The *NEULING* Model

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Abstract

This paper introduces the fundamental electricity market model *NEULING*, which provides least-cost dispatch and re-dispatch calculations in high temporal resolution. The underlying methodology is discussed in the context of the existing literature. Hereby, a special focus is on the classification of the applied DC load flow approach. Furthermore, the equations of the dispatch and re-dispatch models are presented in detail.

1. Some Conceptual Foundations

Electricity market models are a well established tool to analyze the fundamental principles governing the operation and the development of power markets. According to Ventosa et al. (2005), three modeling streams can be distinguished. First, simulation models aim at the reproduction of observed market conditions and results, for example to identify strategic behavior on the part of market participants. An example for this approach can be found in von Hirschhausen et al. (2007). Second, optimization models compute the market outcome with respect to a specified normative goal, for example profit maximization or welfare maximization. Profit-maximizing models usually reflect the perspective of a single price-taking market participant, as in Gatzen (2008). In contrast, welfare maximization reflects the perspective of a social planner or, according to the first welfare theorem, the result of a perfectly competitive market. A welfare maximizing approach is e. g. implemented in Leuthold et al. (2012). The third category according to Ventosa et al. (2005) comprises equilibrium models, which explicitly consider the strategic behavior of individual market players. Thereby, the models may consider one- or multi-stage settings (for an application, see Shanbhag et al. (2011)).

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The electricity market model *NEULING* presented in this chapter has been developed for the purpose of an in-depth analysis of the European electricity market. The focus of the model applications is on the identification of fundamental drivers of the market outcome such as the local generation and transmission network capacities. Furthermore, the impact of market design choices and congestion management shall be analyzed under consideration of physical power flows. In this context and under consideration of the large scale of the problem at hand, the method of choice is a normative social planner approach. The following discussion thus focuses on fundamental optimization models as well as on approaches to power flow modeling.

1.1. Fundamental Electricity Market Models

Fundamental electricity market models optimize the satisfaction of demand by a set of power plants from the point of view of a social planner. The supply side may be defined by the available technologies (e.g. renewable, conventional and storage plants), their capacities and specific costs, as well as by technical constraints which restrict the plants' operation. Demand is either modeled as invariant and exogenously given, or as cost sensitive and flexible.[2] Both approaches reflect different assumptions regarding the price elasticity of electricity demand. Furthermore, demand my be differentiated according to products such as electricity and balancing reserves. The complexity of the optimization problem is finally driven the structure and number of the technical or economic constraints, by intertemporality and the number of variables, which among others depends on the spatial and temporal resolution as well as on the modeling of electricity transmission (see section 1.2). The central results of the optimization comprise information on the spatial and temporal allocation of demand and supply, on regional power exchange, the costs and benefits of electricity supply as well as on the marginal values of production and transmission.

Fundamental electricity market models may be distinguished in cost minimization and welfare maximization approaches. The concepts usually imply different assumptions on demand elasticity. Generally, cost minimizing models make use of fixed (i. e. completely inelastic) demand values and therefore

[2]Since (marginal cost-based) prices are an ex-post interpretation of the market clearing constraint's dual variable, demand cannot be represented by a direct function of the market price. Instead, the value of consumption is calculated as the space below the inverted demand function in dependency of the supplied quantity.

only optimize supply. In contrast, welfare maximizing models usually compute the optimal difference between the utility and the cost functions. Therefore, the objective function implicitly comprises information on the slope of the inverted demand function, and thus on elasticity. Independent of these classical model setups, both mathematical problem formulations can theoretically be adapted to inelastic and elastic demand functions. The combination of welfare maximization and inelastic demand however requires a specification of the value of lost load (VOLL), in order to provide the solvers with a bounded solution space. Irrespective of the assumed demand elasticity, both setups are connected via the duality of the welfare maximization problem and the expenditure minimization problem of the social planner. The concept of duality implies that the objective function of one problem forms a constraint of the other and vice versa. Furthermore, it ensures that the optimal solutions of the primal variables are identical in both problems and that the models' dual variables take the reciprocal values of each other.[3] Concerning the social planner approach in general, the first welfare theorem implies that the socially optimal outcome can be implemented in a perfectly competitive market. In the context of the electricity market, this is illustrated in Schweppe et al. (1988).

Both cost minimization and welfare maximization approaches have been widely applied. For the case of welfare maximization, examples can be found in Weigt (2006) and Green (2007), whereas cost minimization models are applied in Bartels (2009) and Nicolosi (2012). With regard to the model specification and parameterization, welfare maximizing models require a nonlinear problem formulation and the definition of assumptions on demand elasticity. Since the real-time elasticity of electricity demand is assumed to be small, the simplification applied in cost-minimizing models is often accepted in exchange for less computational effort.[4]

Non-linear elements may also be introduced into fundamental electricity market models by considerations of unit commitment, as in Leuthold et al. (2012). With the help of mixed-integer problem formulations, technical ramping constraints an minimum load conditions may be considered in greater detail. Again, this increases the computational complexity of the models. However, the alternative of applying linearized operational con-

[3]A general comment on duality can e. g. be found in Chiang and Wainwright (2005).

