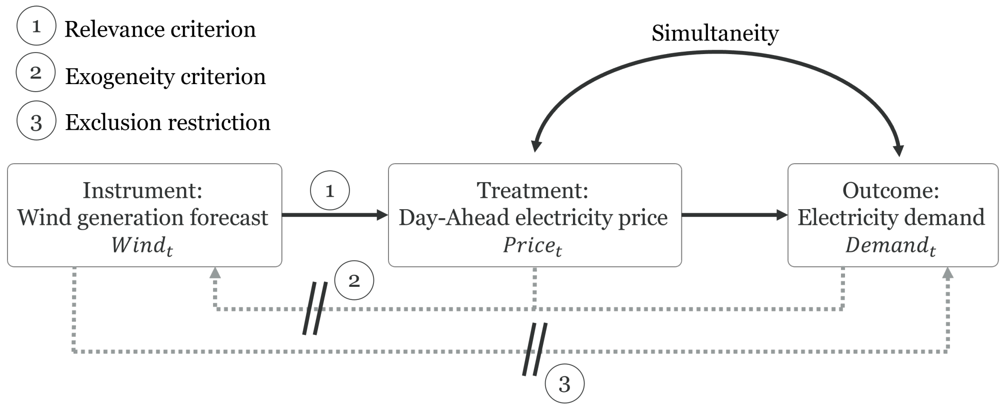


Figure 3: Instrumental variable estimation approach.

the day before delivery, they are not driven by the realized wind generation but rather by the market participants' expectation of the wind generation of the following day. The grid operators in Germany are obliged to publish a forecast for the wind power generation of the following day.[4] The forecast is transmitted daily at 6 pm. Thus, it represents the best publicly available approximation of the generation expectation at the time of price formation and is therefore used as IV.[5].

As the second criterion, the IV must be exogenous with respect to the instrumented and explained variables. The wind power generation forecast is unaffected by price and demand changes. It depends only on the installed capacity of the wind turbines and the weather forecast for the delivery time.

Third, the exclusion restriction must apply to the IV. That is, the wind power generation forecast does not affect electricity demand in any way other than through the price of electricity. This should be true in principle, but there might be correlations between wind power generation and electricity demand due to weather coincidences. For example, it is conceivable that periods of high wind generation (i.e. in winter) coincide with periods of increased electricity demand for heating applications. To isolate these weather correlations from the estimation, dummies for months and hours of a day as well as heating and cooling degrees are introduced in the estimation model.

[4] Art. 14 Par. 1 c) Commission Regulation (EU) No. 543/2013

[5] Alternatively, using the realized wind power generation as IV would also be possible (e.g., Hirth et al. (2023)) However, the actual wind power generation is unknown at the moment of pricing. Also, data on actual generation includes curtailment, e.g. due to grid congestion. However, using actual wind power generation as IV hardly changes the results of the present analysis, as is shown in Appendix B

As an alternative or addition to the selected IV, day-ahead wind generation forecast, one could utilize the photovoltaic (PV) generation or the day-ahead PV generation forecast as instruments. However, there are two reasons to assume that the exclusion restriction of a valid IV is not met for PV generation (and the forecast of it). First, small rooftop PV systems do not consistently provide hourly metering data, so the grid operators' data are partly based on estimates. Therefore, measurement errors that correlate with prices are conceivable and would distort the price-demand response estimation (Hirth et al., 2023). Second, Frondel et al. (2022) show that there exists a so-called solar rebound effect, i.e. more electricity is consumed when solar electricity is generated. A potential correlation between PV generation and demand would violate the exclusion restriction. However, to account for the potential correlation between solar generation and demand, the day-ahead PV generation forecast is included in the model as a control variable.[6]

Equation 6 and 7 show the simultaneous equation model. Here a linear relationship between $Demand_t$ and $Price_t$ is assumed. As alternative model specification logarithmized variables are utilized, following the discussion in Section 3.2.

$$Price_t = \gamma_0 + \gamma_1 \cdot Wind_t + \boldsymbol{\gamma_2} \cdot \boldsymbol{C_t} + \boldsymbol{\gamma_3} \cdot \boldsymbol{D_t} + \epsilon_t \tag{6}$$

$$Demand_t = \beta_0 + \beta_1 \cdot Price_t + \boldsymbol{\beta_2} \cdot \boldsymbol{C_t} + \boldsymbol{\beta_3} \cdot \boldsymbol{D_t} + \mu_t \tag{7}$$

Due to the simultaneity of the two equations, the model is solved via the two-stage least squares (2SLS) approach. In the first stage, $Price_t$ is the response variable, and the IV $Wind_t$ is the primary explanatory variable. In the second stage, $Price_t$ is then replaced with the predicted values from the first stage to estimate the causal effect of $Price_t$ on $Demand_t$. In both stages, there are further controls included in the model. C_t is a vector of covariates, including the hourly day-ahead forecast for PV generation, daily deflated gas, coal and European emission allowances (EUA) prices, as well as hourly heating and cooling degrees. D_t is a vector of dummy controls. Yearly dummies account for changes in generation capacity over time, while monthly dummies correct for seasonal effects in

[6] As a sensitivity analysis, in Appendix B, the estimations are run with the day-ahead PV generation forecast as an additional instrument. The results differ only slightly from the main result, with the estimators for the demand response being slightly lower.

demand and prices. Dummies for the different days of the week and for the different hours of a day control for varying demand structures over time.[7] ϵ_t and μ_t are error terms.

For a precise understanding and interpretation of the model results, it is crucial to acknowledge the presence of autocorrelation in variables within the model. The selected estimation model does not only estimates the demand response to the price in a specific hour but also implicitly includes intertemporal cross-price elasticities with the preceding and subsequent hours (Hirth et al., 2023). This implies that the effects of price fluctuations extend beyond a single hour, capturing interdependencies across time. For instance, due to the possibility of load shifts, the expectation of high prices in a particular hour may influence demand in preceding and subsequent hours.

Following the discussion in Section 3.2, the model is further estimated with logarithmized variables, namely as a log-log specification. For heating and cooling degrees as well as PV generation, a constant of 1 is added beforehand, owing to the presence of zero values. Since the natural logarithm of negative and zero values is not defined, respective data points of negative prices are removed. This eliminates 1.4% (909) of the data points under consideration. Thus, information about demand response to negative prices is lost. However, this is not a problem since the present analysis focuses on demand response when prices are high.

To account for the serial correlation in the data, heteroscedasticity and autocorrelation (HAC) robust standard errors are calculated, for all model specifications. Specifically, the standard errors are calculated using Newey-West kernel with automatic bandwidth selection.[8].

5. Data

The basis of the analysis is an hourly data set for 2015-2022, with a total of 70,024 observations.[9] To ensure that special effects of public holidays and bridge days do not distort the estimation, corresponding data points and the period between Christmas and New Year were removed from the data set, leaving 65,920 observations.

The hourly values of the dependent variable $Demand_t$ are taken from the *Total Load* timeseries published by the European Network of Transmission System Operators (ENTSO-E)(ENTSO-E,

[7] Appendix C shows the effect of adding each dummy control to the model

[8] Estimates are carried out using *ivregress* from StataCorp (2021).

[9] The data set does not include the first 96 hours of 2015, as data on day-ahead electricity prices is not available.

2023).[10] Figure 4 depicts the temporal variation in German electricity demand in the dataset. Over the course of a day demand is highest around 11:00 am and lowest at night at around 2:00 am. Demand is lower at weekends than on weekdays and lowest on Sundays. The electricity demand also shows a seasonal trend, with demand being higher in winter than in summer. When comparing the years, two years, 2020 and 2022, stand out, in which the median electricity demand deviates downward from the other years. For 2020, the effect of the Covid-19 pandemic can be assumed as the cause; for 2022, the reason could be the energy crisis and the associated high electricity prices.

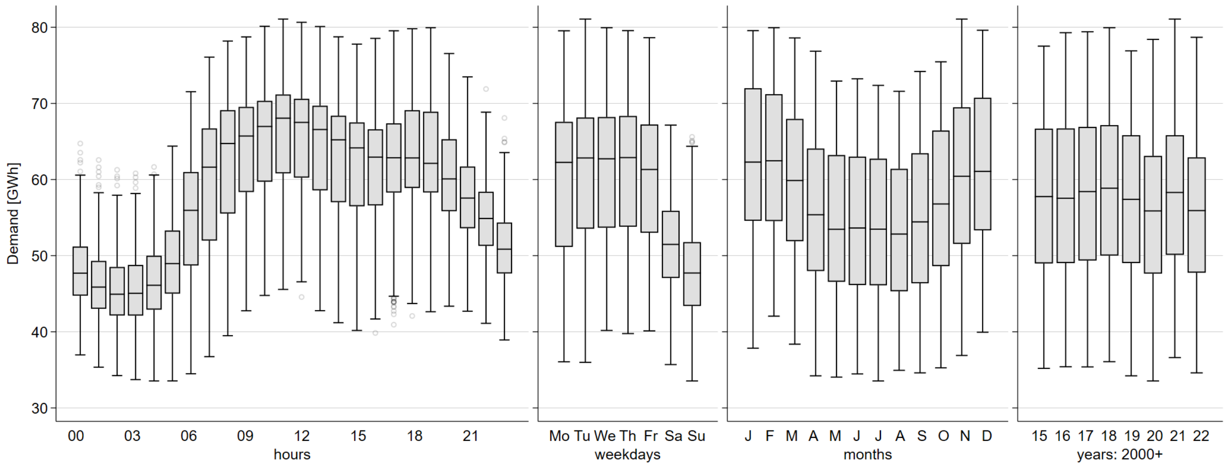


Figure 4: Temporal variation in the variable $Demand_t$, 2015-2022, from (ENTSO-E, 2023).

