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Cointegration of Output, Capital, Labor, and Energy

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Abstract: Cointegration analysis is applied to the linear combinations of the time series of (the logarithms of) output, capital, labor, and energy for Germany, Japan, and the USA since 1960. The computed cointegration vectors represent the output elasticities of the aggregate energy-dependent Cobb-Douglas function. The output elasticities give the economic weights of the production factors capital, labor, and energy. We find that they are for labor much smaller and for energy much larger than the cost shares of these factors. In standard economic theory output elasticities equal cost shares. Our heterodox findings support results obtained with LINEX production functions.

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1 Introduction

Standard economic theory assumes that the markets of the production factors capital, labor, and energy operate in an equilibrium state, where the cost share of each production factor is equal to its output elasticity (defined in eq. (3)), which reflects the productive power of the respective factor. In this equilibrium producers supposedly can maximize profit without any technological constraints on factor combinations.

Recently, however, Kümmel et al. [1] showed that in the presence of technological constraints on factor combinations, optimization leads to a different equilibrium state. In this state output elasticities must be equal to a *modification* of the usual factor cost shares, where shadow prices due to the constraints add to factor prices.

The question whether or not output elasticities and factor cost shares must be equal (*'equality assumption'*) is crucial for the understanding of economic growth. According to standard theory, the role of energy as a production factor is marginal (see, e.g., [2]), because energy only accounts for five per cent of the total factor costs in industrialized countries, while labor accounts for 70–75%, and capital for 20–25%.

However, economic models based on the *equality assumption* have the problem of the Solow residual. The Solow residual accounts for that part of output growth that cannot be explained by the input growth rates weighted by the factor cost shares. It amounts to more than 50 percent of total growth in many cases. Standard neoclassical economics attributes this difference formally to technological progress. This, however, "has lead to a criticism of the neoclassical model: it is a theory of growth that leaves the main factor in economic growth unexplained" [3]. Furthermore, these models cannot explain the economic recessions 1973-75 and 1979-81, known as the energy crises due to the first and the second oil price shock. On the other hand, if the *equality assumption* is dismissed, LINEX production functions,[2] describe economic growth in Germany, Japan, and the USA without Solow residual, and the energy crises of the 1970s are reproduced well [4, 5, 6, 7, 8, 1]. Their time-averaged output elasticities are for labor much smaller and for energy much larger than the factor cost shares.

In order to substantiate the understanding of economic growth as a process subject to technological constraints that originate from limits to capacity utilization and automation, it is desirable to check the output elasticities by a method that per se is independent from the concept of the aggregate production function. This method is cointegration analysis.

Simply speaking, cointegration analysis checks, whether a linear combination of a number of non-stationary time series is a stationary time series itself. If this is the case, the time series variables are said to be cointegrated, meaning that they are statistically significantly connected. In other words, there is no accidental correlation between the variables, as it was the case, when the number of babies in Sweden decreased like the number of storks in that country.

Conceptually, the present work is similar to Di Matteo et al. [9], and Shao et al. [10], in the sense that we try to identify economic relations by looking at the joint statistical properties of economic data.

We proceed in Section 2 with a short and not exhaustive literature review on cointegration analysis of output and production factors. In Section 3, we present some basic notations of growth theory necessary for the the interpretation of our results. Section 4 tests for unit roots in the time series of the logarithms of output, capital, labor, and energy for Germany, Japan, and the USA. In Section 5 we test for cointegration within a sub-space of economically meaningful cointegration vectors, which are identical to Cobb-Douglas output elasticities. Summary and discussion in Section 6 conclude the paper.

[2] The simplest LINEX function for industrial systems is $q = q_0 \; e \exp\left[a\left(2 - \frac{l+e}{k}\right) + ac\left(\frac{l}{e} - 1\right)\right]$, where k, l, e, and q are capital, labor, energy, and output, normalized to their values in a base year; a, and c are technology parameters, representing essentially efficiency and energy demand of the capital stock.

2 Literature Review

Yu and Jin (1992) [11] were the first to perform bivariate cointegration tests of energy and output, followed by Masih and Masih [12]. Their results are inconclusive: while the first do not find that energy consumption and an index of industrial production in the USA cointegrate, the latter do find cointegration between energy and gross domestic product (GDP) in India, Pakistan, and Indonesia, but no cointegration in Malaysia, Singapore, or the Philippines. However, finding no bivariate cointegration does not imply that there cannot exist multivariate cointegration between output, energy, and other variables. Indeed, from the point of view of production theory, one would expect multivariate cointegration of output and *all* relevant production factors rather than bivariate cointegration of output and energy only.

More recently, bivariate cointegration between energy consumption and GDP is found by Soytas and Sari [13] for the G-7 countries and leading emerging markets and by Lee and Chang [14] for Taiwan. These authors also obtain different directions of causality[3] between GDP and energy consumption and conclude that "energy conservation may harm economic growth", especially if causality runs from energy consumption to GDP. This conclusion is problematical. "Energy conservation" means innovations and efficiency improvements that observe the energy-savings potentials indicated by thermodynamics and economics and reduce the amount of primary energy required for a given quantity of energy services. In fact, because of energy conservation economic growth continued after the relatively short recessions of the first and second oil-price shocks despite of a significantly reduced overall growth of energy inputs. Of course, once the thermodynamic limits to energy conservation will have been reached, further reduction of energy input will harm economic growth, indeed. Thus, growth theory that incorporates thermodynamics and cointegration analysis should complement each other in working out the true economic role of energy. In this sense, the present paper tries to complement the studies of Kümmel et al. [1, 6] and Lindenberger [7].