[4]On the real-time elasticity of electricity demand, see Lijesen (2007).

straints as e. g. discussed in Kuntz and Müsgens (2007) has been shown not to impair the quality of the results significantly (see Abrell et al. (2008)).

Further issues influencing the complexity of the model are the considered time horizon as well as the applied temporal resolution. The time horizon is especially relevant for investment models which consider the economic impact of additional capacities over their respective lifetime.[5] With regard to dispatch models or dispatch components of investment models, the temporal resolution influences the representativeness of the results. Instead of computing the operation of power plants over an entire year, the definition of type-days allows for a reduction of the computational effort. Type-days are usually chosen to reflect typical combinations of load and feed-in from renewable energy sources (RES) and thus often neglect extreme events. This approach is applied both in dispatch as well as in combined dispatch and investment models, such as in Nüßler (2012) and EWI (2011). In contrast, Gatzen (2008) chooses a model specification with reflects 8760 consecutive hours of a given year for the purpose of the assessment of the profitability of pumped-storage plants. As demonstrated in Nicolosi (2012), the temporal resolution may influence the results of investment models considerably, especially in systems with a high share of RES.

Time is also an important factor in stochastic electricity market models, which allow for the representation of a gradual disclosure of market-relevant information. In contrast to models which are based on the assumption of perfect foresight, unit commitment or investment decisions may be made without full knowledge about future developments. This may e. g. concern the production of RES or the evolution of demand. Corresponding examples may be found in Meibom et al. (2006) and Abrell and Kunz (2012).

Furthermore, fundamental electricity market models can be differentiated according to the representation of the transmission network. An overview of common approaches is given in the following.

1.2. The Power Flow Model

The consideration of electricity exchange is an important property of fundamental electricity market models which aim at a realistic representation of an interconnected electricity system. A simple approach to the analysis of power exchange is the definition of one-node regions and the use of variables

[5] Cf. EWI (2011).

which define bi-directional flows between the regions. In this setup, the limits of power exchange are commonly defined by so-called net transfer capacities (NTC). Since the physical flows in the real world are governed by Kirchhoff's and Ohm's laws, the NTC-approach is a substantial simplification. Given that the constellation of local consumption and production influences the power flows on the entire network, the results of NTC-based models will be sub-optimal or even infeasible in reality.

The available alternatives are alternating current approaches (AC), direct-current approaches (DC) and the use of power transfer distribution matrices (PTDF). The properties of the models are discussed in detail in Schweppe et al. (1988) and Stigler and Todem (2005) (both AC and DC), as well as in Waniek (2010) (PTDF) and Nüßler (2012) (all approaches). The AC approach constitutes the most detailed representation of power flows, since it considers both active and reactive power as well as transmission losses. Thus, the flows on highly complex networks can be reproduced. However, solving the AC models requires an iterative process which may not converge at all times.[6] Furthermore, the integration of an AC approach into large-scale electricity market models is challenging. Thus, applications as in Barth (2007) are rare.

The DC approach provides a linear approximation of power flows and can thus be directly incorporated into electricity market models. The linearization is achieved by a neglect of reactive power flows, by the assumption of small phase angles and by a normalization of the voltage magnitudes to one. According to Schweppe et al. (1988), all of the assumptions become more inaccurate for lower voltage and distribution lines, as well as in times of high line loading. Furthermore, assuming a constant rather than a time-variant resistance-reactance ratio will lead to approximation errors. As stated in Schweppe et al. (1988), the last assumption may however facilitate the interpretation of a market model's results. Purchala et al. (2005) test the validity of the DC assumptions and conclude that the overall performance of the approximation is good, although errors on individual lines may occasionally be significant.

Power transfer distribution factors also allow for a linear representation of power flows. Just as the DC approach, PTDF matrices can neither reflect reactive power flows nor losses. The underlying factors may either be

[6] Cf. Groschke et al. (2009).

derived from AC or from DC models and reflect the load flows in a specific consumption and production constellation. Thus, the matrices vary with the use of the network. However, the matrices are usually defined on the basis of a representative constellation rather than for each each of them. Baldick (2002) and Lui and Gross (2002) demonstrate theoretically and empirically that this approximation is usually justifiable. However, Duthaler et al. (2008) highlight that the approximation error becomes substantial if the calculations are based on a zonal model. This is especially the case, if the zones are not defined in accordance with the fundamental market structure. In general, a new calculation of PTDF matrices is inevitable in the case of a change in the network topology.

Both DC and PTDF approaches have been applied in various settings. The DC approach is often applied in the context of nodal pricing models. For example, Overbye et al. (2004) demonstrate that the DC-based power flow calculation in locational marginal pricing models is fairly close to the AC solution using the case of the Midwest U.S. transmission system. Further applications in nodal pricing scenarios are among others presented in Weigt (2006), Green (2007) and Kunz (2009). In contrast, the PTDF approach is often used in the analysis of international power flows. Waniek (2010) demonstrates that the application of PTDF matrices is preferred to NTC-based specifications of international transmission capacities with respect to welfare.