Hourly day-ahead wholesale electricity prices $Price_t$ are taken from *Day-ahead prices* timeseries of ENTSO-E (2023). These prices are the result of the German day-ahead auction at the EPEX power exchange. The nominal prices from ENTSO-E (2023) are deflated using the Consumer Price Index (CPI) from (Destatis) to the level of January 2015 in order to account for the impact of inflation and accurately assess the demand response without the confounding effects of rising prices over time.[11] In Figure 5, the variance of real prices over time is shown. With the exception of the

[10]All data from the ENTSO-E Transparency Platform used in this analysis is on country level "Germany" and was downloaded on 28.02.2023. The latter is important as values are regularly updated, especially on electricity demand. Information on the ENTSO-E data can be found at `https://transparency.entsoe.eu/content/static_content/Static%20content/knowledge%20base/knowledge%20base.html`

[11]For the same purpose, other price data (gas, coal and EUA prices) are also adjusted for inflation using the CPI. Thus, with the exception of the literature results, all EUR values presented reflect real values based on the reference point of January 2015. The central results of the analysis remain unchanged even if nominal prices are used. However, the linear estimates for the price variables are slightly lower in that case.

annual figure, the price distributions are characterized by a large number of positive outliers. These occur due to the extremely high electricity prices in 2021 and 2022 compared to previous years. Prices above 100 EUR/MWh occurred very rarely in the years 2015 to 2020, which is evident in the annual figure. Due to this significant difference in price levels and variance, the analysis performed is based on the split of the data set into the low-price years (2015-2020) and high-price years (2021-2022). According to the literature (e.g. Wolff and Feuerriegel (2017)), one would expect prices to be higher in winter than summer, as demand increases and solar generation declines. Based on the figure, this relationship can not be clearly demonstrated, due to the extremely high prices in July to October 2022. Prices tend to be lower on weekends than during the week, and there are two price peaks during the day, once at 8:00 am and once at 7:00 pm.

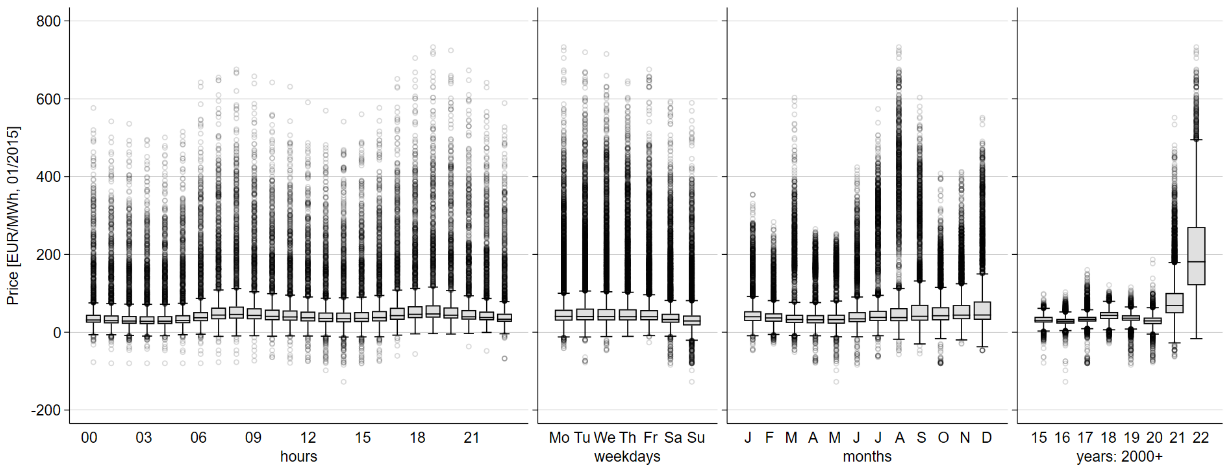


Figure 5: Temporal variation in the variable $Price_t$, 2015-2022, from (ENTSO-E, 2023).

As described in the methodological discussion in Section 4, the day-ahead wind generation forecast of the grid operators $Wind_t$ is used as IV for the endogenous variable $Price_t$. Together with the forecast for solar generation, the forecast values are available from ENTSO-E (2023) as the hourly time series *Day-Ahead generation Forecast Wind and Solar*. The forecast of solar generation PV_t is used as a covariate in the model, as PV generation is potentially correlated with the demand (see Section 4). For both variables, a negative effect on the electricity price is to be expected. Figure 6 shows the time variation of the two variables. Both variables show an increasing trend over the past years, corresponding to the increase in generation capacities. However, due to the weather dependency, this is subject to a wide dispersion. Both variables also show a strong seasonal structure: While electricity generation from wind is particularly high in winter, solar electricity generation is high in summer. The intra-day structure is much more pronounced for solar electricity than wind generation: while no PV electricity is generated at night, most electricity is generated at noon.

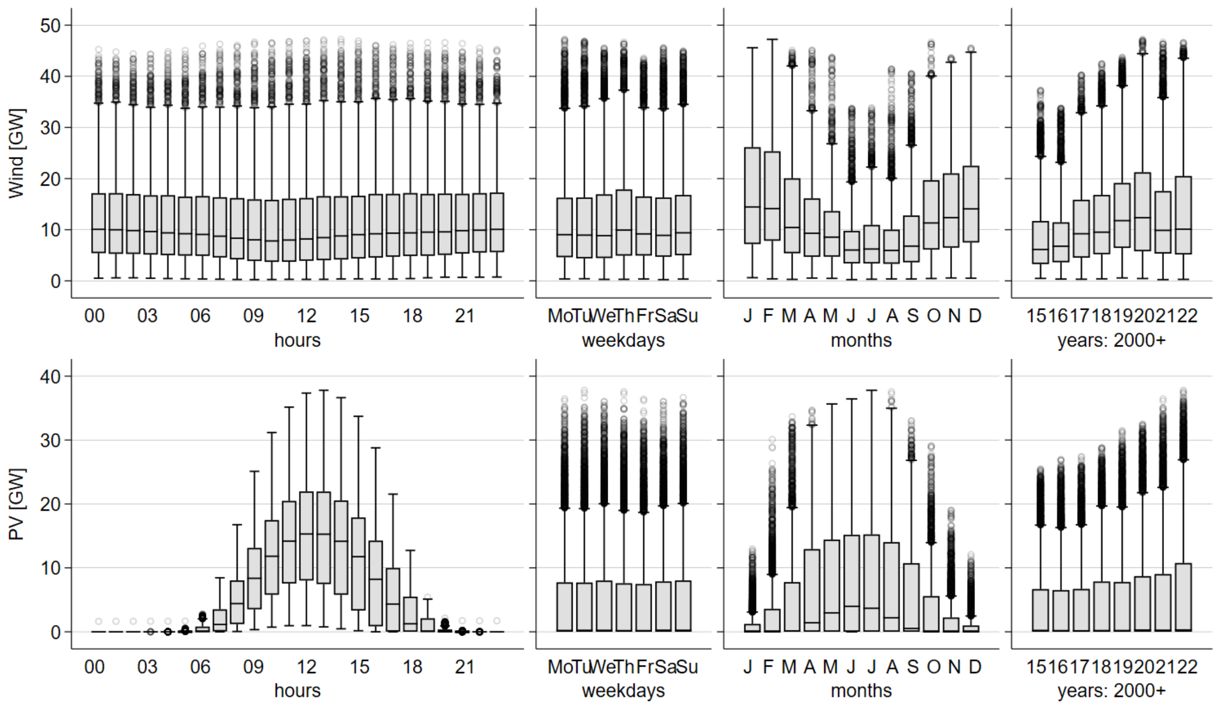


Figure 6: Temporal variation in the variables $Wind_t$ (top) and PV_t (bottom), 2015-2022, from (ENTSO-E, 2023).

Besides the day-ahead PV generation forecast, several additional covariates are used in the model. Further covariates that affect the supply side of electricity pricing are fuel prices and prices for emission allowances, which determine the marginal costs of conventional power plants. As they are not available on hourly basis, they further do not qualify as IV in this setting. Since the marginal cost of generating electricity in conventional power plants increases with the fuel and certificate prices, a positive effect on the electricity price can be expected for all three variables. For the coal price $Coal_t$ the daily prices for coal imports at the Amsterdam-Rotterdam-Antwerp (ARA) trading point are utilized.[12] As gas price, the daily prices for natural gas at the reference trading point TTF are included.[13] Daily prices for European Emission allowances (EUA) are taken from ICAP (2023). Figure 7 shows the deflated fuel and certificate prices compared to daily averages of the deflated day-ahead electricity price. The increase in prices in 2021 and especially in 2022 is particularly noteworthy. As a result of the war in Ukraine, gas prices in particular reached record levels. The electricity price follows the development of the underlying fuel prices. The correlation is particularly evident for the gas price.