Stern [15] performs multivariate cointegration tests of output, capital, labor, and energy in the USA and concludes that "cointegration does occur and that energy input cannot be excluded from the cointegration space"; see also Cleveland et al. [16]. Ghali and El-Sakka [17] come to similar results for Canada. On the other hand, when restricted to a Cobb-Douglas production function without a time trend and under the condition that the output elasticities of capital and labor (but not energy) have to sum up to unity, Stern [15] does not find cointegration anymore.

The 1929 Great Crash and the 1973 Oil-Price Shock are the topics of unit-root tests for more than ten economic time series like GNP, industrial production, employment and wages by Perron [18] and Zivot and Andrews [19]. Energy, however, was not considered. While Perron concludes that most variables are stationary around a deterministic trend function with a change in the intercept in 1929 and a change in the slope after 1973, Zivot and Andrews do not treat the great crash and the oil price shock as exogeneous events. They rather treat break points as endogeneous and find less evidence against the unit-root hypothesis for many of the data series, on the one hand, and stronger evidence against it for some like industrial production and GNP, on the other hand.[4]

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[3] Lee and Chang interpret weak exogeneity in a cointegrated system as causality.

[4] If one wants to go beyond our aim of checking the magnitude of the output elasticities, one should subject energy time series to the corresponding tests, too, and then test for cointegration between output and inputs with a method that allows for structural breaks in the intecept and trend of the cointegrated regression. Such a method is presented by Westerlund [20]. However, before applying it to our time series between the early 1960s and 1990s, one would have to examine the small-sample properties for sample sizes of $T \leq 37$, while [20] only conducts Monte Carlo experiments for $T = 100$ and 200. This is left for future studies.

3 Basic Growth Dynamics

Assume that economic output q is a twice differentiable function of the production factors capital k, labor l, and energy e, which in turn depend on time t:

$$q = q(k(t), l(t), e(t)). \tag{1}$$

Output and production factors are normalized with respect to their values in a base year. The total time derivative of q, multiplied by $\mathrm{d}t/q$, yields the 'growth equation':

$$\frac{\mathrm{d}q}{q} = \alpha\frac{\mathrm{d}k}{k} + \beta\frac{\mathrm{d}l}{l} + \gamma\frac{\mathrm{d}e}{e} \tag{2}$$

with the *output elasticities*

$$\alpha \equiv \frac{k}{q}\frac{\partial q}{\partial k}, \quad \beta \equiv \frac{l}{q}\frac{\partial q}{\partial l}, \quad \gamma \equiv \frac{e}{q}\frac{\partial q}{\partial e}, \tag{3}$$

which give the weights by which the marginal ralative changes of the production factors contribute to the marginal relative change of output. In this sense they measure the productive powers of capital, labor, and energy.[5]

Factor-independent output elasticities and linear homogeneity of the production function, which implies $\gamma = 1 - \alpha - \beta$, lead to the simplest integral of eq. (2):[6]

$$q_{CDE} = q_0 k^{\alpha} l^{\beta} e^{1-\alpha-\beta}, \tag{4}$$

the energy dependent Cobb-Douglas function with constant returns to scale.

For our purpose it is convenient to consider the time series of the logarithms of output and production factors,

$$\tilde{q}_t \equiv \ln q(t),\ \tilde{k}_t \equiv \ln k(t),\ \tilde{l}_t \equiv \ln l(t),\ \tilde{e}_t \equiv \ln e(t). \tag{5}$$

With these definitions the logarithm of eq. (4) becomes:

$$\tilde{q}_{CDE} = \tilde{q}_0 + \alpha\tilde{k}_t + \beta\tilde{l}_t + (1-\alpha-\beta)\tilde{e}_t. \tag{6}$$

In order to be meaningful economically, the constant output elasticities of the Cobb-Douglas function must be non-negative:

$$\alpha \geq 0, \quad \beta \geq 0, \quad \gamma = 1 - \alpha - \beta \geq 0. \tag{7}$$

This constraint is based on the assumption that the sum of all entrepreneural decisions will always lead to a state of the economic system where the increase of an input never results in a decrease of output.

Equation (6) is the starting point for cointegration analysis of the economic systems whose growth has been previously investigated by Kümmel et al. [6, 1].[7] They are: Germany's total economy (FRG TE) and industrial sector "Warenproduzierendes Gewerbe" (FRG I),[8] Japan's "Industries" (Japan I), the sector "Industries" of the USA (USA I), and

[5] If the production function depends explicitly on time, an additional term, $\frac{dt}{q}\frac{\partial q}{\partial t}$, occurs in eq. (2).

[6] In general, $\alpha = \alpha(k, l, e)$, $\beta = \beta(k, l, e)$, and $\gamma = \gamma(k, l, e)$ are functions of the production factors. Linear homogeneity of the production function in k, l, and e means that at any fixed time t an increase of *all* inputs by the same factor λ must increase output by λ. This leads to $\gamma = 1-\alpha-\beta$ (see [1]). Economists refer to this condition as "constant returns to scale". For a more complete treatment of growth theory, see [6, 1].