Both approaches are also common in the fundamental analysis of re-dispatch. Re-dispatch is a congestion management approach applied within bidding zones that resolves internal congestion by the means of adjusting power plant generation schedules. Just as in the case of dispatch models, the re-dispatch optimization problem can be specified by a social planner approach. Thereby, the accurate representation of power flows is of special importance. Nüßler (2012) uses a PTDF approach to calculate re-dispatch in the German market. Also Linnemann et al. (2011) use a PTDF approach and furthermore demonstrate a method to integrate (n-1)-security constraints into a re-dispatch model by the means of an iterative process. DC approaches are applied in Kunz (2011) and Görner et al. (2008), who also emphasize the possibility of network topology optimization as a congestion management option.

1.3. Classification of the NEULING Model

The New European Linear Investment and Grid Model *NEULING* as e. g. applied in Burstedde (2012) calculates the cost-minimal generation dispatch and re-dispatch of a given power plant portfolio for 8760 hours of a year. The high temporal resolution allows for the consideration of general structural patterns such as seasonal variations in load as well as of extreme events.

The spot market dispatch of conventional power plants is determined such that the residual demand (exogenous system demand including network losses less electricity produced by renewable energy sources and less must-run generation from combined heat and power plants) is met in each hour. Furthermore, the demand for positive and negative balancing reserve needs to be covered. The conventional power plant fleet including hydro storage (Hyd-S), pumped storage (Hyd-PS) and compressed air energy storage (CAES) plants is grouped into 25 so-called vintage classes according to primary fuel, age and technological characteristics such as efficiency. Each vintage class is then dispatched under consideration of variable and ramping costs, as well as linearized minimum- and part-load restrictions.

The cost-minimal nodal dispatch is subject to network restrictions implemented by a DC power flow model. The DC power flow gives an approximation of the physical flows over the high-voltage alternating current transmission network in the so-called core model regions. Thereby, both losses as well as reactive power are neglected in order to keep the problem linear. The lines' physical capacities are standardized and multiplied with a factor of 0.8 in order to account for a security margin. Furthermore, interconnectors to and in between so-called satellite regions are implemented by the use of net transfer capacities (NTC).

The basic spatial resolution of *NEULING* gives a nodal representation of the core model regions. Depending on the object of investigation, the nodes can be aggregated into zones. Although the zonal dispatch assumes internal copperplates, the information on the nodal net injections is preserved. Thus, the usage of the internal grid can be determined ex-post and the necessary re-dispatch in case of violations of network constraints can be calculated based on the plants' operating status and their utilization.

The optimal flow-based re-dispatch is represented by the least-cost combination of upward and downward ramping, which at the same time relieves the overloaded line and keeps the balance between demand and supply at each node. The marginal costs of a plant's upward re-dispatch are given by its variable fuel and ramping costs, while the marginal savings of its downward re-dispatch equal the avoided costs of generation. This basic setup corresponds to a cost-based mechanism in which the generators are either compensated or charged as to render them indifferent with regard to being re-dispatched.

2. Technical Model Description: Dispatch

In the following, the dispatch problem as solved by *NEULING* is introduced in detail. Its core is defined by cost equations as well as by market clearing and load flow constraints. Further equations restrict the dispatch solution by imposing technical limits on power plant operation. Additionally, the provision of positive and negative reserve may set upper or lower bounds to spot market production.

2.1. Total Costs of Electricity Production

NEULING minimizes total costs of electricity production TC over all model regions n for a given year y. Thereby, TC comprises variable costs of production VC^{PROD}, variable ramping costs VC^{RTO} and fixed operation and maintenance costs FC^{OM}:

$$\min_{y}! TC_y = VC_y^{PROD} + VC_y^{RTO} + FC_y^{OM}. \tag{1}$$

The variable costs of production equal the sum of fuel and CO_2 costs $fuel\_c$ as well as miscellaneous variable costs $other\_vc$ multiplied by electricity production GEN over regions n and all generation technologies t. The set of technologies includes both conventional (nuclear and thermal) as well as storage technologies such as pumped hydro storage, hydro reservoirs and compressed air energy storage (CAES). The latter subset may be addressed separately by the use of the index st. Compared to producing units, relatively higher costs apply to the portion of capacities which are kept ready to operate without actively contributing to the hourly demand-supply-balance in order to avoid start-ups or to provide positive balancing reserve: These capacities ($CAP\_RTO - GEN$) are priced with part load fuel costs $fuel\_c^{PL}$ and $other\_vc$. Furthermore, compressors of pumped storage or CAES plants operating ($COMP$) and those only ready to operate (CAP_RTO_COMP) also induce $other\_vc$. The associated consumption of energy is priced implicitly with the marginal cost of electricity supply since it raises the demand for electricity.