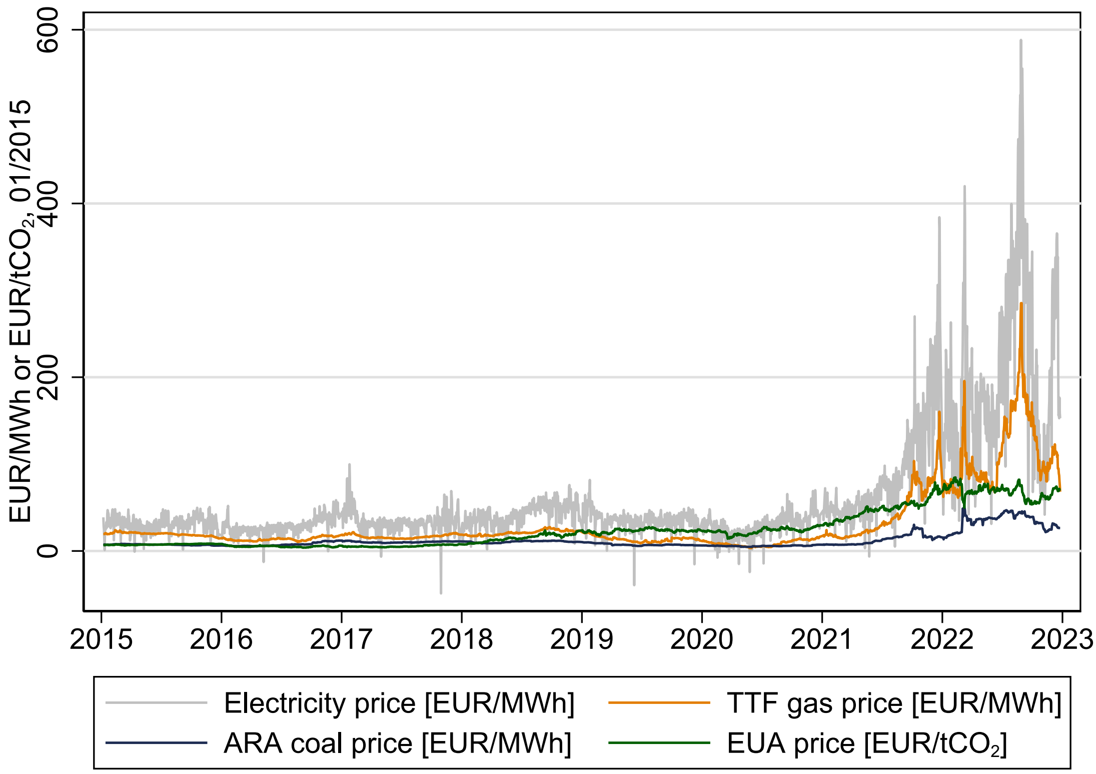


Figure 7: Daily average electricity price, gas, coal and EUA price in 2015-2022.

Heating and cooling degrees are used as covariates on the demand side. The need for heating or cooling implies correspondingly higher electricity demand. Therefore, a positive correlation between electricity demand and heating or cooling degrees can be expected. For the calculation of hourly heating degrees and cooling degrees temperature data by region is taken from Copernicus Climate

[12] Coal (API2) CIF ARA (ARGUS-McCloskey).
[13] Dutch TTF Natural Gas Futures.

Change Service (2020)[14]. By weighting the temperatures with the region specific population from Eurostat (2023), I derive population weighted average temperatures for Germany. Hourly heating and cooling degrees are then calculated based on a temperature threshold of 15 °C.

Table 1 gives an overview over descriptive statistics of the used data.

Table 1: Descriptive statistics, 2015-2022 (N = 65,920)

Variable	Mean	Median	SD	Min	Max	Source
Demand (MW)	57348	57393	9963	33542	81077	ENTSO-E
Price (EUR/MWh, 01/2015)	61.84	37.44	75.64	-127.35	732.52	ENTSO-E
Wind (GW)	11.95	9.17	9.39	0.24	47.23	ENTSO-E
PV (GW)	4.88	0.21	7.48	0	37.78	ENTSO-E
Gas (EUR/MWh, 01/2015)	31.02	17.55	37.33	3.25	285.27	ICE Dutch TTF
Coal (EUR/MWh, 01/2015)	11.98	8.88	9.37	4.71	49.13	API2 CIF ARA
EUA (EUR/tCO_2, 01/2015)	24.46	19.19	21.73	3.81	84.68	ICAP
HD (°C)	5.84	4.86	5.63	0	27.04	CDS
CD (°C)	1.43	0	2.98	0	20.75	CDS

6. Empirical results

I estimate the short-run elasticity of electricity demand for Germany using the 2SLS approach described in Section 4, both for the linear and the log-log model specification. First, the results for the first stage are examined concerning the validity of the approach before the second-stage model results are discussed. I distinguish between low-price and high-price years to investigate whether the elasticities differ for the time periods and, thus, for the different price ranges. In addition, further sensitivities are discussed.

6.1. First stage

Table 2 shows the results of the model in linear (1a) and log-log (1b) specification. In both variants, the estimator for the instrumental variable $Wind_t$, i.e., the day-ahead wind generation forecast, is significant. In the linear model, 1 GW of additional wind generation, ceteris paribus, leads to a price decrease of around 1.8 EUR/MWh. Similarly, the second model results state that a one percent

[14] Air temperature at an altitude of 2m for NUTS3 regions.

increase in wind generation leads to a 0.31 % decrease in price.[15] The partial R squared shows for both models that $Wind_t$ explains a relevant part of the variance of the price. In addition, the Montiel-Pflueger robust weak instrument test was performed for both model specifications. The F-statistics of the tests at 5% confidence level are 11,151 and 8,110, respectively, which is well above the critical value of the test of 37.42. Based on the test result and the significance at the first stage, I conclude that $Wind_t$ is indeed a strong instrument for $Price_t$. The estimators of the other covariates have the expected signs and magnitudes. PV power generation has a negative impact on the electricity price, similar in magnitude to wind power generation. Fuel and certificate prices enter positively, with the gas price having the largest effect on the price at 1.7 EUR/MWh. This order of magnitude is also intuitively plausible if one assumes that an average gas-fired power plant has an efficiency of about 50%. When gas-fired power plants determine the price of electricity, an increase in the price of gas of 1 EUR leads to an increase in the price of electricity of 2 EUR. Since gas-fired power plants do not determine the price at every hour, the estimated value is lower.

Table 2: First stage results

	(1a) linear		(1b) log-log	
Wind [GW]	-1.765***	[-1.80,-1.73]	-0.314***	[-0.32,-0.31]
PV [GW]	-2.495***	[-2.58,-2.41]	-0.298***	[-0.31,-0.29]
Gas [EUR/MWh]	1.707***	[1.66,1.75]	0.537***	[0.51,0.56]
Coal [EUR/MWh]	0.259**	[0.10,0.41]	0.163***	[0.13,0.20]
EUA [EUR/tCO2]	0.925***	[0.84,1.01]	0.268***	[0.24,0.30]
Heating degrees [°C]	0.555***	[0.48,0.63]	0.079***	[0.07,0.09]
Cooling degrees [°C]	1.341***	[1.21,1.47]	0.061***	[0.05,0.07]
Dummy variables				
Hours	Yes		Yes	
Weekdays	Yes		Yes	
Months	Yes		Yes	
Years	Yes		Yes	
Fit statistics				
Partial R^2 Wind	0.215		0.187	
Adjusted R^2	0.869		0.684	
Observations	65920		65011	

95% confidence intervals in brackets. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.
Coefficients of dummy variables can be found in Appendix E.

[15] The within-variance of the price, after accounting for the dummy control variables, is shown in Appendix D.

6.2. *Second stage*

6.2.1. *Linear model results*

Table 3 shows the second stage results for the linear specification. For Model (2a), where data from all eight years under consideration was utilized, the estimator for the linear demand response to price is –61.81 , i.e. when the price increases by 1 EUR/MWh, demand decreases by 61.81 MW. In contrast, the data set used in Model (3a) includes only the low-price years (2015-2020), and Model (4a) includes only the high-price years (2021-2022). The estimation results differ significantly from the results of the joint estimation. While the estimator for the low-price years is significantly higher (–109.4), the estimator for the high-price years is lower (–34.35). The intuition behind this result follows the explanations in Section 3.2. In contrast to the linear relationship between price and demand assumed in the model, the absolute demand response decreases with the price level. At low prices, a price increase of 1 EUR is relatively large, and consumers in the market are willing or able to not consume or postpone their consumption at this price increase. In contrast, the same price increase in years with high prices is relatively small. Consumers who buy electricity at the price level of high price years have already exhausted their potential to reduce consumption: demand can hardly be reduced any more, e.g. because the delivery is contractually fixed. Demand at this price level is less flexible in absolute terms. Calculating demand response as a linear model across all price levels ignores this decrease in absolute flexibility as prices rise. Therefore, the joint Model (2a) estimate forms a weighted average over the demand flexibility of different price levels. The results, thus, confirm Hypothesis 2.