[7] We use the data of these authors.

[8] We consider the Federal Republic of Germany (FRG) only until 1989, because the merger of the east German planned economy with the west German market economy at reunification in 1990 and the associated structural change cannot be described by the energy-dependent Cobb-Douglas function with its constant output elasticities. Things are different with the LINEX function [1].

the total economy of the USA (USA TE). Our procedure is similar to that of Schröer and Stahlecker [21], who asked, whether the aggregate Cobb-Douglas production function represents a cointegrating relation for the time series of output, capital, and labor, and the work of Stern [15], and Cleveland, Kaufmann, and Stern [16], who also include energy.

4 Unit Roots

Before we proceed with cointegration analysis we have to check, whether the considered time series exhibit unit roots, that is, if they are non-stationary. We perform standard augmented Dickey-Fuller (DF) tests [22, 23] which model the time dependence of the variable u_t by the stochastic process

$$u_t = \rho u_{t-1} + \sum_{i=1}^{p} \theta_i \Delta u_{t-i} + \varepsilon_t. \tag{8}$$

Here, and only within the present Section, u_t stands for one of the variables $\tilde{q}_t$, $\tilde{k}_t$, $\tilde{l}_t$, and $\tilde{e}_t$. $\Delta u_{t-i} \equiv u_{t-i} - u_{t-i-1}$ is the so-called first difference of u_{t-i}; and ε_t is supposed to be independent and identically distributed, following a zero-mean, constant-variance normal distribution, in short: $\varepsilon_t \sim$ i.i.d.N$(0, \sigma^2)$.

The regression coefficients ρ and θ_i are estimated by the method of ordinary least squares (OLS). The OLS estimate $\hat{\rho}$ and its standard error $\hat{\sigma}_\rho$ define the quantity

$$\tau = \frac{\hat{\rho} - 1}{\hat{\sigma}_\rho}. \tag{9}$$

The distribution of τ for time series with a unit root, i.e. $\rho = 1$, is known from Monte Carlo simulations of Dickey and Fuller [22] and MacKinnon [24].

The Dickey-Fuller procedure is also suitable to test for stationarity around a deterministic trend. For that purpose, a constant c, and optionally a linear time trend gt, are added to the right hand side of eq. (8). The quantity τ defined in eq. (9) for these test specifications is re-named as τ_μ (if c is included), and τ_τ (if c and gt are included). Dickey and Fuller [23] also calculate critical values for the tests of significance of the terms c and gt added to the r.h.s. of eq. (8). These tests are denoted $\tau_{\alpha\mu}$-test, and $\tau_{\beta\tau}$-test, respectively, because Dickey and Fuller used the greek letters α and β, which we replaced by c and g to avoid confusion with the output elasticities.

In order to save space in Table 1, we only present the results for a lag length $p = 1$ in the augmented Dickey-Fuller (ADF) test. This test specification is preferable to the (not augmented) DF test with $p = 0$, because autocorrelations in the residuals of the regressions are reduced. However, we do not apply information criteria to find the optimal lag length in order not to shorten our time series of $T = 28$ to 37 observations too much.

The results in Table 1, where one, two, or three asterisks indicate the 90%, 95%, or 99% level of confidence for stationarity, show that all of the considered time series for Germany's total economy and industrial sector exhibit unit roots, with the exception of $\tilde{k}_t$, which is trend-stationary for FRG TE. In the Japanese sector Industries $\tilde{q}$, $\tilde{k}$, and $\tilde{e}$ may be trend-stationary. For USA I the unit root hypothesis is only rejected at 90% confidence level for the variable $\tilde{q}$, if a time trend is included. There is also some likelihood for trend stationarity in variables of USA TE. These results fully justify the application of cointegration analysis in the cases of Germany's and the United States' industies sectors, and they justify the application of cointegration analysis for the other systems, as long as no time trends are included. Consequences of possible trend-stationarity of some of the variables for cointegration including time trends are discussed in Section 6.