$$
\begin{aligned}
VC_y^{PROD} = &\sum_{h,n,t} GEN_{y,h,n,t} \cdot (fuel\_c_{y,h,n,t} + other\_vc_{n,t}) \\
&+ \sum_{h,n,t} (CAP\_RTO_{y,h,n,t} - GEN_{y,h,n,t}) \\
&\quad \cdot (fuel\_c^{PL}_{y,h,n,t} + other\_vc_{n,t}) \\
&+ \sum_{h,n,st} COMP_{y,h,n,st} \cdot other\_vc_{n,st} \\
&+ \sum_{h,n,st} (CAP\_RTO\_COMP_{y,h,n,st} - COMP_{y,h,n,st}) \cdot other\_vc_{n,st}
\end{aligned} \tag{2}
$$

Variable costs of ramping procedures are approximated by the part of capacity ramped-up in a given hour CAP_UP and CAP_UP_COMP, multiplied by fuel and attrition costs ($start\_attr$) incurred during the required start-up time $start\_time$. Whereas ramping up is thus associated with costs, ramping down is assumed to be free. However, reducing generation comes at the price of opportunity costs of ramping up again at a later point in time. Ramping costs are relevant for generation units of conventional technologies as well as for turbines and compressors of storage plants.

$$
\begin{aligned}
VC_y^{RTO} = &\sum_{h,n,t} CAP\_UP_{y,h,n,t} \\
&\quad \cdot (fuel\_c_{y,h,n,t} + start\_attr_t) \cdot start\_time_{y,h,n,t} \\
&+ \sum_{h,n,st} CAP\_UP\_COMP \cdot start\_attr_t \cdot start\_time_{y,h,n,t}
\end{aligned} \tag{3}
$$

Additionally to the variable costs of electricity supply, fixed operation and maintenance costs fom_c per installed MW of capacity $instcap$ enter the calculation of the total annual costs.

$$
FC^{OM} = \sum_{n,t} instcap_{y,n,t} \cdot fom\_c_t \tag{4}
$$

2.2. Market Clearing and Network Capacity Constraints

The central constraint in the optimization is the balance equation which ensures that demand is met at all times. The hourly demand values of *load* include both final consumption and network losses, but not the consumption of storage plants which is modeled endogenously. Energy produced by renewable energy sources *renewables* is provided exogenously to the model and reduces total demand to residual demand *res_load*:

$$res\_load_{y,h,n} = load_{y,h,n} - renewables_{y,h,n} \tag{5}$$

The residual load is then provided for by domestic generation and power exchange between model regions. Exchange between so-called core regions ($NETINPUT$) is represented by a DC load flow, whereas trade with and between so-called satellite regions EXC is based on net transfer capacities (NTC). Consumption of storage plants increases the required production, while curtailment of electricity produced by renewable energy sources (RES-E) decreases total supply. Curtailment comes at costs of zero and is thus applied when there is excess supply, either due to renewable infeed or ramping constraints and balancing market obligations of conventional capacities. It is restricted by the hourly infeed from renewables, such that $CURTAIL_{y,h,n}$ is smaller or equal $renewables_{y,h,n}$. The balance equation is now given by:

$$\begin{aligned}&\sum_t GEN_{y,h,n,t} - \sum_{st} COMP_{y,h,n,t} \qquad (6)\\&- NETINPUT_{y,h,n} + \sum_m EXC_{y,h,m,n}\\&= res\_load_{y,h,n} + \sum_m EXC_{y,h,n,m} + CURTAIL_{y,h,n}\end{aligned}$$

Thereby, $NETINPUT$ is specified as follows, where $DELTA$ is a free variable representing the phase-angle at a given node and b is the network susceptance matrix which represents the susceptances of all lines connecting nodes n and m:[7]

$$NETINPUT_{y,h,n} = \sum_m b_{y,n,m} \cdot DELTA_{y,h,m} \tag{7}$$

The DC load flow is restricted by the available line capacities:

$$-av\_sec \cdot cap\_line_{y,l} \leq \sum_n k_{y,n,l} \cdot DELTA_{y,h,n} \leq av\_sec \cdot line\_cap_{y,l} \tag{8}$$

[7]In a DC load flow, the susceptance of a line defines the degree to which power fed in at a connected node flows through the given line. It is determined by the reactance and resistance of the line.