Figure 8 illustrates the difference between the estimators via linear demand functions. The estimator of the linear model corresponds to the slope of the function. In addition to the three estimated values from Table 3, the figure includes linear estimators of the demand response from Hirth et al. (2023) (–79.6) and based on Knaut and Paulus (2016) (–99.1).[16] The estimations from Models (2a)-(4a) and their confidence intervals differ substantially. The estimate based on Knaut and Paulus (2016) aligns closely in magnitude with the estimate for the low-price years. This similarity

[16] In Knaut and Paulus (2016), the demand response is derived individually for the different hours of the day. The values range from –42.1 at 10:00 am to –201.8 at 5:00 pm. For comparability, I assume the non-weighted daily average of these values (–99.1).

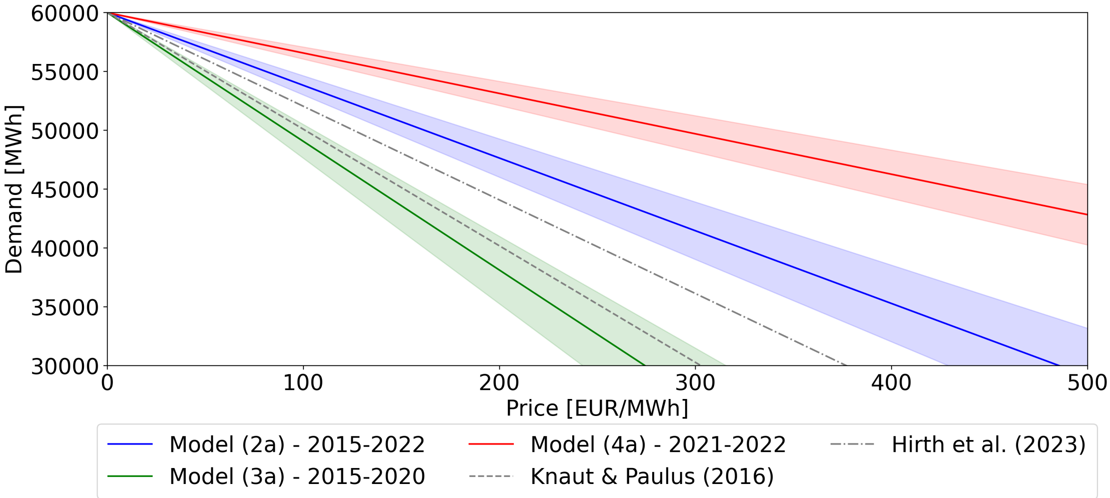
Table 3: Second stage results of the linear specification

	(2a) 2015-2022	(3a) 2015-2020	(4a) 2021-2022
Price [EUR/MWh]	-61.81***	-109.4***	-34.35***
	[-69.95,-53.66]	[-123.75,-95.07]	[-39.50,-29.21]
PV [GW]	-204.0***	-164.1***	-211.8***
	[-231.02,-176.93]	[-191.23,-137.00]	[-248.38,-175.29]
Gas [EUR/MWh]	86.41***	226.5***	53.43***
	[65.83,106.98]	[151.79,301.26]	[39.21,67.65]
Coal [EUR/MWh]	61.79	76.99	83.41***
	[-3.53,127.12]	[-77.66,231.64]	[34.63,132.20]
EUA [EUR/tCO2]	150.8***	110.8**	140.0***
	[115.42,186.16]	[37.22,184.30]	[116.10,163.93]
Heating degrees [°C]	379.1***	351.3***	416.0***
	[337.44,420.73]	[310.82,391.72]	[357.24,474.71]
Cooling degrees [°C]	176.4***	144.0***	153.8***
	[140.67,212.10]	[109.39,178.58]	[81.72,225.81]
Dummy variables			
Hours	Yes	Yes	Yes
Weekdays	Yes	Yes	Yes
Months	Yes	Yes	Yes
Years	Yes	Yes	Yes
Fit statistics			
Adjusted R^2	0.868	0.885	0.883
Observations	65920	49434	16486

95% confidence intervals in brackets. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.
Standard errors are calculated as Newey-West HAC robust standard errors.
Coefficients of dummy variables can be found in Appendix E.

is not surprising, considering that the estimate is derived from 2015 data, which corresponds to the low-price period. On the other hand, the estimate from Hirth et al. (2023) is, in absolute terms, lower than the estimate for the low-price year, despite being based on data from 2015-2019. There are two main reasons for this disparity. Firstly, the estimate does not include data from 2020, a year characterized by exceptionally low prices. Secondly, the study does not account for inflation in the price data, which, in comparison to my estimation, also leads to, in absolute terms, lower demand response estimates.

In all three model variants, the PV generation forecast enters negatively. A potential explanation may be that small-scale PV generation volumes are partially estimated by the transmission system operators (see Section 4). As a result, part of the small-scale PV generation appears as a reduction



Note: For the illustration of the price-demand functions, an example intercept of 60,000 MW of demand was assumed for a price of 0 EUR/MWh.

Figure 8: Linear demand functions based on estimation results from different models. The shaded areas represent the 95%-confidence intervals of the estimators.

in $Demand_t$. Moreover, the negative sign indicates that this effect of measurement errors dominates a potential solar rebound effect in demand.[17]

6.2.2. Log-log model results and average elasticities

As discussed in Section 3.2, the log-log specification is a formulation of the demand function that considers that the absolute demand response decreases with the price level. Models (2b)-(4b) are based on logarithmized data for the variables. The estimator for the demand response in Model (2b), i.e., using the entire data set, is –0.045 . This estimator can be directly interpreted as a constant elasticity, i.e., for a price increase of 1 %, demand decreases by –4.5 %. The elasticities for the two subsets are –4.2 % and –5.3 %, respectively. The differences between elasticities for the low and high price years and the entire data set are much smaller than for the linear formulation. A look at the confidence intervals shows that these even overlap significantly. A Chow test shows that the estimators of the Models (3b) and (4b) differ at the 5% level but not at higher significance levels.

[17] This is in line with findings in Hirth et al. (2023).

In the linear formulation, estimation results exhibit a strong dependence on the subset of data considered, highlighting the sensitivity of the estimates to the price range. In contrast, the estimators in the log-log specification demonstrate less pronounced variations. Consequently, when comparing the two models that cover the entire data set, (2a) and (2b), the log-log formulation (2b) provides a better representation of the underlying relationships across the entire price range. The empirical findings, thus, align with Hypothesis 2, as they indicate that the log-log formulation captures the true relationships more effectively than the linear specification when considering the entire data set.

Table 4: Second stage results of the log-log specification

	(2b) 2015-2022	(3b) 2015-2020	(4b) 2021-2022
Price [EUR/MWh] (log)	-0.0455***	-0.0418***	-0.0533***
	[-0.05,-0.04]	[-0.05,-0.04]	[-0.06,-0.04]
PV [GW] (log+1)	-0.0240***	-0.0212***	-0.0288***
	[-0.03,-0.02]	[-0.02,-0.02]	[-0.03,-0.02]
Gas [EUR/MWh] (log)	0.0355***	0.0722***	0.0401**
	[0.02,0.05]	[0.06,0.09]	[0.02,0.06]
Coal [EUR/MWh] (log)	0.0006	-0.0061	0.0353*
	[-0.02,0.02]	[-0.03,0.01]	[0.01,0.06]
EUA [EUR/tCO2] (log)	0.0400***	-0.0034	0.0805***
	[0.02,0.06]	[-0.02,0.01]	[0.05,0.11]
Heating degrees [°C] (log+1)	0.0279***	0.0244***	0.0314***
	[0.02,0.03]	[0.02,0.03]	[0.03,0.04]
Cooling degrees [°C] (log+1)	0.0225***	0.0207***	0.0241***
	[0.02,0.02]	[0.02,0.02]	[0.02,0.03]
Dummy variables			
Hours	Yes	Yes	Yes
Weekdays	Yes	Yes	Yes
Months	Yes	Yes	Yes
Years	Yes	Yes	Yes
Adjusted R^2	0.885	0.895	0.880
Observations	65011	48670	16341

95% confidence intervals in brackets. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.
Standard errors are calculated as Newey-West HAC robust standard errors.
Coefficients of dummy variables can be found in Appendix E.

Figure 9 shows the log-log demand functions based on the estimators from Table 4. The figure also includes the average elasticities derived from the linear models of Knaut and Paulus (2016) (–5.4 %) and Hirth et al. (2023) (–5.1 %) as well as average elasticities obtained from the results of the linear formulation (Table 3, Model (2a): –6.7 %, Model (3a): –6.4 %, Model (4a): –8.9 %).[18] It is evident that the curves are much closer to each other compared to the linear functions (Figure 8), particularly at higher price levels. The plotted confidence intervals for models (2b)-(4b) overlap, further highlighting the consistency of the log-log estimators. Moreover, the average elasticity estimates from Knaut and Paulus (2016) and Hirth et al. (2023) fall within the range of values obtained from the log-log specification.