System		τ_τ	τ_μ	τ	$\tau_{\beta\tau}$	$\tau_{\alpha\mu}$
FRG TE	$\tilde{q}$	-1.85 $^{-}$	-1.88 $^{-}$	1.56 $^{-}$	1.44 $^{-}$	3.16 **
1960-89	$\tilde{k}$	-4.57 ***	-8.49 ***	0.12 $^{-}$	1.72 $^{-}$	8.96 ***
	$\tilde{l}$	-1.69 $^{-}$	-1.58 $^{-}$	0.87 $^{-}$	-1.35 $^{-}$	-2.46 $^{-}$
	$\tilde{e}$	-1.72 $^{-}$	-1.95 $^{-}$	0.81 $^{-}$	1.02 $^{-}$	2.66 **
FRG I	$\tilde{q}$	-1.83 $^{-}$	-2.28 $^{-}$	1.21 $^{-}$	1.00 $^{-}$	3.06 **
1960-89	$\tilde{k}$	-1.93 $^{-}$	-1.79 $^{-}$	0.30 $^{-}$	0.83 $^{-}$	1.84 $^{-}$
	$\tilde{l}$	-2.89 $^{-}$	-0.84 $^{-}$	0.95 $^{-}$	-2.75 $^{-}$	-1.94 $^{-}$
	$\tilde{e}$	-1.63 $^{-}$	-2.06 $^{-}$	0.41 $^{-}$	0.66 $^{-}$	2.55 *
Japan I	$\tilde{q}$	-3.42 *	-2.28 $^{-}$	0.48 $^{-}$	2.92 **	2.69 **
1965-92	$\tilde{k}$	-4.06 **	-2.15 $^{-}$	-0.04 $^{-}$	3.67 **	2.51 *
	$\tilde{l}$	-1.65 $^{-}$	0.57 $^{-}$	1.57 $^{-}$	2.16 $^{-}$	0.98 $^{-}$
	$\tilde{e}$	-3.63 **	-2.95 *	0.58 $^{-}$	2.16 $^{-}$	3.23 **
USA TE	$\tilde{q}$	-3.33 *	-2.06 $^{-}$	1.55 $^{-}$	3.05 **	3.64 ***
1960-96	$\tilde{k}$	-0.77 $^{-}$	-3.00 **	1.22 $^{-}$	0.53 $^{-}$	6.15 ***
	$\tilde{l}$	-4.88 ***	-046 $^{-}$	2.10 $^{-}$	4.85 ***	2.28 *
	$\tilde{e}$	-2.63 $^{-}$	-2.63 *	1.02 $^{-}$	1.03 $^{-}$	1.53 $^{-}$
USA I	$\tilde{q}$	-3.35 *	-1.44 $^{-}$	1.79 $^{-}$	3.15 **	2.93 **
1960-93	$\tilde{k}$	-0.87 $^{-}$	-2.41 $^{-}$	-0.19 $^{-}$	0.72 $^{-}$	3.35 **
	$\tilde{l}$	-2.77 $^{-}$	-0.66 $^{-}$	1.72 $^{-}$	2.71 *	2.94 **
	$\tilde{e}$	-2.36 $^{-}$	-2.96 $^{-}$	0.63 $^{-}$	0.75 $^{-}$	3.47 ***

Table 1: Augmented ($p = 1$) Dickey-Fuller unit-root tests of the time series of the logarithms of output and production factors. 90% $^{(*)}$, 95% $^{(**)}$, and 99% $^{(***)}$ critical values for τ_τ, τ_μ, and τ statistics are taken from [24] for the respective lengths $(T-p)$ of the time series. For $(T-p) = 30$ they are: τ_τ: -3.22, -3.57, -4.29; τ_μ: -2.62, -2.96, -3.67; τ: -1.62, -1.95, -2.64; they differ little for somewhat smaller or larger $(T-p)$. 90% $^{(*)}$, 95% $^{(**)}$, and 99% $^{(***)}$ critical values for the $\tau_{\beta\tau}$ and $\tau_{\alpha\mu}$ statistics for $(T-p) = 25$ from [23]: $\tau_{\beta\tau}$: 2.39, 2.85, 3.74; $\tau_{\alpha\mu}$: 2.20, 2.61, 3.41; again, there is little sensitivity to small changes of $(T-p)$.

5 Cointegration

5.1 Constraints of the Cointegration Space

In the following analysis, we look into the question for which values of the constants α, β, and γ the linear combination

$$u_t = \tilde{q}_t - \alpha\tilde{k}_t - \beta\tilde{l}_t - \gamma\tilde{e}_t, \tag{10}$$

is stationary and likely to represent a cointegrating relation. In Subsection 5.2, a constant, and in Subsection 5.3, a constant and a linear trend will be introduced into eq. (10), which we omit for convenience here. The r.h.s. of eq. (10) can be rewritten as the scalar product of the cointegration vector $(1, -\alpha, -\beta, -\gamma)'$, and the vector of variables $(\tilde{q}_t, \tilde{k}_t, \tilde{l}_t, \tilde{e}_t)'$.

The statement that eq. (10) represents a cointegrating relation is another way of saying that the time series of $\tilde{q}_t$ can be expressed by the linear combination $\alpha\tilde{k}_t + \beta\tilde{l}_t + \gamma\tilde{e}_t$. This allows the conclusion that, if the hypothesis of cointegration can be accepted and one identifies the cointegration vector components α, β, and γ with the output elasticities of capital, labor, and energy, then the energy-dependent Cobb-Douglas function (4) is a valid description of output as a function of the production factors.

However, the interpretation of a cointegrating relation as a Cobb-Douglas production function does only make sense if the cointegration vector is economically meaningful, and if the residuals u_t fulfill some basic requirements. Thus, the available space for the components of the cointegration vector is severely restricted by the following constraints: a) the requirement of constant returns to scale, $\gamma = 1 - \alpha - \beta$, which reduces the number of independent components of the cointegration vector by one, b) the requirement that the output elasticities α, β, and $\gamma = 1 - \alpha - \beta$ have to be non-negative; and the conditions that c) the Durbin-Watson coefficient (d_W) of autocorrelation deviates as little as possible from its best value 2, and d) the Residual Sum of Squares (RSS) is minimum.