where av_sec times $line\_cap$ equals the thermal capacity of line l reduced by a security margin which is a linear substitute for a (n-1)-security criterion. k represents the network transfer matrix which relates the susceptance of a line to its start- and end-nodes. In order to guarantee the solvability of the DC load flow problem, $DELTA$ is set to zero at the so-called slack node via a non-zero parameter *slack*. If all bus injections were specified, the energy balance constraint would almost certainely not be satisfied due to an overspecification of the model (cf. Schweppe et al. (1988)). Thus, the following equation is specified:

$$slack_n \cdot DELTA_{y,h,n} = 0 \tag{9}$$

The positive variable EXC is restricted via the following constraint to the available net transfer capacity ntc_cap, which typically varies per season:

$$EXC_{y,h,n,m} \leq av\_ntc_h \cdot ntc\_cap_{y,n,m} \tag{10}$$

2.3. Balancing Reserve Market Constraints

NEULING only considers the capacity provision of balancing reserve markets, not the call. Thus, the model calculates the least-cost alternatives for positive and negative reserve which implies a reduction of available capacities for spot market production and an increase of must-run generation respectively. The central balancing market constraints balance national supply and demand for each reserve product (here: secondary reserve SR and tertiary reserve TR) under consideration of prequalification constraints. Thereby, demand is specified per country c.

The example for positive secondary reserve SR_pos shows that standing and spinning reserves (POS_SR_STAND and POS_SR_SPIN) are differentiated. Standing reserves may be provided by (extramarginal) generating units not operating at the spot market, which start up quickly enough as to ensure timely production. Less flexible technologies may only serve as spinning reserve, i.e. when they are already generating and guarantee sufficiently quick increases in production levels. Accordingly, technology specific binary prequalification parameters (av_SR_stand and av_SR_SPIN) are assigned under consideration of the plants' flexibility and the lead times for the given reserve product. Furthermore, a separate variable for compressors of storage plants participating in the balancing market, POS_SR_COMP, is introduced. Compressors may only provide positive reserve by ramping-down (spinning) units, thus reducing consumption.

$$
\begin{aligned}
SR\_pos_c = &\sum_{n \in c,t} av\_SR\_stand_t \cdot POS\_SR\_STAND_{y,h,n,t} \\
&+ \sum_{n \in c,t} av\_SR\_spin_t \cdot POS\_SR\_SPIN_{y,h,n,t} \\
&+ \sum_{n \in c,st} av\_SR\_spin_{st} \cdot POS\_SR\_COMP_{y,h,n,st}
\end{aligned}
\tag{11}
$$

The case of negative secondary reserve SR_neg is different to the extent that it can either be provided by spinning generation capacities ramping down or by both standing and (not yet fully used) spinning compressor capacities increasing their consumption.

$$
\begin{aligned}
SR\_neg_c = &\sum_{n \in c,t} av\_SR\_spin_t \cdot NEG\_SR_{y,h,n,t} \\
&+ \sum_{n \in c,t} av\_SR\_spin_{st} \cdot NEG\_SR\_SPIN\_COMP_{y,h,n,st} \\
&+ \sum_{n \in c,st} av\_SR\_stand_{st} \cdot NEG\_SR\_STAND\_COMP_{y,h,n,st}
\end{aligned}
\tag{12}
$$

Both equations 11 and 12 can be reproduced for the case of tertiary reserve.

2.4. Storage Constraints

Storage plants require separate equations accounting for the intertemporal constraints on storage levels. The storage volume is implicitly given by the installed capacity $inst\_cap$ of the generating units multiplied by a volume factor vol_factor, which gives the average regional ratio of capacity and storage. The storage level ($STORAGE\_LEVEL$) at the beginning of the modelled time period is determined by an additional factor, $initial\_level$:

$$
\begin{aligned}
&STORAGE\_LEVEL_{y,h=1,n,st} \\
&= initial\_level_{n,st} \cdot vol\_factor_{n,st} \cdot inst\_cap_{y,n,st}
\end{aligned}
\tag{13}
$$

The upper and lower bounds of the storage level are given by the size of the storage and buffers for additional charging and discharging due to calls of secondary or tertiary balancing reserve (SR, TR).

$$\begin{aligned}&STORAGE\_LEVEL_{y,h,n,st}\\&\leq vol\_factor_{n,st}\cdot inst\_cap_{y,n,st}\\&- av\_SR\_spin_{st}\cdot NEG\_SR\_SPIN\_COMP_{y,h,n,st}\\&- av\_SR\_stand_{st}\cdot NEG\_SR\_STAND\_COMP_{y,h,n,st}\\&- av\_TR\_spin_{st}\cdot NEG\_TR\_SPIN\_COMP_{y,h,n,st}\\&- av\_TR\_stand_{st}\cdot NEG\_TR\_STAND\_COMP_{y,h,n,st}\end{aligned} \tag{14}$$

$$\begin{aligned}&STORAGE\_LEVEL_{y,h,n,st}\\&\geq av\_SR\_spin_{st}\cdot POS\_SR\_SPIN_{y,h,n,st}\\&+ av\_SR\_stand_{st}\cdot POS\_SR\_STAND_{y,h,n,st}\\&+ av\_TR\_spin_{st}\cdot POS\_TR\_SPIN_{y,h,n,st}\\&+ av\_TR\_stand_{st}\cdot POS\_TR\_STAND_{y,h,n,st}\end{aligned} \tag{15}$$