The similarity between the average elasticities obtained through the linear formulation in these studies and those derived from the log-log formulation supports Hypothesis 1. It suggests that, for low price ranges, the linear approximation of the demand curve does not deviate too much from the log-log formulation. The average elasticity based on the linear estimators of the models (2a) and (3a) are somewhat higher, in absolute terms, than the corresponding constant elasticities of the models (2a) and (2b). However, this contrasts with the average elasticity estimated using the linear model specification for high prices (Model (4a)), which is considerably higher, in absolute terms, than the constant elasticities determined with the log-log specification. This much larger disparity arises because the assumption of a linear demand response is invalid, particularly at higher price levels.

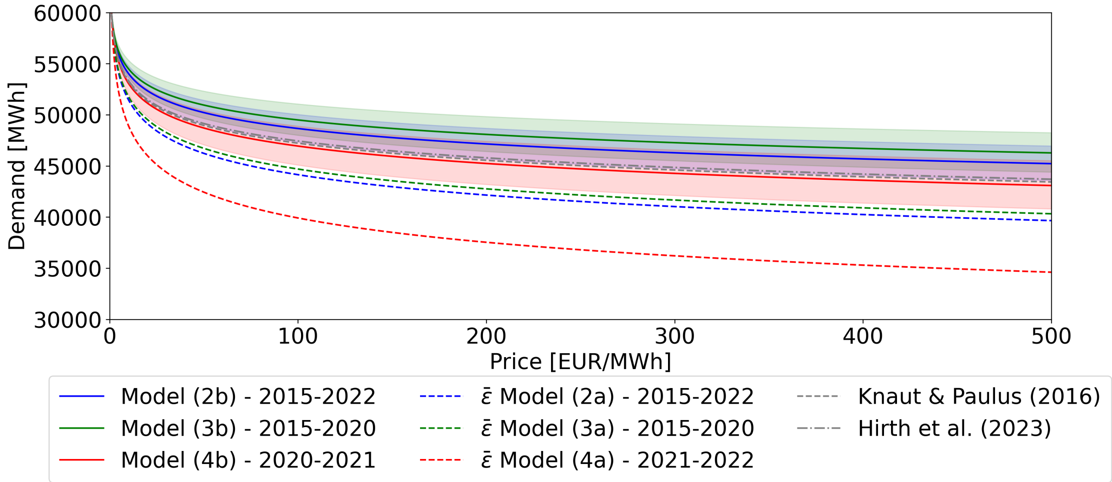
6.3. Further model specifications

A key aspect of the present analysis is the separation of the data into low-price and high-price years. However, this separation is not perfectly unambiguous. As shown in Figure 5, the price of electricity has increased steadily since the beginning of 2021. Alternative separation dates for the start of the high-price period are, for example, after the first or second quarter of 2021. However, choosing a separation date that differs from the one in the main specification does not drastically affect the analysis results.[19]

In addition to the selected covariates, there may be other potentially beneficial covariates to be added, particularly on the supply side. For example, the availability of hydroelectric or nuclear power generation in Germany and neighbouring countries influences the electricity price, as does the

[18] The average elasticity is derived from the linear model specifications with Equation 3.

[19] Appendix F.1 contains the results for alternative separation dates. The results strengthen the validity of statements based on the main model specifications.



Note: For the illustration of the price-demand functions, an example intercept of 60,000 MW of demand was assumed for a price of 0 EUR/MWh.

Figure 9: Log-log demand functions based on estimation results from different models. The shaded areas represent the 95%-confidence intervals of the estimators.

available coal-fired power plant capacity. Including possible proxies for these influencing variables hardly changes the results.[20] That is because these effects are already absorbed by the corresponding time dummy variables in the main model specification.

In similar price elasticity analyses, realized wind generation is used as the instrumental variable for price instead of the day-ahead forecast (e.g., Hirth et al., 2023). However, this alternative model specification does not greatly impact the model results.[21]

In the primary model specifications, fuel and certificate prices are utilized as covariates in both estimation stages. This approach aims to enhance the precision of estimation in the first stage, in identifying the relationship between the forecast of wind power generation and the electricity price. However, the potential for endogeneity issues arises when incorporating these covariates in the second stage: It is conceivable that electricity demand influences gas, coal, and EUA prices, given the impact of electricity demand on the demand for these commodities. An alternative model specification is employed to address this concern, wherein these price data are omitted from both stages. Although this may reduce the precision of estimation in the first stage, the estimation

[20] Appendix F.2 contains the results for model specifications, including covariates for the availability of nuclear, hydro and coal-fired power generation.

[21] Appendix B contains the results for model specifications using wind electricity generation as an instrument.

remains fundamentally valid as long as the instrumental variable requirements are met, and a strong relationship exists between the instrument and the variable being explained (Angrist and Pischke, 2008). The corresponding model results (Appendix F.3) confirm the presented results of the primary model specifications (Models 1 to 5). Even without the price covariates, the wind energy forecast remains a strong instrument, and the estimated demand responses exhibit only minimal changes. These results further validate the analysis presented, indicating that any potential endogeneity concerns arising from the price covariates have negligible impact on the main findings.

7. Conclusion

In a power system with a large share of intermittent generators, it is increasingly important to understand how demand responds to price signals, especially when prices are high. Electricity prices in 2021 and 2022 were higher than ever. This offers the opportunity to better understand how demand responds under these circumstances. Against this backdrop, this paper focuses on examining the short-term (hourly) elasticity of electricity demand in Germany. By analyzing the new observations of high prices, I aim to explore the functional relationship between demand response and price levels across a broader range of price dispersion. I analyze both linear and log-log demand-price relationships, exploring the dynamics and complexities of demand response to varying price levels. To this end, I first examine the characteristics of the two functional forms of the demand curve and establish three hypotheses for empirical analysis. I employ a two-stage least squares (2SLS) approach to address the simultaneity between demand and prices. The hourly demand response to hourly day-ahead prices is determined using the day-ahead wind power generation forecast as an IV for the price. I calculate separate models for the entire 2015-2022 period, as well as subsets of the high- and low-price years.

In the linear model that includes all observation years, an increase in price by 1 EUR/MWh, all else equal, leads to a decrease in demand of about 62 MW. The estimate of the decrease in demand for the low-price years is higher (–109.4) and lower (–34.35) for the high-price years. The estimates for the different time subsets in the log-log model are closer together. All else equal, the 1 % increase in price leads to a 4.5 % decrease in demand when considering all observations, 4.2 % for the low-price years, and 5.3 % for the high-price years. Whereby the confidence intervals of the individual estimators overlap.

The quantitative findings provide confirmation for the formulated hypotheses. Consistent with Hypotheses 1 and 2, the results demonstrate that while a linear relationship between demand and price can yield similar average elasticities to the assumption of constant elasticities for low prices, this is no longer the case for high-price years. The linear demand response decreases with the price. Therefore, the linear estimators for the high-price and low-price years differ significantly, and this also carries over to the average elasticity values derived from them. In contrast, as anticipated by Hypothesis 3, the use of the log-log formulation effectively captures the decrease in absolute demand response (per price change of 1 EUR/MWh), making it a preferable approach when dealing with substantial price spreads.

For researchers and policymakers, the results imply that using elasticities based on the linear approximation of the demand curve should be critically questioned. Possible applications for such elasticity estimates could be electricity market, energy system or price forecasting models. In particular, if the application purpose includes the occurrence of high prices, the decreasing linear demand response to prices should be accounted for.

The estimated elasticity values can be used in further research, e.g. in electricity and energy market models, to address diverse inquiries. In addition to the already mentioned example of estimating necessary controllable capacities, this could also include other questions of infrastructure planning, market design and operational decisions in the electricity sector. It is worth noting that the potential influence of autocorrelation on the estimation results should be duly considered, as the estimators capture time-crossing effects of price movements on demand. This presents an avenue for further investigation into isolating the individual effects and quantifying their magnitude. This paper limits its consideration to two possible assumptions for specifying the demand function, linear and log-log. The idea behind this is that these two represent the most common specifications that are widely used. However, both can, of course, only be approximations of reality. In further research, the use of piece-wise linear models or quantile regression models would be conceivable to investigate the dependence of the demand response on the price level in a more detailed way. Moreover, the present research is limited to the investigation of short-term elasticities. Long-term elasticities and the extent to which they have changed as a function of price levels would be a promising topic for further research.

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Appendix A. OLS estimation

Table A.5 shows the results of the main models (2a) and (2b) compared to the results of simple OLS estimations. The results of the OLS estimation are biased due to the simultaneity issue between demand and price. Accordingly, the estimators show non-intuitive results. Both OLS models have positive estimated demand responses to a price increase. The results confirm that using an instrumental variable or a comparable approach is necessary to obtain unbiased results.

