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Service production functions

by

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von

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Abstract:

The paper derives production functions designed to model the evolution of service industries. The derivation is based on specifying the output elasticities of the factors according to differential equations and asymptotic technological boundary conditions in factor space. The derived functional forms incorporate labor, capital, energy, and technological parameters, whose time changes model innovation and structural change. The model is applied to the evolution of the German market-determined services 1960-1989.

Keywords: Production Factor Energy; Production Functions; Structural Change

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1 Introduction

The economic importance of service production increases. Recent economic growth in the industrialized countries has been largely based on expanding service sectors. This trend is likely to continue. Service growth is expected to provide a major source of employment. On the other hand, the largest potentials of automation are identified in trade, banking, insurance, and public administration. For instance, Thome (1997) estimates 6.7 million potential job losses in Germany by the diffusion of state-of-the-art IT-technology in the mentioned service industries.

In view of these and related issues, this paper develops production functions designed to model production and technological change in service industries. The methodological approach has been applied previously to derive production functions modeling industrial evolution (Kümmel et al. 1985, 2002). The approach takes into account the production factor energy, which enables to incorporate potential progress of automation, i.e. the substitution of labor by energy and (increasingly information processing) capital, in the production function. In addition, technological change is modelled via capital-related efficiency parameters.

Within the conventional production function approach, technological progress is exogenous and the production factor energy, if taken into account as a factor, is attributed minor importance. Regarding the minor technological importance of energy in production theory, it is noteworthy to recall the historical development of the neoclassical model. When the concept of marginal utility was introduced in the 19th century, the primary focus was on a theory of value, price, and exchange. One started with a 'model of pure exchange' of goods, without considering their production. When the marginal-utility concept was extended to apply to production, however, this implied the technological assumption of sufficient factor-input substitutability such that, at given factor prices, the cost-minimizing production optimum lies in the interior –and not on the boundary– of the region in factor space accessible to the production system according to the state of technology (at a given point in time). With this technological assumption of an interior optimum in factor space, marginal factor productivities equal factor prices, and, in the resulting production model, the output elasticities of the factors equal their cost shares. It is however not a-priori obvious, whether the assumption of sufficient factor-input substitutability and interior production optima is justified empirically. Alternatively, one might expect that routine labor can be substituted by energy and capital only to the extent to which technological progress removes technological restrictions and makes the corresponding factor combinations accessible.

In the industrialized countries the factor cost shares are typically 0.7 (labor), 0.25 (capital) and 0.05 (energy). With these shares as technological factor-input weights neither the recessions during the energy crises in the 1970s, nor long-term economic growth can be explained. Large residuals remain. They are associated with a time-dependent multiplier in the aggregate production function and interpreted as the effects of "technical progress" (Solow, 1957). In most cases the residual plays a more important role than the explanatory factors, which, according to Gahlen (1972), makes the neoclassical theory of production tautological. Solow (1994, p. 48) comments: "This ... has led to a criticism of the neoclassical model: it is a theory of growth that leaves the main factor in economic growth unexplained".

Motivated by the Solow residual, 'new' growth theories emerged since Romer (1986) and Lukas (1988). Their key feature is to drop the assumption of diminishing returns to capital by introducing knowledge and human capital, arguing for positive spillovers of knowledge. Along this line, many important contributions have been made, e.g. by endogenizing R&D-activity, the process of innovation, monopolistic competition, or international trade (see, e.g., Grossman and Helpman, 1989; Agion and Howitt, 1992; Romer, 1994). While these and related contributions have offered new theoretical arguments, they have, however, lead to only limited empirically-based insights, because a commonly accepted definition and measurement prescription of knowledge and human capital is still out of sight. For instance, Howard Pack (1994, p. 55) concludes: “But have the recent theoretical insights succeeded in providing a better guide to explaining the actual growth experience? This is doubtful.”

The present paper follows a complementary approach, keeping the conventional notion of physical capital and the assumption of diminishing returns. The approach takes into account the production factor energy in the production function, which enables to explicitly include production possibilities associated with increasing automation.