The intertemporality of storage plant dispatch is embodied by the following dynamic equation relating the hourly change of the storage level to the preceding compressor and generator operation. Thereby, the efficiencies $(eff,\ eff\_comp)$ of both components are accounted for, as well as natural inflow ex_inflow in the case of hydro storage.

$$\begin{aligned}&STORAGE\_LEVEL_{y,h,n,st}\\&= STORAGE\_LEVEL_{y,h-1,n,st}\\&+ eff\_comp_{st}\cdot COMP_{y,h-1,n,st} - \frac{1}{eff\_comp_{st}}\cdot GEN_{y,h-1,n,st}\\&+ ex\_inflow_{h-1,n,st}\cdot vol\_factor_{n,st}\cdot inst\_cap_{y,n,st}\end{aligned} \tag{16}$$

Furthermore, a yearly cycle may be defined for pumped storage and CAES plants by the use of the following equation:

$$\begin{aligned}&STORAGE\_LEVEL_{y,h=1,n,st}\\&= STORAGE\_LEVEL_{y,h=8760,n,st}\\&+ eff\_comp_{st}\cdot COMP_{y,h=8760,n,st} - \frac{1}{eff\_comp_{st}}\cdot GEN_{y,h=8760,n,st}\end{aligned} \tag{17}$$

2.5. Operational Constraints

The operation of power plants at the spot market is physically limited by their technical availability av_tech (typically varying per season) and potential balancing market obligations. This is reflected via the capacity ready to operate CAP_RTO. Its lower bound is given by the level of spot market production and the contracted positive reserve provided by spinning units. The latter ensures that the corresponding capacities are ready to operate in the (hypothetical) case of a call.

$$\begin{aligned} CAP\_RTO_{y,h,n,t} \geq\ & GEN_{y,h,n,t} \\ & + av\_SR\_spin_t \cdot POS\_SR\_SPIN_{y,h,n,t} \\ & + av\_TR\_spin_t \cdot POS\_TR\_SPIN_{y,h,n,t} \end{aligned} \tag{18}$$

The upper bound of CAP_RTO is defined by the plants availability and the provision of positive standing reserves. The latter ensures that production may be increased in case of a call, but does not require the capacity to be ramped-up already.

$$\begin{aligned} CAP\_RTO_{y,h,n,t} \leq\ & av\_tech_{h,n,t} \cdot inst\_cap_{y,n,t} \\ & - av\_SR\_stand_t \cdot POS\_SR\_STAND_{y,h,n,t} \\ & - av\_TR\_stand_t \cdot POS\_TR\_STAND_{y,h,n,t} \end{aligned} \tag{19}$$

The level of capacity ready to operated can be influenced on an hourly basis by upward and downward ramping, CAP_UP and CAP_DOWN. Thus, the following dynamic constraint holds:

$$\begin{aligned} CAP\_RTO_{y,h,n,t} =\ & CAP\_RTO_{y,h-1,n,t} \\ & + CAP\_UP_{y,h-1,n,t} - CAP\_DOWN_{y,h-1,n,t} \end{aligned} \tag{20}$$

The variable CAP_RTO eventually restricts the level of spot market production. While the upper level of production is implicitly stated by 18, its lower bound is set by an approximation of minimal load levels ($min\_load \cdot CAP\_RTO$) plus obligations on negative reserve markets. The additive connection ensures that a call of negative reserve does not lead to the plant being switched off completely. Furthermore, the connection between CAP_RTO and generation indirectly ensures that spinning reserves are only provided by producing units.

$$
\begin{aligned}
GEN_{y,h,n,t} \geq\ & min\_load_t \cdot CAP\_RTO_{y,h,n,t} \\
& + av\_SR\_spin_t \cdot NEG\_SR_{y,h,n,t} \\
& + av\_TR\_spin_t \cdot NEG\_TR_{y,h,n,t}
\end{aligned}
\tag{21}
$$

The restrictions on and imposed by capacity ready to operate also apply to compressor units. Thereby, the specific definitions of reserves provided by compressors have to be accounted for.

$$
\begin{aligned}
& CAP\_RTO\_COMP_{y,h,n,st} \\
& \geq COMP_{y,h,n,st} \\
& + av\_SR\_spin_{st} \cdot NEG\_SR\_SPIN\_COMP_{y,h,n,st} \\
& + av\_TR\_spin_{st} \cdot NEG\_TR\_SPIN\_COMP_{y,h,n,st}
\end{aligned}
\tag{22}
$$

$$
\begin{aligned}
& CAP\_RTO\_COMP_{y,h,n,st} \\
& \leq av\_tech_{h,n,st} \cdot inst\_cap_{y,n,st} \\
& - av\_SR\_stand_{st} \cdot NEG\_SR\_STAND\_COMP_{y,h,n,st} \\
& - av\_SR\_stand_{st} \cdot NEG\_TR\_STAND\_COMP_{y,h,n,st}
\end{aligned}
\tag{23}
$$

$$
\begin{aligned}
CAP\_RTO\_COMP_{y,h,n,st} =\ & CAP\_RTO\_COMP_{y,h-1,n,st} \\
& + CAP\_UP\_COMP_{y,h-1,n,st} \\
& - CAP\_DOWN\_COMP_{y,h-1,n,st}
\end{aligned}
\tag{24}
$$

$$
\begin{aligned}
COMP_{y,h,n,st} \geq\ & min\_load_{st} \cdot CAP\_RTO\_COMP_{y,h,n,st} \\
& + av\_SR\_spin_{st} \cdot POS\_SR_{y,h,n,st} \\
& + av\_SR\_spin_{st} \cdot POS\_TR_{y,h,n,st}
\end{aligned}
\tag{25}
$$