Table A.5: Comparison of main model results and results for OLS estimation.

	linear		log-log	
	(2a) 2SLS	(5a) OLS	(2b) 2SLS	(5b) OLS
Price [EUR/MWh]	-61.81***	16.14***	-0.0455***	0.0134***
Wind [GW]		133.87***		0.0184***
PV [GW]	-204.0***	-9.533**	-0.0240***	-0.0065***
Gas [EUR/MWh]	86.41***	-46.66***	0.0355***	0.0039**
Coal [EUR/MWh]	61.79	41.60***	0.0006	-0.0089***
EUA [EUR/tCO2]	150.8***	78.66***	0.0400***	0.0243***
Heating degrees [°C]	379.1***	335.8***	0.0279***	0.0233***
Cooling degrees [°C]	176.4***	71.83***	0.0225***	0.0188***
Dummy variables				
Hours	Yes	Yes	Yes	Yes
Weekdays	Yes	Yes	Yes	Yes
Months	Yes	Yes	Yes	Yes
Years	Yes	Yes	Yes	Yes
Fit statistics				
Adjusted R^2	0.868	0.914	0.885	0.914
Observations	65920	65920	65011	65011

95% confidence intervals in brackets. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

Appendix B. Alternative IV specifications

Figure B.10 shows the estimators and confidence intervals for the main model specification and two sensitivities. Adding the day-ahead PV generation forecast as IV does not fundamentally change the results. Compared to the main model specification, the estimators of the price effect are slightly lower in all models. This finding is consistent with Hirth et al. (2023). Using the actual wind generation instead of the forecast hardly influences the model's results.

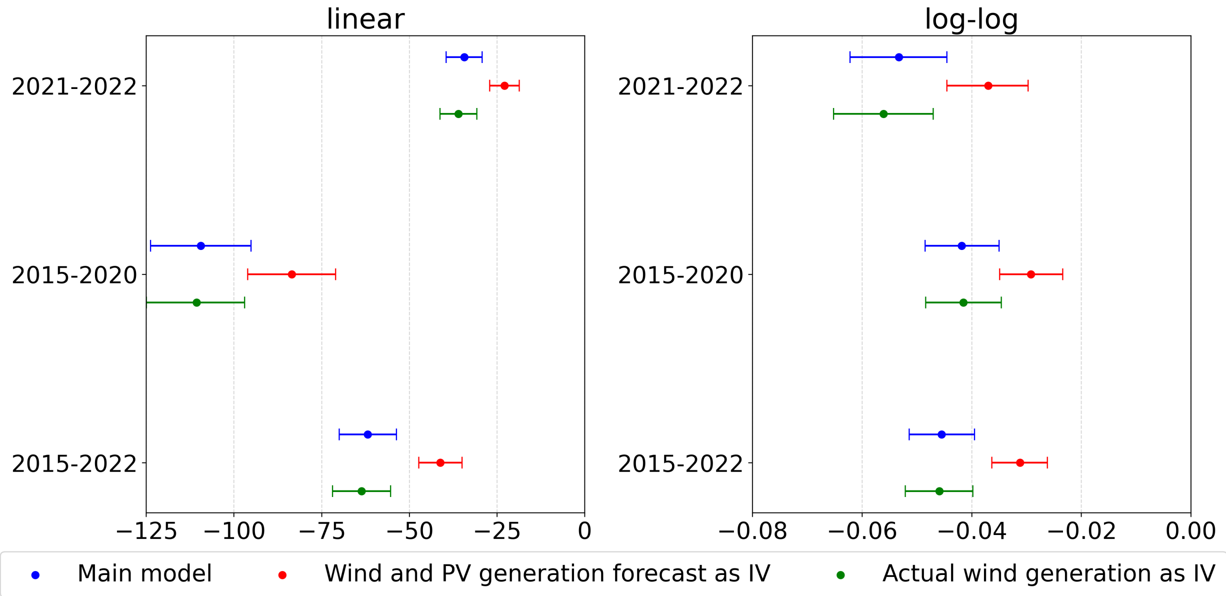

Figure B.10: Estimators for the price effect on demand for the linear (left) and log-log model specification (right). Results are shown for the main model specification (Table 3 and Table 4) (blue), a model specification where the day-ahead PV generation forecast was included as additional IV (red) and a model specification where the actual wind generation was used instead of the forecast. Whiskers indicate the 95% confidence interval.

Appendix C. Effect of dummy controls

Table C.6 shows the main model results of the linear model specification for the years 2015-2022 (2a) compared to the results of model specifications with less dummy control variables (6)-(9). The model without dummy controls (6) does not produce plausible results, e.g. the influence of PV generation on demand is strongly positive due to temporal correlation. Adding dummy controls for the hour of a day (7) can partially correct for this: the sign of the estimator becomes negative. Model (8) further includes dummies for the day of the week. Since there are major differences in the demand structure of weekends and weekdays, the Adjusted R squared and, thus, the model's fit increase significantly. Adding monthly dummies to the adjustment for seasonal effects (8) reduces, in particular, the estimated impact of weather effects (Heating and cooling degrees, PV generation). Finally, the addition of the yearly dummies (main model (2a)) especially affects the estimators of the impact of fuel and emission prices since the influence of changing power plant capacities over time is captured in these dummies.

Table C.6: Comparison of main linear model results for the years 2015-2022 and results of model specifications with less dummy control variables.

	(6)	(7)	(8)	(9)	(2a)
Price [EUR/MWh]	-123.5***	-72.71***	-73.08***	-58.98***	-61.81***
PV [GW]	472.4***	-307.3***	-289.6***	-212.1***	-204.0***
Gas [EUR/MWh]	228.5***	108.5***	103.8***	80.38***	86.41***
Coal [EUR/MWh]	-172.7***	-2.982	10.84	22.47***	61.79***
EUA [EUR/tCO2]	23.72***	25.12***	26.39***	15.66***	150.8***
Heating degrees [°C]	715.9***	560.2***	567.6***	389.2***	379.1***
Cooling degrees [°C]	296.5***	227.0***	222.7***	190.4***	176.4***
Dummy variables					
Hours	No	Yes	Yes	Yes	Yes
Weekdays	No	No	Yes	Yes	Yes
Months	No	No	No	Yes	Yes
Years	No	No	No	No	Yes
Fit statistics					
Adjusted R^2		0.505	0.836	0.861	0.868
Observations	65920	65920	65920	65920	65920

95% confidence intervals in brackets. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

Appendix D. Within-variance of the covariates

I exploit the within-variation of the explanatory variables after controlling for time effects when estimating their impacts on the dependent variable. To this end, I include time-related dummy controls in my model. I calculate the variation of the covariates after controlling for time effects by regressing them on the control dummies. Table D.7 shows the standard deviations of the corresponding residuals, calculated for the linear model specification (2a) and the log-log specification (2b). The within-variation is still considerable after controlling for the time effects. For example, the standard deviation of the main explanatory variable, $Price_t$, is much bigger than the discussed treatments of 1 EUR/MWh. The listed values in Table D.7 may support the interpretation of the estimated treatment effects of the explanatory variables (Mummolo and Peterson, 2018).

Table D.7: Within standard deviation of the variables

Model:	(2a)	(2b)
Price [EUR/MWh]	45.44	0.61
Wind [GW]	8.41	0.80
PV [GW]	3.88	0.45
Gas [EUR/MWh]	17.20	0.27
Coal [EUR/MWh]	3.58	0.20
EUA [EUR/tCO2]	4.55	0.15
Heating degrees [°C]	2.89	0.52
Cooling degrees [°C]	2.15	0.52

Appendix E. Estimators for dummy variables

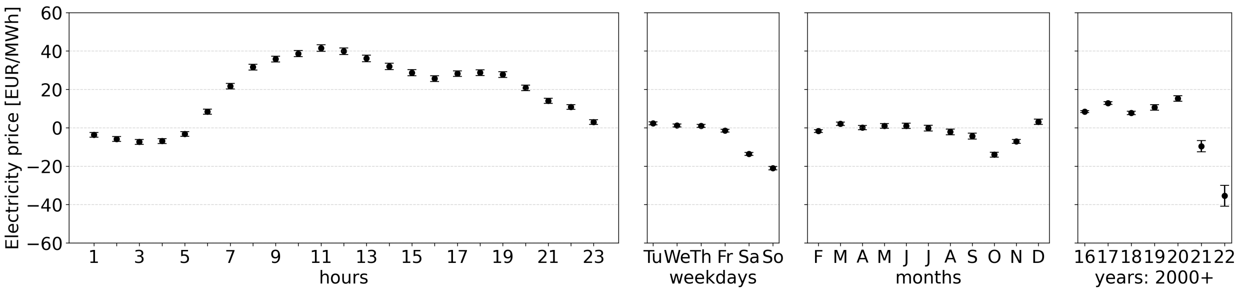

Figure E.11: Time dummies in the linear first stage (1a)