In fact, the issue of automation, and, more general, engineering foundations in production theory, although apparently crucial (as such and in view of the potentialities of and limits to technological progress), are virtually absent in the literature on the level of macro-economic production functions. As Dorfman, Samuelson, and Solow (1958, p. 131) remark, “...there seems to have been a misunderstanding somewhere because the technologists do not take responsibility for production functions either. They regard the production function as an economist’s concept, and, as a matter of history, nearly all the production functions that have actually been derived are the work of economists rather than of engineers“. Admittedly, some work on engineering production functions has been done, meanwhile. This work, however, focuses rather on specific industrial production processes than on the macro-level. To give an example, Gow (2002) derives production functions depending on capital, labor, energy, and specific material inputs for olefin alkylation processes in refinery engineering, taking into account thermodynamic laws and process constitutive equations[1]. From this perspective, the present paper, which has its roots in the work of Kümmel (1980, 1982), tries to

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[1] E.g., kinetic rate or stoichiometric relations.

contribute to bridging the gap between thermodynamics and engineering on the one hand and macro-economic production functions on the other.[2] Whereas the (macro-) production functions of Kümmel et al. (1985) focus on industrial production, the present paper proposes new functional forms designed to model the evolution of service industries.

The commonly used macro-economic production functions, apart from general convexity requirements, do not exploit any specific technological a-priori information. Translog functions, as second-order Taylor expansions of any (log-linear) functional form, are as such general and flexible – but the resulting relatively large number of their free parameters reduces their explanatory power from a statistical perspective. In view of the principle trade-off between generality and flexibility on the one hand and explanatory power on the other, it would be desirable to reduce the degrees of freedom of production functions not only through general convexity requirements, but by imposing specific technological boundary conditions, while introducing parameters with a well defined physical/technological interpretation. This is the strategy of the present paper. It derives energy-dependent production functions by specifying technological boundary conditions for the elasticities of production, and then obtains production functions by integration. This way, Section 2 derives production functions for service industries. Section 3 provides a numerical example by applying the model to the market-determined services in Germany for the period 1960-1989. Section 4 provides a summary and conclusions.

[2] "The need to reintegrate the natural sciences with economics" is discussed by Hall et al. (2001).

2 Deriving production functions from technological boundary conditions

In modelling the evolution of production systems we start from the following total time-derivative of a production function *q(k,l,e,t)*:

$$\frac{dq}{dt} = \frac{\partial q}{\partial k}\frac{dk}{dt} + \frac{\partial q}{\partial l}\frac{dl}{dt} + \frac{\partial q}{\partial e}\frac{de}{dt} + \frac{\partial q}{\partial t}. \qquad (1)$$

Here *q, k, l,* and *e* are appropriate measures of value-added, capital, labor, and energy, respectively. Taking into account the production factor energy is necessary, if production functions are to have a physically sound interpretation. Energy is measured in energetic units, e.g. petajoule per year, labor in hours worked per year. Ideally, one would like to measure capital by the amount of work performance and information processing that capital is able to deliver when fully activated by energy and labor. Likewise, the output might be measured by the work performance and information processing necessary for its generation. The detailed, quantitative technological definitions of capital and output are given in (Kümmel, 1980, 1982). Since, however, the corresponding physical measurements are not available empirically, proportionality between them and the constant currency data is assumed. In Eq. (1), all quantities are normalized to their absolute values Q_0, K_0, L_0, E_0 in a base year ´*0*´, i.e., $q=Q/Q_0$, $k=K/K_0$, $l=L/L_0$, $e=E/E_0$. We include an explicit time-dependence of *q* to model structural change and innovation, e.g. improvements of organisational and energy conversion efficiencies, see below.