As said before, constraint a) implies that at any fixed time t an increase of *all* inputs by the same factor λ must increase output by λ, and constraint b) is based on the assumption that the sum of all entrepreneural decisions will always lead to a state of the economic system where the increase of an input never results in a decrease of output. Conditions c) and d) reduce the likelihood that one has forgotten an important factor of production, and that the empirical data is not reproduced adequately by the model. The LINEX functions in [1] with factor- and time-dependent output elasticities have d_W between 1.64 and 1.9 for the systems considered in the present paper. Of course, one cannot expect such d_W for Cobb-Douglas functions with their constant output elasticities. But the center of the d_W-contours in Fig. 1, in combination with the center of the RSS-contours, fixes nearly uniquely the possible α, β, and γ from a statistical point of view in the plane of non-negative output elasticities with constant returns to scale.

In the remaining parts of this section, we perform cointegration analysis following Engle and Granger [25], because their method can be easily applied under the constraint of non-negative output elasticities. Furthermore, it allows a graphical representation of the results for the entire sub-space and thus a comparison of the regions with highest probability of cointegration with the regions of the best values of d_W and RSS (Fig. 1).

Further evidence for the hypothesis that a single linear cointegrating relation exists between the variables $\tilde{q}$, $\tilde{k}$, $\tilde{l}$, and $\tilde{e}$ is provided in Appendix A. There, we apply the Johansen procedure to our data, which is the appropriate method for the multivariate case if the number of cointegrating relations is unknown.

In Subsection 5.2 we analyse FRG TE, FRG I, Japan I, and USA I. Here we do not allow for time trends in the cointegration vector. This way we can see, whether the variables represented by the logarithms of output, capital, labor, and energy are linearly cointegrated, even if explanations for economic growth other than changes of the factor inputs are excluded.[9]

The case USA TE is treated in Subsection 5.3. There, we have to include a time trend, because it is the only production system considered in this paper with an overall growth of output that is (slightly) greater than the growth of any of the three production factors.[10]

5.2 Cointegration Analysis for Germany, Japan, and USA Industries

We start from eq. (10), with an additional constant $\tilde{q_0}$ and γ replaced with $(1-\alpha-\beta)$:[11]

$$u_t = \tilde{q}_t - \tilde{q_0} - \alpha\tilde{k_t} - \beta\tilde{l_t} - (1-\alpha-\beta)\tilde{e}_t, \tag{11}$$

Following Engle and Granger [25], we perform (augmented) Dickey-Fuller tests as described in Section 4, with the only difference that u_t in eq. (8) now stands for the quantity defined in eq. (11), instead of a single macroeconomic variable.[12] Engle and Granger [25] point out that, if the constants $\tilde{q_0}$, α, and β in eq. (11) have to be estimated, the distribution of τ for the hypothesis of stationarity is not identical with the distribution of τ for the hypothesis that the constants form a cointegration vector. They give critical values of the distribution of τ from Monte Carlo simulations for the latter hypothesis.

We use two methods, M_a and M_b, of investigating the values of α and β that make eq. (11) a cointegrating relation. M_a is standard and straightforward, M_b is a new graphical generalization of M_a.

Method M_a first estimates eq. (11) by the OLS method and produces a time series of residuals u_t. Then, (augmented) DF tests of stationarity are applied to the residuals u_t. The estimated coefficients $\alpha, \beta, \gamma = 1-\alpha-\beta$, and $q_0 = \exp(\tilde{q_0})$ are shown in Table 2. Also shown are the coefficient of determination R^2, the Durbin-Watson coefficient of autocorrelation d_W, and the values of τ for $p = 0, 1$, i.e the results of the DF tests. According to the information criteria of Akaike [26] and Schwarz, the best choice for all systems is $p = 1$; only for Japanese Industries $p = 0$ is preferable according to the Schwarz criterion, but not the Akaike criterion. One, two or three asterisks indicate the 90%, 95% or 99% level of confidence that the considered time series of u_t is stationary according to the distributions given by MacKinnon [24]; one, two or three daggers indicate the 90%, 95% or 99% level of confidence that eq. (11) with the estimated constants $\tilde{q_0}$, α, and β is a cointegration relation according to the distributions given by Engle and Granger [25].

The results in Table 2 show that the time series of the logarithms of output, capital, labor, and energy can be accepted to be cointegrated, with the estimated α, β, $\gamma = 1-\alpha-\beta$ as components of cointegration vectors, for the total economy and the industrial sector of Germany, and for US Industries. Only for Japanese Industries the 90% confidence level for cointegration is not reached, but the time series of the residuals u_t is still stationary on 95% confidence level, so the evidence against cointegration is not very strong.

Method M_b plots confidence intervals for stationarity and contours of both the Residual Sum of Squares $RSS = \sum_{t=1}^{T} u_t^2$ and the Durbin-Watson coefficients d_W in that part of

[9]We do not expect that a time trend in a Cobb-Douglas function that represents a cointegrating relation will have an overall impact on growth that is larger than the small impact of the explicit time dependence of the LINEX function computed by Kümmel et al. [6, 1]. Furthermore q_{CDE} tends to underestimate energy's output elasticity. Thus, we should be on the safe side with this procedure of checking deviations from the cost shares.

[10]This can be seen from Fig. 4 of [6]. Therefore, without the inclusion of an additional time trend, the conditions of non-negative output elasticities and constant returns to scale would necessarily lead to large residuals and no-cointegration. Given the possibility of trend stationarity in some of the variables the results of the analysis for USA TE have their meaning mostly in indicating for which output elasticities cointegration is most likely.