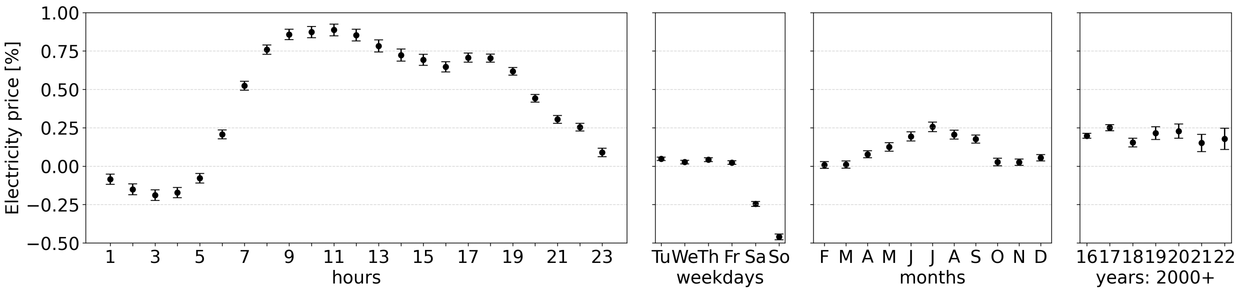

Figure E.12: Time dummies in the log-log first stage (1b)

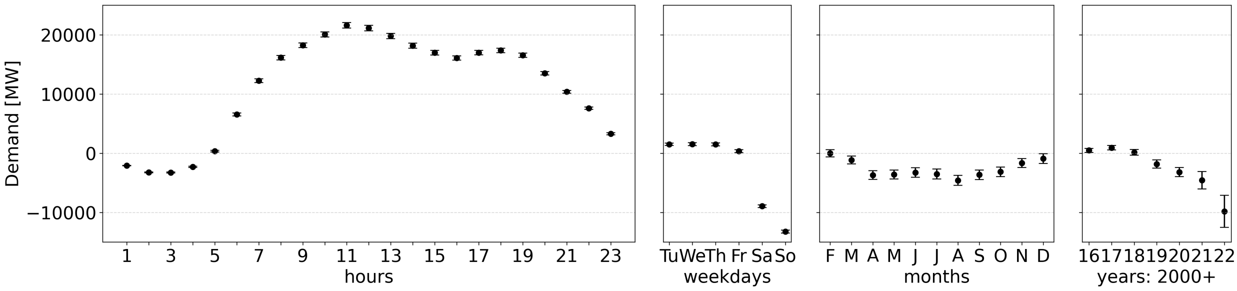

Figure E.13: Time dummies in the linear second stage 2015-2022 (2a)

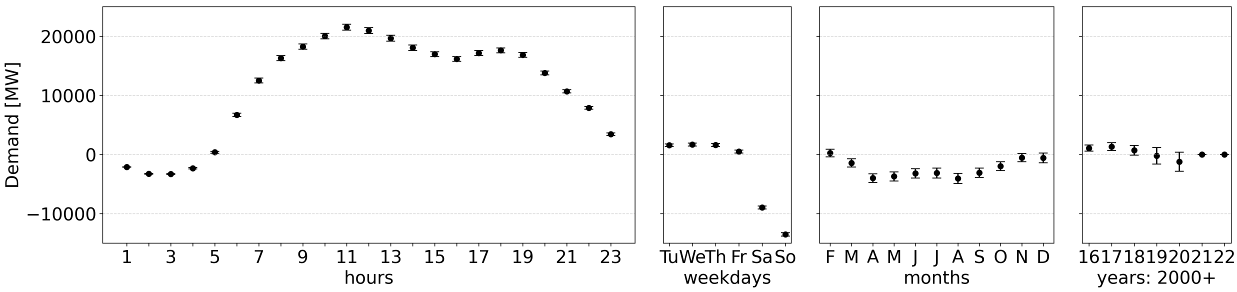

Figure E.14: Time dummies in the linear second stage 2015-2020 (3a)

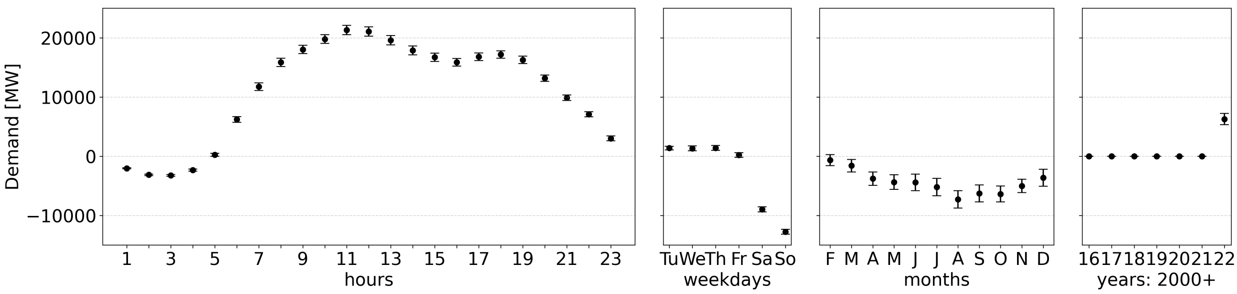

Figure E.15: Time dummies in the linear second stage 2021-2022 (4a)

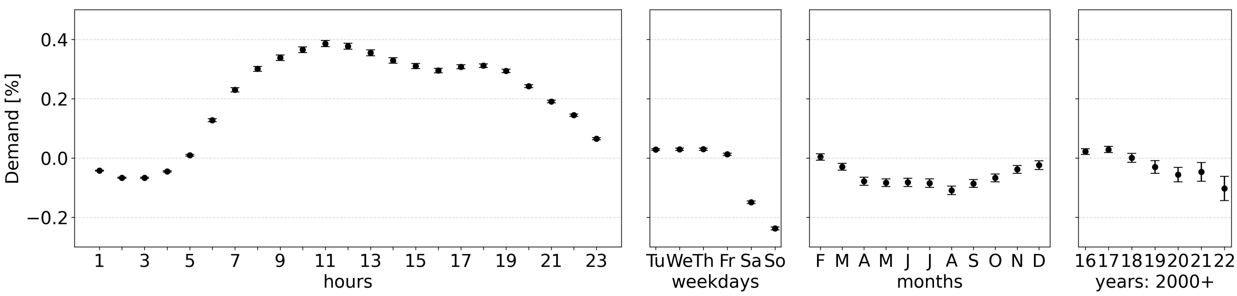

Figure E.16: Time dummies in the log-log second stage 2015-2022 (2b)

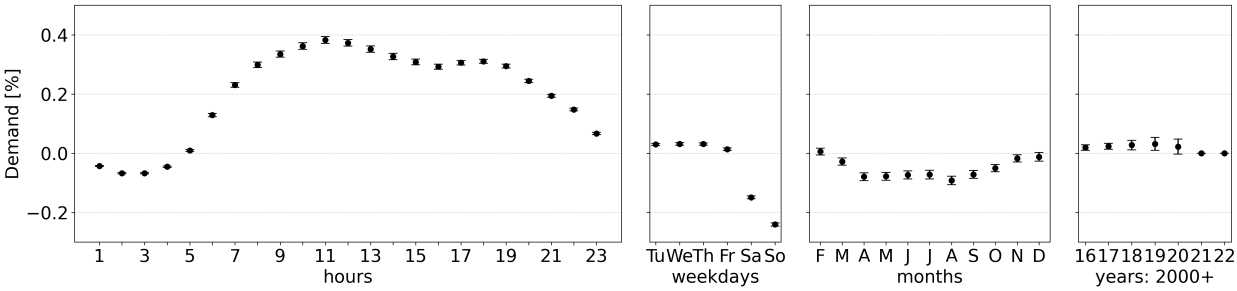

Figure E.17: Time dummies in the log-log second stage 2015-2020 (3b)

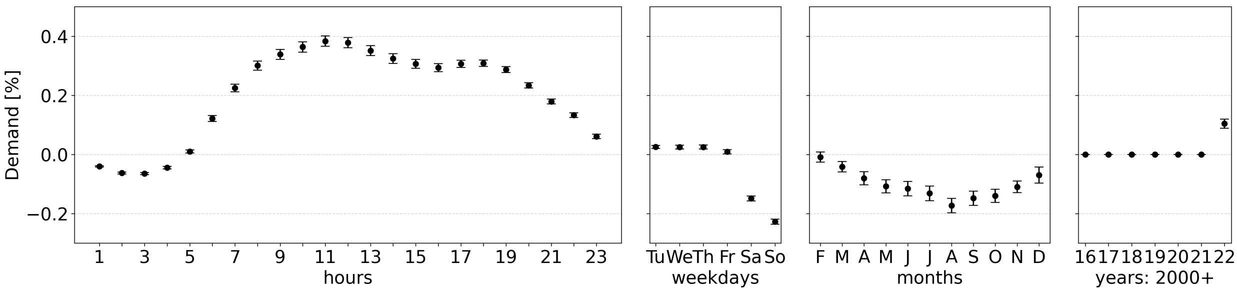

Figure E.18: Time dummies in the log-log second stage 2021-2022 (4b)

Appendix F. Further estimation results

Appendix F.1. Models with alternative separation dates

Figure F.19 shows the estimators and confidence intervals for the main model specification and two alternative models with differing separation dates (X) between the low price time period and high price time period: after the first Quarter of 2021 (red) and after the first half of 2021 (green). The choice of the separation date hardly changes the results. Since the choice of a later date increases the price differences between the subsets, the linear estimates for the subsets diverge even further. In comparison, the estimates for the subsets in the log-log models move closer together. The results thus reinforce the validity of statements made based on the main model specifications.