Rearranging (1) yields the following growth equation (2) that relates the (infinitesimal) relative change of the normalized output, *dq/q*, to the relative changes of the normalized factor inputs, *dk/k, dl/l, de/e*:

$$\frac{dq}{q} = \alpha\frac{dk}{k} + \beta\frac{dl}{l} + \gamma\frac{de}{e} + C, \qquad (2)$$

where $\alpha \equiv (k/q)/(\partial q/\partial k)$, $\beta \equiv (l/q)/(\partial q/\partial l)$, and $\gamma \equiv (e/q)/(\partial q/\partial e)$ are the output elasticities of capital, labor, and energy, and $C \equiv (t/q)(\partial q/\partial t)(dt/t)$ represents the explicit time-dependence of the production function.

As long as the explicit time-dependence vanishes, i.e.,

$$C = 0, \tag{3}$$

capital, labor, and energy are, by definition, all factors of production, and technical causality in their work performance and information processing uniquely determines the output q. The mathematical consequences are that, given Eq. (3), the functions $\alpha(k,l,e)$, $\beta(k,l,e)$, and $\gamma(k,l,e)$ must satisfy the integrability conditions of Eq. (2) (see below) and the integral, i.e. the production function, exhibits constant returns to scale,

$$\alpha + \beta + \gamma = 1. \tag{4}$$

A non-vanishing C, on the other hand, will be associated with the time-dependence of the technological parameters to be introduced on the way of deriving the production function. Through changes of these parameters, one component of technical progress is modelled. The other component of progress, the one based on increasing automation, i.e. the substitution of labor by energy-driven (and increasingly information processing) capital, will be explicitly incorporated into the energy-dependent functional form. Note that energy-dependent production functions are not necessarily confined to the mapping of a set of production possibilities at a given point in time, but may incorporate potential future production possibilities as a result of automation-based technical progress.

The integrability conditions of the production function, i.e. the requirement that the second-order mixed derivatives of q with respect to capital, labor, and energy have to be equal, result in a set of three (cyclically symmetric) coupled partial differential equations for α, β, and γ: $k\partial\beta/\partial k = l\partial\alpha/\partial l$, $l\partial\gamma/\partial l = e\partial\beta/\partial e$, $k\partial\gamma/\partial k = e\partial\alpha/\partial e$. Due to the constant returns to scale, Eq. (4), one of the three elasticities can be eliminated. If one eliminates β, the resulting set of differential equations reads:[3]

$$k\frac{\partial\alpha}{\partial k} + l\frac{\partial\alpha}{\partial l} + e\frac{\partial\alpha}{\partial e} = 0, \tag{5}$$

$$k\frac{\partial\gamma}{\partial k} + l\frac{\partial\gamma}{\partial l} + e\frac{\partial\gamma}{\partial e} = 0, \tag{6}$$

$$k\frac{\partial\gamma}{\partial k} = e\frac{\partial\alpha}{\partial e}. \tag{7}$$

The most general solutions of (5) and (6) are $\alpha = f(l/k, e/k)$ and $\gamma = g(l/k, e/k)$, i.e., the elasticities may be arbitrary differentiable functions *f* and *g* of the factor ratios. Additionally, they have to satisfy the coupling equation (7). The boundary conditions which determine the solutions of this system of partial differential equations unequivocally would require the knowledge of α on a surface and γ on a curve in *kle*-space (Smirnow, 1962; Kümmel, 1980). Although it is practically impossible to obtain such knowledge, this insight accorded from the theory of partial differential equations provides the key to incorporating specific technological information into the production function. The idea is to replace the "real" boundary conditions –which would determine the "true" production function– by approximate or asymptotic technological boundary conditions, specify the elasticities of production correspondingly, and then obtain the production function by integration.