[11]The constant $\tilde{q_0}$ is important, because without it a change of the (arbitrary) base year to which the economic variables are normalized would lead to changes of the residuals u_t, and thus to arbitrary changes of the results of cointegration analysis.

[12]Deterministic trends in u_t have been excluded, using additional $\tau_{\alpha\mu}$- and $\tau_{\beta\tau}$-tests according to [23].

	FRG TE 1960-89	FRG I 1960-89	Japan I 1965-92	USA I 1960-78	USA I 1960-93
α	0.522	0.442	0.433	0.239	0.670
	± 0.029	± 0.050	± 0.014	± 0.142	± 0.031
β	0.094	0.041	0.217	0.098	0.131
	± 0.026	± 0.040	± 0.024	± 0.057	± 0.058
γ	0.384	0.517	0.350	0.663	0.199
	± 0.039	± 0.064	± 0.027	± 0.153	± 0.066
q_0	0.998	1.013	1.039	1.037	1.049
	± 0.005	± 0.010	± 0.014	± 0.014	± 0.014
R^2	0.996	0.992	0.995	0.980	0.992
d_W	0.848	1.165	0.562	0.570	0.595
$\tau_{p=0}$	-2.07 (**,–)	-3.29 (***,††)	-2.01 (**,–)	–	-2.72 (***,†)
$\tau_{p=1}$	-3.04 (***,†)	-3.76 (***,†††)	-2.22 (**,–)	–	-3.40 (***,††)

Table 2: The constants in the linear combination (11), where $\gamma \equiv 1 - \alpha - \beta$, statistical quality measures for the corresponding energy-dependent Cobb-Douglas function, and τ values of the DF ($p = 0$) and ADF ($p = 1$) unit-root tests for u_t. Critical τ values for the Dickey-Fuller stationarity test: * 90% -1.62, ** 95% -1.95, *** 99% -2.64. Critical τ values for the Engle-Granger cointegration test at $p = 0$: † 90% -2.71, †† 95% -3.05, ††† 99% -3.90; at $p = 1$: † 90% -2.91, †† 95% -3.17, ††† 99% -3.73.

the α-β plane, where the constraints of non-negative output elasticities hold. They are shown in Fig. 1.

The overlap of the regions of highest stationarity probability with those within the contours of smallest RSS and highest d_W determines the set of α, β, and $\gamma = 1 - \alpha - \beta$ values for which the linear combination (11) is likely to represent a cointegrating relation and the corresponding energy-dependent Cobb-Douglas production function fits best the empirical data of output. The confidence intervals are obtained in the following way: For each point of a lattice in the plane of non-negative α and β that lies below the $\gamma = 0$ diagonal the parameter $\tilde{q}_0$ in eq. (11) is estimated by the OLS method; then the τ values according to the ADF test with the regression (8) for u_t are calculated for appropriate p values and compared with the critical τ values of [24] for 90%, 95% and 99% stationarity probability.

The graphical method confirms the results of Table 2. The values of α, β, and $\gamma = 1-\alpha-\beta$ that, according to Figure 1, lead to stationary residuals u_t in eq. (11), thus fulfilling the basic requirement for acceptance as possible components of cointegration vectors and, in addition, yield the smallest RSS and largest d_W numbers are close to those listed in Table 2.

It is interesting to note that by looking at the contours of the τ statistics alone, the possibility of existence of two linearly independent cointegrating relations within the economically meaningful sub-space can not be excluded: if β *or* γ are set to zero (that is, on the vertical or diagonal boundaries of the triangular sub-spaces in Fig. 1), the residuals u_t are still stationary for appropriate values of α. In other words, Cobb-Douglas functions without the factor labor *or* energy would still produce stationary residuals. But the d_W and RSS values on the boundaries of the triangular sub-spaces are so far away from their best values in the centers of the respective contours, that these cointegration vectors can be rejected due to their poor performance in fitting the empirical data. Thus, we can confirm our result in Appendix A, that with high probability exactly one unique cointegrating relation exists, for the economically meaningful sub-space.