3. Technical Model Description: Re-dispatch

The concept of re-dispatch may be categorized as curative congestion management. Curative instruments address congestion after the closure of the dispatch markets and the announcement of the resulting generation schedules.[8] Consequently, they imply the invisibility of network constraints within the bidding zone and the potential infeasibility of power flows resulting from trade.

In accordance with common practice, the re-dispatch module of *NEULING* is therefore decoupled from the dispatch problem. The connection between both parts is established by transferring the dispatch results to the re-dispatch model: (Dispatch-) Optimal generation levels, balancing market obligations and the NTC-based exchange between core and satellite regions eventually enter the re-dispatch calculation as exogenous input parameters. The re-dispatch module also requires information on further parameters such as demand, RES-E and variable costs, which remain unchanged with respect to the dispatch. Concerning the modules' inputs, the crucial difference lies in the network parameters. As a basis for re-dispatch, all flows over the entire network have to be calculated, whereas the dispatch only considers transmission between bidding zones. Therefore, the set of transmission lines is more comprehensive in the second model step. Furthermore, the lines' start and end nodes have to be redefined.

In the following, the implementation of re-dispatch in *NEULING* is discussed equation by equation.

3.1. Re-dispatch Costs

The objective function of the re-dispatch module is defined by the sum of the hourly variable costs of re-dispatch, which is minimized in the course of the solution process.

$$\text{min! } RC_y = \sum_{h,n} VC\_REDIS_{y,h,n} \tag{26}$$

Thereby, the variable costs equal the sum of additional fuel and CO_2 costs, other variable costs of production and ramping costs incurred by increasing generation at one point (GEN_REDIS_up) and the corresponding savings realized by decreasing generation at another point (GEN_REDIS_down).[9]

[8]In contrast, preventive congestion management mechanisms aim at influencing the dispatch before market closure as to reduce the risk of overloading network elements.

[9]The model does not explicitly consider any remuneration schemes which compensate re-dispatched generators. This is consistent with the chosen social planner approach. Nonetheless, the variable costs of re-dispatch as calculated by *NEULING* may be thought of as the outcome of perfect cost-based re-dispatch.

In the case of upward re-dispatch, separate variables are introduced for standing and part load generators: Positive re-dispatch of standing generators ($GEN\_REDIS\_up^{STAND}$) is restricted to quick-starting technologies, while re-dispatch from part load operation ($GEN\_REDIS\_up^{PL}$) is priced with part load variable costs (vc^{PL}). Furthermore, the consumption of compressors may be decreased as an option of positive re-dispatch ($COMP\_REDIS\_up$) or increased as a negative re-dispatch option ($COMP\_REDIS\_down$). Since compressors are quick starting units and do not incur significant losses from part load operation, the variables are not distinguished any further. The underlying cost parameters equal those of the dispatch model and are summarized by the production costs vc or vc^{COMP}, and ramping costs vc_rto or vc_rto^{COMP}.

$$\begin{aligned}
&VC_REDIS_{y,h,n} \qquad (27)\\
&= \sum_t GEN_REDIS_up_{y,h,n,t} \cdot (vc_{y,h,n,t} + vc_rto_{y,h,n,t})\\
&+ \sum_t GEN_REDIS_up^{STAND}_{y,h,n,t} \cdot (vc_{y,h,n,t} + vc_rto_{y,h,n,t})\\
&+ \sum_t GEN_REDIS_up^{PL}_{y,h,n,t} \cdot \left(vc^{PL}_{y,h,n,t} + vc_rto_{y,h,n,t}\right)\\
&- \sum_t GEN_REDIS_down_{y,h,n,t} \cdot vc_{y,h,n,t}\\
&+ \sum_{st} COMP_REDIS_down_{y,h,n,st} \cdot \left(vc^{COMP}_{y,h,n,st} + vc_rto^{COMP}_{y,h,n,st}\right)\\
&- \sum_{st} COMP_REDIS_up_{y,h,n,st} \cdot vc^{COMP}_{y,h,n,t}
\end{aligned}$$