Technological boundary conditions of service production

The factor-dependence of the output elasticity of capital can be specified using the law of diminishing returns to capital (Kümmel, 1980): We take into account that capital

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[3] In Kümmel (1980) γ is eliminated, resulting in a corresponding set of equations for α and β.

growth contributes to output growth, as labor and energy, which handle and activate capital, grow correspondingly; without any labor and energy (i.e., at zero capacity utilization) capital cannot be productive. This is reflected by the following asymptotic technological boundary condition on the output elasticity of capital, α:

$$\lim_{l/k \to 0,\, e/k \to 0} \alpha \to 0. \tag{8}$$

The simplest possible ansatz for α as function of factor ratios, as required by (5), and satisfying (8), is

$$\alpha = a_0 \frac{l+e}{k}, \tag{9}$$

where the parameter a_0 measures (half of) the output elasticity of capital in the base year (when $k=l=e=1$) and has to be determined empirically. The additive combination of l/k and e/k in (9) reflects the substitutability of labor-handled by energy-driven capital in the course of automation-increasing technological progress. This way, the ansatz for capital's elasticity of production (eq. (9)) specifies, what Binswanger and Ledergerber (1974, p. 107) expressed qualitatively as follows: "energy substitutes for and complements labor, while enhancing capital's productive capacity".

Whereas in manufacturing a state of *total* automation can be, at least in principle, considered technologically feasible, in service production, by its very nature, the potentials of automation are much more limited. However, it is still possible to substitute routine labor by energy and (increasingly information-processing) capital. In fact, in the medium term most substitution of labor by computer-based information processing is expected in trade, banking, insurance, and public administration (Thome, 1997). Therefore, it is plausible to assume that in service production a state of *maximum* automation is possible. We incorporate the corresponding production possibilities in the production function by employing the law of diminishing returns: It is assumed that the approach towards the state of maximum automation in service production is associated with decreasing returns to energy utilization; in the limiting state the increase of output due to (additional) energy input vanishes. Introducing the demands of energy e_m and capital k_m in the state of maximum automation, this imposes the following asymptotic boundary condition on the elasticity of production of energy, γ:

$$\lim_{e \to e_m,\, k \to k_m} \gamma \to 0. \tag{10}$$

The simplest ansatz for the elasticity of energy as function of the factor ratios, satisfying the boundary condition (10) and the differential equation (7), which couples it to the elasticity of capital, Eq. (9), is:

$$\gamma = a_0 \left(c_m - \frac{e}{k} \right), \tag{11}$$

where we have introduced the technological parameter c_m as a measure of the energy demand of the capital stock, such that $c_m \equiv e_m / k_m$.

Inserting these elasticities of production, (9) and (11), together with $\beta = 1 - \alpha - \gamma$ from (4), into the growth equation (2), while observing $C=0$, and integrating with the integration constant q_0, yields the service production function q_{S1}:

$$q_{S1}(k,l,e) = q_0\, l \left(\frac{e}{l} \right)^{a_0 c_m} \exp\left\{ a_0 \left(2 - \frac{l+e}{k} \right) \right\}. \tag{12}$$

A non-zero C in Eq. (2) makes the technology parameters c_m, a_0, and q_0 time-dependent.

The above procedure of deriving production functions by specifying factor-dependences of output elasticities subject to a set of partial differential equations and asymptotic technological boundary conditions in factor space may be employed to derive further new functional forms. We give one more example.

In order to emphasize the labor-dependence of service production, one may introduce another boundary condition on the output elasticity of capital, in addition to those given above:

$$\lim_{l/k \to 0} \quad \alpha \to 0. \tag{13}$$

The simplest output elasticities, which satisfy both the system of partial differential equations (5)-(7) and the boundary conditions (8), (10), and (13), are

$$\alpha = 2a_0 \frac{l}{k}\left(\frac{e}{k}+1\right), \tag{14}$$

$$\beta = 1-\alpha-\gamma, \tag{15}$$

$$\gamma = a_0\left(c_m^2 \frac{l}{e}-\frac{le}{k^2}\right). \tag{16}$$

Inserting (14)-(16) into Eq. (2), observing (3), and integrating yields the service production function q_{S2} of the form

$$q_{S2}(k,l,e) = q_0\, l \exp\left\{a_0\left(3-2\frac{l}{k}-\frac{le}{k^2}\right)+a_0 c_m^2\left(1-\frac{l}{e}\right)\right\}. \tag{17}$$

Again, a non-zero C in Eq. (2) makes the technology parameters c_m, a_0, and q_0 time-dependent.