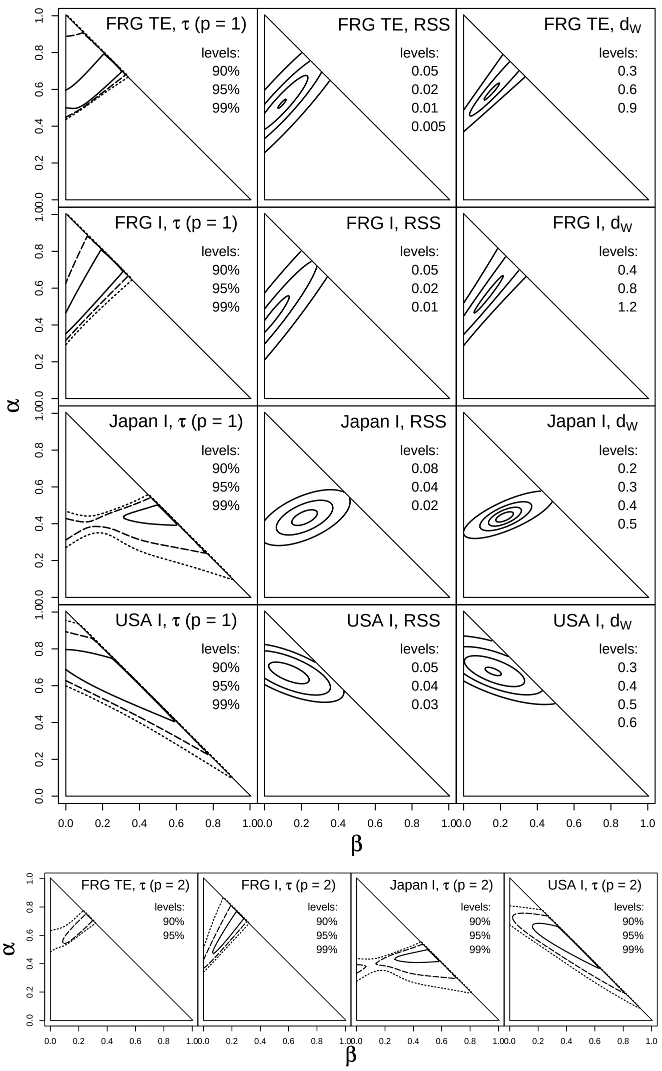

Figure 1: *Upper figure:* Contours in the α-β plane of: *left:* the confidence intervals of stationarity according to the τ statistics in the ADF test with $p = 1$; *middle:* the Residual Sum of Squares RSS; *right:* the Durbin-Watson coefficients d_W. The dotted, dashed, and solid contours of the ADF test represent the 90%, 95%, and 99% confidence levels, respectively. The levels of the RSS and d_W contours are given in the legends. From inside to outside, RSS values increase, and d_W values decrease. The region of the α-β plane, where output elasticities are non-negative, is bounded by a solid line. *Lower figure:* ADF-test τ statistics for $p = 2$.

5.3 Cointegration Analysis for USA, Total Economy

As mentioned above, the system "USA, Total Economy" can not be analysed under the constraints of eq. (7) without a time trend because the output q grows faster than any of the production factors. An exponential time trend changes the energy-dependent Cobb-Douglas function (4) to

$$q^*_{CDE} = q_0 k_t^\alpha l_t^\beta e_t^{1-\alpha-\beta} \exp\left(\theta \, \frac{t - t_0}{t_e - t_0}\right) . \tag{12}$$

Here t_0 is the time of the first observation (in our case, 1960), t_e is the time of the last observation (1996), and θ represents the strength of the time trend.

We estimate the parameters of

$$\tilde{q}_t = \tilde{q}_0 + \alpha \tilde{k}_t + \beta \tilde{l}_t + (1 - \alpha - \beta)\tilde{e}_t + \theta \frac{t - t_0}{t_e - t_0} + u_t, \tag{13}$$

with the residual u_t. In so doing, we determine θ in two ways. A) by estimating it together with q_0, α, and β. B) by choosing it in such a way that the time average $\bar{\delta}$ of

$$\delta \equiv \frac{t - t_0}{q} \frac{\partial q}{\partial t} \tag{14}$$

is the same for the LINEX function of Kümmel et al. [1] and for q^*_{CDE}, where the LINEX $\bar{\delta}$ is 0.1. If we insert q^*_{CDE} into eq. (14) and calculate the time average between t_0 and t_e we obtain $\bar{\delta} = \theta/2$. Thus the $\bar{\delta}$ of LINEX and that of q^*_{CDE} are equal for $\theta = 0.2$. The results of cointegration analysis for cases A and B are shown in Table 3.

System: USA TE, 1960-1996		
Case:	A	B
α	0.228 ± 0.134	0.561 ± 0.024
β	0.472 ± 0.117	0.195 ± 0.045
γ	0.301 ± 0.177	0.244 ± 0.051
q_0	1.032 ± 0.008	1.036 ± 0.009
θ	0.351 ± 0.059	0.2
$R2$	0.999	0.997
d_W	0.432	0.584
$\tau_{p=0}$	-2.16 $^{(-,-)}$	-2.82 $^{(*,\dagger)}$
$\tau_{p=1}$	-2.90 $^{(*,-)}$	-3.88 $^{(***,\dagger\dagger\dagger)}$

Table 3: Estimated parameters of Eq. (13) for "USA Total Economy", 1960-96 (where $\gamma = 1 - \alpha - \beta$), and τ values of the Dickey-Fuller ($p = 0$) and augmented Dickey-Fuller ($p = 1$) tests for the residual u_t. Case A: θ is estimated. Case B: θ is determined by the LINEX $\bar{\delta}$. Critical τ_μ values for $(T - p) = 35$ [24]: * 90% -2.61, ** 95% -2.95, *** 99% -3.62. Critical values of the Engle-Granger cointegration test see Table 2.

In case B, when θ is fixed by the $\bar{\delta}$ criterion, the residuals are stationary according to the Dickey-Fuller stationarity test, and cointegration is likely according to the Engle-Granger test.[13] For $p = 1$, which is the preferable test specification according to the Akaike information criterion in both cases, the null hypothesis of no cointegration is rejected on 99% confidence level. In case A, it can not be rejected on 90% confidence level; the d_W is also worse than in case B.

Thus, if the average contributions to economic growth from the explicit time dependence of the LINEX function and of the Cobb-Douglas function q^*_{CDE} are the same, the logarithms of output, capital, labor, and energy of the total economy of the USA can be accepted to be linearly cointegrated under the constraints of non-negative elasticities. In this case, the LINEX elasticities α, β, and γ of [1, 6] and the elasticities in Table 3 agree within the error margins, too.