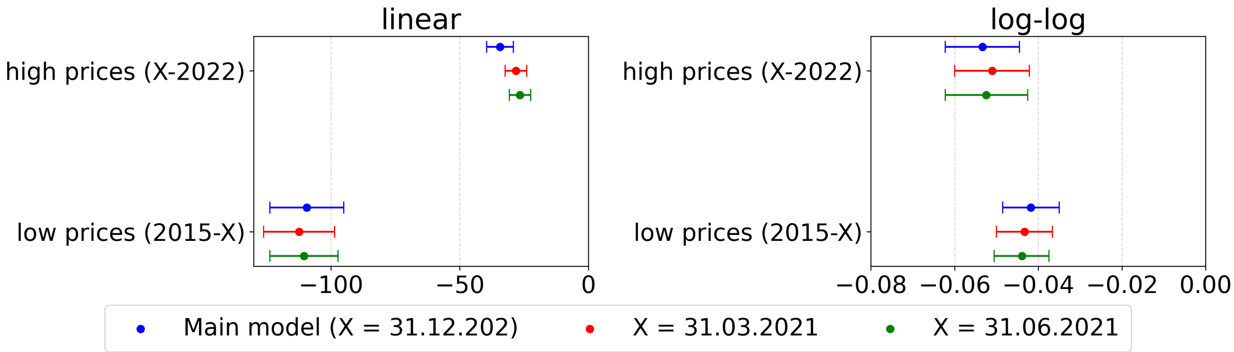

Figure F.19: Estimators for the price effect on demand for the linear (left) and log-log model specification (right), separately for low and high price time subsets. Results are shown for the main model specification (Table 3 and Table 4) (blue) and two model specifications with differing separation dates (X) between the low price time period and high price time period. Whiskers indicate the 95% confidence interval.

Appendix F.2. Models with further supply-side covariates

Figure F.20 shows the estimators and confidence intervals for the main model specification and an alternative model specification that includes additional supply-side covariates representing German and French available nuclear capacities and German available hydro-power and coal-power capacities. As data on power plant capacities and availability are unavailable, I use proxies based on the hourly generation data from ENTSO-E (2023). To approximate weekly/monthly available capacities, I use the weekly/monthly generation maxima. I cannot use the hourly or even daily values of generation directly as available capacities, as the generation output of these plants reacts to electricity prices, which would lead to endogeneity issues. I use the weekly maximum for nuclear and hydro generation, as the electricity generation from these plants is less reactive to prices. To approximate the development of available coal-power capacities in Germany, I use the monthly maximum generation values. Including these supply-side covariates hardly changes the results.

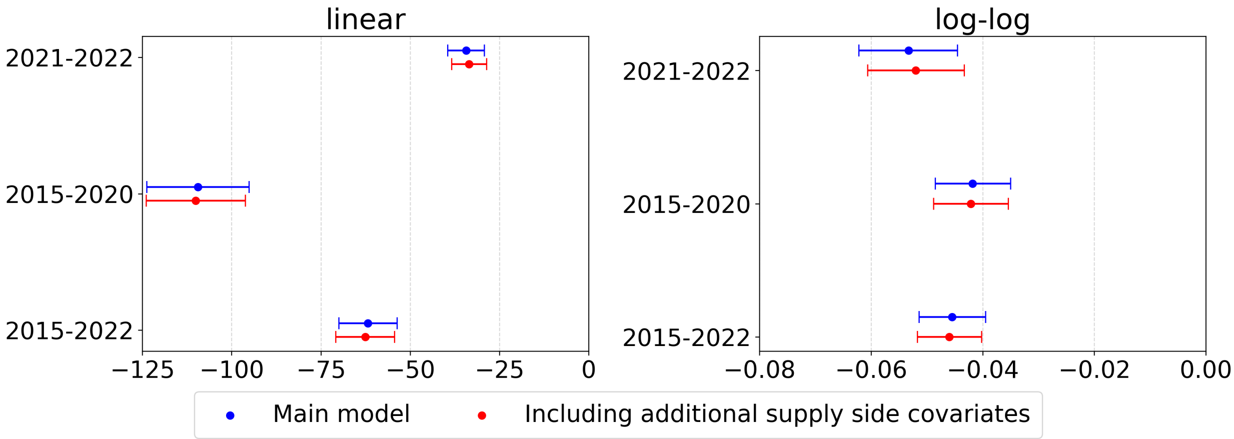

Figure F.20: Estimators for the price effect on demand for the linear (left) and log-log model specification (right). Results are shown for the main model specification (Table 3 and Table 4) (blue), and a model specification including additional supply-side covariates (red). Whiskers indicate the 95% confidence interval.

Appendix F.3. Models without price covariates

Table F.8 shows the estimators and confidence intervals for the first stage of model specifications without the utilization of gas, coal and EUA prices as covariates. The estimators for the influence of $Wind_t$ on the $Price_t$ are slightly higher than in the main model specifications (Model 1a and 1b). The partial R squared still shows for both models (linear and log-log) that $Wind_t$ explains a relevant part of the variance of the price. The F-statistics of the Montiel-Pflueger robust weak instrument test at 5% confidence level are 9,254 and 7,924, respectively, which is well above the critical value of the test of 37.42. Based on the test result and the significance at the first stage, I conclude that also without the usage of the price covariates $Wind_t$ remains a strong instrument for $Price_t$.

The Tables F.9 and F.10 show the estimators and confidence intervals for the second stage of model specifications without the utilization of gas, coal and EUA prices as covariates. The estimators of the Models 11 to 13 are of very similar magnitudes to the estimators of the main model specifications (Model 3 to 5). The results thus confirm the analysis results, suggesting that any potential endogeneity issue resulting from the price covariates is inconsequential.

Table F.8: First stage results of model specifications without price covariates

	(10a) linear		(10b) log-log	
Wind [GW]	-1.978***	[-2.02,-1.94]	-0.319***	[-0.33,-0.31]
PV [GW]	-2.482***	[-2.59,-2.37]	-0.300***	[-0.31,-0.29]
Heating degrees [°C]	0.978***	[0.87,1.09]	0.0897***	[0.08,0.10]
Cooling degrees [°C]	1.734***	[1.54,1.93]	0.0669***	[0.06,0.08]
Dummy variables				
Hours	Yes		Yes	
Weekdays	Yes		Yes	
Months	Yes		Yes	
Years	Yes		Yes	
Fit statistics				
Partial R^2 Wind	0.132		0.174	
Adjusted R^2	0.702		0.643	
Observations	65920		65011	

95% confidence intervals in brackets. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

Table F.9: Second stage results of the linear specification without price covariates

	(11a) 2015-2022	(12a) 2015-2020	(13a) 2021-2022
Price [EUR/MWh]	-56.33***	-109.1***	-34.14***
	[-64.28,-48.39]	[-124.00,-94.26]	[-40.61,-27.68]
PV [GW]	-188.7***	-171.0***	-212.8***
	[-216.16,-161.21]	[-199.52,-142.52]	[-256.20,-169.46]
Heating degrees [°C]	390.9***	362.9***	435.2***
	[339.34,442.53]	[322.40,403.47]	[334.82,535.60]
Cooling degrees [°C]	190.4***	149.5***	274.6***
	[144.50,236.33]	[112.24,186.85]	[179.26,369.97]
Dummy variables			
Hours	Yes	Yes	Yes
Weekdays	Yes	Yes	Yes
Months	Yes	Yes	Yes
Years	Yes	Yes	Yes
Fit statistics			
Adjusted R^2	0.844	0.881	0.836
Observations	65920	49434	16486

95% confidence intervals in brackets. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

Standard errors are calculated as Newey-West HAC robust standard errors.

Table F.10: Second stage results of the log-log specification without price covariates

	(11b) 2015-2022	(12b) 2015-2020	(13b) 2021-2022
Price [EUR/MWh] (log)	-0.0454***	-0.0421***	-0.0536***
	[-0.05,-0.04]	[-0.05,-0.03]	[-0.06,-0.04]
PV [GW] (log+1)	-0.0243***	-0.0218***	-0.0286***
	[-0.03,-0.02]	[-0.03,-0.02]	[-0.03,-0.02]
Heating degrees [°C] (log+1)	0.0285***	0.0252***	0.0331***
	[0.02,0.03]	[0.02,0.03]	[0.02,0.04]
Cooling degrees [°C] (log+1)	0.0228***	0.0210***	0.0266***
	[0.02,0.03]	[0.02,0.02]	[0.02,0.03]
Dummy variables			
Hours	Yes	Yes	Yes
Weekdays	Yes	Yes	Yes
Months	Yes	Yes	Yes
Years	Yes	Yes	Yes
Adjusted R^2	0.880	0.890	0.853
Observations	65011	48670	16341

95% confidence intervals in brackets. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.
Standard errors are calculated as Newey-West HAC robust standard errors.