It is important to note that the elasticities of production must be non-negative in order to make economically sense. In this way, limits to factor-input substitutability are taken into account. The requirement of a positive γ (in (11) or (16)) implies that one cannot feed more energy into the machines and electric devices of the capital stock than they can receive according to their technical design. The requirement of non-negative α, β, and γ imposes restrictions on the admissible factor quotients in the elasticities of production and the production functions.

3 Numerical example

In order to illustrate the principle working of the derived functional forms, the service production function q_{S1} of Eq. (12) is applied to the evolution of the German sector "market-determined services" 1960-1989. The result of fitting q_{S1} to the empirical time-series of capital, labor, energy, and output is depicted in Fig. 1. The numerical values of the three technology parameters for the periods until 1977 and after 1978 have been determined by non-linear OLS subject to the constraints of non-negative elasticities of production, using the Levenberg-Marquardt method (Press et al., 1992).

For the parameters and their standard errors we find: q_0=0.96 (0.01), a_0=0.36 (0.04), c_m=1.37 (0.28) for the period 1960-1977, and q_0=0.62 (0.14), a_0=0.71 (0.08), c_m=0.84 (0.20) between 1978 and 1989.[4] The increase of the capital efficiency parameter a_0 and the decrease of the energy demand parameter c_m reflect significant innovation during the considered three decades. The reduced c_m results from investments into more energy-efficient technology in response to the oil-price shocks in the 1970s. Although, in our illustrative example, the time-dependence of these parameters is modeled in the simplest possible way, i.e. by a one-year pulse between 1977 and

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[4] Like the Deutsche Bundesbank (Federal Reserve Bank of Germany, 1996) in its macro-econometric multi-country model, we present here the standard econometric quality measures, namely the coefficients of determination, R^2, and Durbin Watson coefficients of autocorrelation, dw. We find R^2=0.989, dw=0.89 for the period until 1977 and R^2=0.871, dw=0.46 after 1978. The positive autocorrelations are due to the necessarily approximate character of the boundary conditions on the elasticities of production, and thus, of the production function. When estimating the German GDP 1974-1995, using a CD function of capital and labor with cost share weighting and exponential time-dependence, the econometricians of the Deutsche Bundesbank (1996, p. 47) obtain R^2=0.97 and dw=0.24.

1978, the growth of service production over three decades, including the recession during the energy crisis 1973-75, is reproduced with only minor residuals.[5] Thus, it seems that the capital and energy-based progress of electronic information processing in the service industries, e.g. in trade, banking, and insurance industries, is modelled satisfactorily by the derived functional form.

Computation of production elasticities by inserting the factor inputs into the corresponding equations using the relevant technological parameters before and after 1977/78 results in the following time-averaged elasticities of production for capital, labor, and energy: $\overline{\alpha}=0.54$, $\overline{\beta}=0.31$, $\overline{\gamma}=0.15$ until 1977, and $\overline{\alpha}=0.53$, $\overline{\beta}=0.26$, $\overline{\gamma}=0.21$ after 1978.

Although we do not want to claim too much from our simple numerical example, the results indicate that the elasticities of production of capital and energy significantly exceed their cost shares, whereas for labor the opposite holds. These discrepancies induce technological progress towards the observed direction of substituting expensive routine labor by energy and (increasingly information processing) capital. As a number of recent studies[6] have shown, a similar disequilibrium prevails in industrial production. Given the observed factor prices, the production systems appear to be operating in boundary –and not interior– cost minima in factor space, where the boundaries, at a given point in time, are established

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[5] Of course, innovations and structural change penetrate the market gradually, which might favor a model with continuous time-dependent parameters. However, there is the unavoidable trade-off between increases in model flexibility (with a necessarily increased number of fit parameters) on the one hand and the associated loss of explanatory power from a statistical perspective on the other. In view of the indicated statistical quality measures, our approach of dividing the considered three decades into two characteristic periods before and after the energy crisis seems to provide a reasonable approximation in modelling innovation and structural change in the considered service industries.