3.2. Balance and Network Capacity Constraints

In order to quantify the need for re-dispatch, the power flows resulting from the dispatch have to be calculated once more. Since in a zonal setup the dispatch model only considers a limited set of network constraints, the true flows are obscured. Therefore, the re-dispatch model uses the nodal results for generator and compressor operation (*gen*, *comp*), NTC-based trade (*exc*), the curtailment of RES-E (*curtail*), as well as the nodal residual demand parameters as inputs for the calculation of the associated load flow-based nodal net exchange $NETINPUT\_DIS$. The net exchange is defined analogously to section 2.2.

$$\begin{aligned} &\sum_t gen_{y,h,n,t} - \sum_{st} comp_{y,h,n,st} \\ &- NETINPUT\_DIS_{y,h,n} + \sum_m exc_{y,h,m,n} \\ &= res\_load_{y,h,n} + \sum_m exc_{y,h,n,m} - curtail_{y,h,n} \end{aligned} \tag{28}$$

Again, the load flow calculation requires the fixation of the dispatch-related phase angle ($DELTA\_DIS$) at the slack node.

$$slack_n \cdot DELTA\_DIS_{y,h,n} = 0 \tag{29}$$

After these preparatory load flow calculations, the re-dispatch balance equation can be defined. It implicitly requires the modifications of the DC power flows not to impair the local balance of supply and residual demand. Thus, all re-dispatch related changes in the nodal balance need to net out exactly. Changes may be induced by ramping domestic generators and compressors up or down, by additional RES-E curtailment (RES_down) and/or by modifying the load flow-based net exchange $NETINPUT\_REDIS$.

$$\begin{aligned} &\sum_t \left(GEN\_REDIS\_up_{y,h,n,t} + GEN\_REDIS\_up^{STAND}_{y,h,n,t}\right) \\ &+ \sum_t \left(GEN\_REDIS\_up^{PL}_{y,h,n,t} - GEN\_REDIS\_down_{y,h,n,t}\right) \\ &- \sum_{st} \left(COMP\_REDIS\_down_{y,h,n,st} - COMP\_REDIS\_up_{y,h,n,st}\right) \\ &- RES\_down_{y,h,n} - NETINPUT\_REDIS_{y,h,n} \\ &= 0 \end{aligned} \tag{30}$$

For the calculation of the re-dispatch induced power flows, a separate slack condition has to be defined:

$$slack_n \cdot DELTA\_REDIS_{y,h,n} = 0 \tag{31}$$

The final goal of the re-dispatch is to relieve overloaded transmission lines. Thus, the sum of dispatch and re-dispatch related load flows needs to respect the individual line capacities.

$$-av\_sec \cdot cap\_line_{y,l} \leq \sum_n k_{y,n,l} \cdot DELTA\_DIS_{y,h,n} + \sum_n k_{y,n,l} \cdot DELTA\_REDIS_{y,h,n} \leq av\_sec \cdot line\_cap_{y,l} \tag{32}$$

3.3. Operational Constraints

The last part of the re-dispatch model consists of the capacity constraints which limit the ramping of generators and compressors. First the upward re-dispatch out of part load generation is restricted to the capacity currently in part load operation (cap^{PL}), which is an output of the dispatch.

$$GEN\_REDIS\_up^{PL}_{y,h,n,t} \leq cap^{PL}_{y,h,n,t} \tag{33}$$

Furthermore, the positive re-dispatch of capacities standing or in full load operation is limited to the unused capacity *cap_unused*. Again, *cap_unused* is an output from the dispatch model and equals the part of capacity neither used for generation nor reserved for balancing services. As stated above, capacities standing as a result of the dispatch may only start up if the technology is flexible enough.

$$GEN\_REDIS\_up^{STAND}_{y,h,n,t} + GEN\_REDIS\_up_{y,h,n,t} \leq cap\_unused_{y,h,n,t} \tag{34}$$

In the case of downward re-dispatch, *GEN_REDIS_down* is limited by the dispatch level of generation as well as by technological or reserve-related minimum load levels (*min_gen*).

$$GEN\_REDIS\_down_{y,h,n,t} \leq gen_{y,h,n,t} - min\_gen_{y,h,n,t} \tag{35}$$

Analogously, the re-dispatch from compressors is limited:

$$COMP\_REDIS\_up_{y,h,n,st} \leq comp_{y,h,n,st} - min\_comp_{y,h,n,st} \tag{36}$$

$$COMP\_REDIS\_down_{y,h,n,st} \leq cap\_comp\_unused_{y,h,n,st} \tag{37}$$

Concerning the use of storage technologies, positive re-dispatch of the generators requires sufficiently high storage levels. The hourly storage level values (*storage*) are given by the dispatch.

$$\left(GEN\_REDIS\_up^{STAND}_{y,h,n,t} + GEN\_REDIS\_up_{y,h,n,t}\right)\frac{1}{eff_{st}} \leq storage_{y,h,n,st} \tag{38}$$

Finally, RES-E curtailment in the course of congestion management is restricted to the remaining in-feed after dispatch.

$$RES\_down_{y,h,n} \leq renewables_{y,h,n} - curtail_{y,h,n} \tag{39}$$

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