6 Ayres (2001), Ayres and Warr (2001), Beaudreau (1998), Hall et al. (2001), Kümmel et al. (1985, 2000, 2002), Lindenberger (2000), and Lindenberger et al. (2001).

by the state of technology in information processing and automation, and prevent the systems from sliding at once into the absolute cost minimum with considerably lower labor input.

4 Summary and conclusions

This paper proposed production functions designed to model the evolution of service industries. The method of their derivation, which may be utilized to deduce further new functional forms, is based on specifying the output elasticities of labor, capital, and energy according to a set of differential equations, and subject to asymptotic technological boundary conditions in factor space. These boundary conditions allow to incorporate potential technological change and progress of automation, where routine-labor is substituted by energy and (increasingly information processing) capital, in the production function. In view of the diffusion of IT-technology, this is especially important in the traditional service industries like trade, banking, insurance, or public administration. The corresponding production possibilities are explicitly incorporated in the derived functional forms.

The methodology applied enables to introduce parameters with a well-defined physical/technological interpretation in the production function, e.g. the energy demand of the capital stock. Whereas in the conventional production function approach the accounting of technical progress requires the specification of a certain neutrality hypothesis (Hicks, Harrod, Solow, Sato-Beckmann neutral progress or other), functional forms containing technological parameters with a well-defined physical interpretation enable to analyse the actual direction of technological progress and change. Application of the model to the German market-determined services over three decades yields shifts of capital efficiency and energy demand parameters that indicate structural change and the market penetration of increasingly energy efficient technology in response to the energy price-hikes in the 1970s, a finding consistent with the empirically well known massive investments in energy saving technology since the oil-price crises.

The numerical results presented indicate that capital's and energy's output elasticities systematically exceed the respective factor cost shares, whereas labor's elasticity is below its factor share. This mismatch, which was also found in a number of similar studies, reflects the replacement of expensive routine labor by energy-driven and increasingly information processing capital in the course of technological progress in the observed direction of increasing automation, while the Solow residual is mostly resolved.

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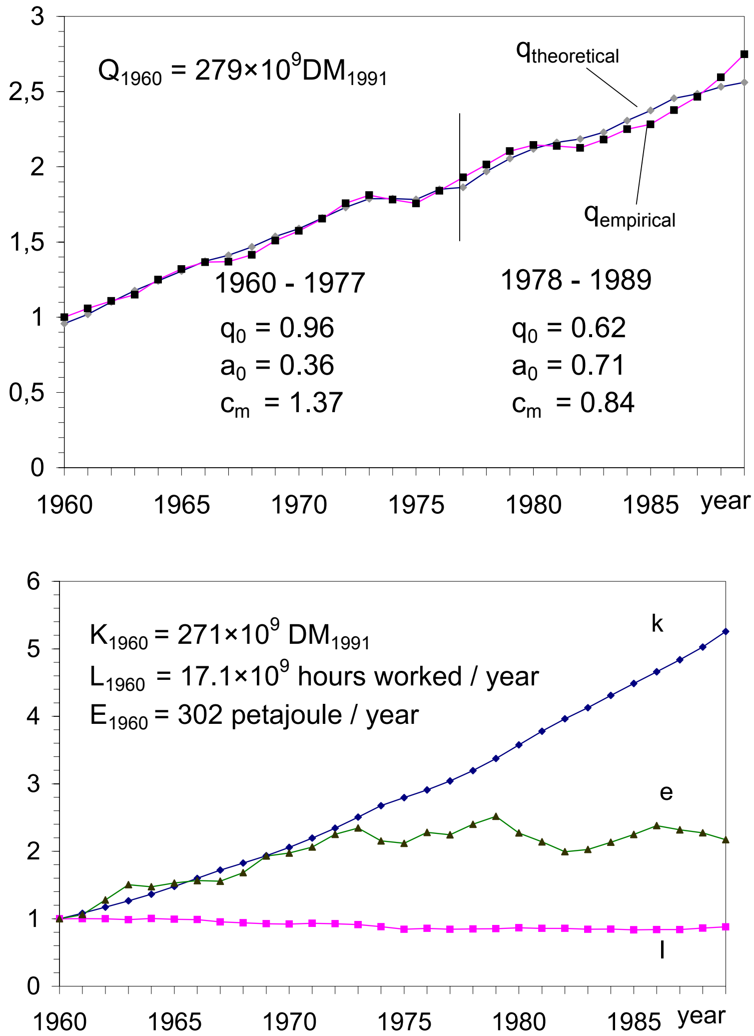
Figure 1: *Top:* Evolution of (normalized) gross value-added $q=Q/Q_{1960}$ of German "market-determined services" 1960-1989: empirical values (dark squares) and theoretical values calculated with q_{S1} (light diamonds). *Bottom:* Evolution of the empirical (normalized) factor inputs of capital $k=K/K_{1960}$, labor $l=L/L_{1960}$, and (final) energy $e=E/E_{1960}$.



Appendix: Data

The considered German 'market-determined services' include banking and insurance, wholesale and retail, the labor-intensive building and construction industry, and 'other services' of the German National Accounts (VGR). Capital and output data are taken from VGR (1994) in constant prices of 1991, labor data from labor statistics in hours worked per year (IAB, 1988, 1996), and energy data from energy balances in petajoule consumed per year (VWEW, 1995). In the German energy balances, 'market-determined services' are included in the category 'small consumers'. The energy consumption of 'market-determined services' was approximated by assuming that its share developed like the value-added share within 'small consumers'. The absolute quantities of capital, labor, energy, and output for market-determined services in the base year 1960 are given in Fig. 1. The numerical data underlying Fig. 1 is given in Tab. 1.

Table 1: Normalized inputs of capital, labor, energy, and output (empirical and theoretical) underlying Fig. 1.

year	k	l	e	q_emp	q_theo
1960	1,000	1,000	1,000	1,000	0,957
1961	1,082	1,001	1,061	1,058	1,020
1962	1,171	0,999	1,279	1,108	1,102
1963	1,265	0,985	1,505	1,149	1,176
1964	1,364	1,004	1,475	1,250	1,241
1965	1,478	0,992	1,530	1,320	1,307
1966	1,599	0,988	1,566	1,366	1,372
1967	1,720	0,953	1,555	1,369	1,411
1968	1,824	0,940	1,682	1,414	1,467
1969	1,934	0,926	1,930	1,509	1,537
1970	2,057	0,921	1,973	1,574	1,589
1971	2,195	0,932	2,063	1,655	1,659
1972	2,342	0,925	2,250	1,758	1,730
1973	2,505	0,912	2,344	1,811	1,788
1974	2,675	0,882	2,153	1,781	1,789
1975	2,795	0,843	2,118	1,756	1,782
1976	2,908	0,857	2,279	1,840	1,850
1977	3,041	0,843	2,244	1,930	1,863
1978	3,195	0,848	2,400	2,015	1,969
1979	3,373	0,853	2,517	2,104	2,055
1980	3,575	0,865	2,270	2,144	2,119
1981	3,778	0,857	2,140	2,138	2,162
1982	3,963	0,856	1,994	2,125	2,184
1983	4,127	0,843	2,027	2,180	2,229
1984	4,308	0,846	2,133	2,250	2,307
1985	4,486	0,834	2,248	2,282	2,374
1986	4,659	0,838	2,379	2,376	2,455
1987	4,837	0,840	2,318	2,465	2,485
1988	5,026	0,861	2,273	2,595	2,532
1989	5,256	0,878	2,170	2,748	2,561