[13] We use the τ_μ critical values (see Section 4) in Table 3 because a linear time trend is included in eq. (13), which necessarily leads to a vanishing drift in the estimate of u_t of eq. (8).

6 Summary and Discussion

The hypothesis of cointegration of the logarithms of output, capital, labor, and energy can not be rejected for the examined economic systems of Germany, Japan, and the USA. In the cases of the Japanese Industries sector and USA Total Economy, evidence for cointegration is somewhat weaker because in the first case, the 90% confidence level for cointegration is not reached, although the residuals are still stationary (see Table 2), and in the second case, possible trend-stationarity of some of the macroeconomic variables might weaken the significance of the results of cointegration analysis including a linear trend. The cointegration vectors represent the output elasticities of the energy-dependent Cobb-Douglas function. They are for labor much smaller and for energy much larger than the cost shares of these factors. This confirms the results of growth modelling based on the LINEX production function.

Numerical discrepancies between the LINEX elasticities of [1] and the cointegration vector components are due to the lower sensitivity of the energy-dependent Cobb-Douglas function to the efficiency improvements that occurred after the energy crises. The graphs of the Cobb-Douglas growth curves for Japanese and US Industries do not reproduce the recessions and recoveries in conjunction with the energy crises, but rather average them out, and the Durbin-Watson coefficients deviate more from the best value, 2, than the d_W numbers of LINEX.[14]

The problem, whether one may also do cointegration analysis based on the LINEX function by looking into the stationarity of $u_t = \ln(q/e) - [\ln q_0 + 2a] + a\frac{l+e}{k} - ac(\frac{l}{e} - 1)$, is left for the future.

The physically and economically important result of this cointegration analysis is the confirmation of the main finding of prior heterodox economic studies: energy is a much more powerful factor of production and routine labor a much weaker factor than the cost shares of these factors indicate in standard growth theory. This removes the Solow residual, reproduces the energy crises, and elucidates the forces behind the pressure towards increasing automation and unemployment: Cheap, powerful energy/capital combinations are substituted for productively weak, expensive routine labor. Furthermore, influential analyses of the economic impacts of climate change like the DICE model of Nordhaus assume that “Output is produced by a Cobb-Douglas production function in capital, labor, and energy” [27], where in the standard neoclassical approach energy has just the tiny cost-share weight of about five percent. Giving energy a weight of the order of the output elasticities γ in Tables 2 and 3 may change the conclusions and recommendations of economic climate-change analyses substantially.

[14] On the other hand, the Cobb-Douglas fit to the time series “USA I, 1960-78” – which is too short for calculating τ values in Table 2 – yields an energy output elasticity quite close the average LINEX elasticity for “USA I, 1960-93”, and it does reproduce the first energy crisis quite well, as shown by [4].

A Cointegration Ranks

In this Appendix we perform cointegration analysis with the Johansen procedure [28, 29, 30, 31, 32] in order to answer the question, if the time series of the variables $\tilde{q}$, $\tilde{k}$, $\tilde{l}$, and $\tilde{e}$ are linked by a single cointegrating relation. The Johansen procedure, for the case of possible trends in the levels of the variables, is based on the following Vector Error Correction Model (VECM):

$$\Delta \mathbf{X}_t = \mathbf{c} + \sum_{i=2}^{k} \Gamma_{i-1} \Delta \mathbf{X}_{t-i+1} + (\Pi, \mathbf{g})(\mathbf{X}'_{t-1}, t)' + \varepsilon_t, \qquad (15)$$

where, in our case, $\mathbf{X}_t = (\tilde{q}_t, \tilde{k}_t, \tilde{l}_t, \tilde{e}_t)'$ is a $(N \times 1)$-matrix of the relevant variables ($N = 4$); $\Delta \mathbf{X}_t \equiv \mathbf{X}_t - \mathbf{X}_{t-1}$ is the first difference of this matrix; $\mathbf{c}$ and $\mathbf{g}$ are $(N \times 1)$-matrices of constants, where $\mathbf{g}$ represents deterministic trends in the levels-series; ε_t is a vector of White-Noise terms. The $(N \times N)$-matrices Γ_{i-1} have a similar function as the coefficients θ_i in eq. (8): they take care of possible autocorrelations in the residuals. Finally, the $(N \times N)$-matrix Π is the cointegration matrix.

System	H_0	trace		λ_{max}	
FRG TE	$r \leq 3$	5.49	–	5.49	–
1960-89	$r \leq 2$	20.98	–	15.49	–
	$r \leq 1$	40.96	*	19.98	–
	$r = 0$	99.20	***	58.24	***
FRG I	$r \leq 3$	4.79	–	4.79	–
1960-89	$r \leq 2$	18.13	–	13.34	–
	$r \leq 1$	39.71	*	21.58	–
	$r = 0$	74.08	***	34.37	**
Japan I	$r \leq 3$	10.99	*	10.99	*
1965-92	$r \leq 2$	23.36	*	12.37	–
	$r \leq 1$	42.08	*	18.72	–
	$r = 0$	90.89	***	48.81	***
USA TE	$r \leq 3$	6.30	–	6.30	–
1960-96	$r \leq 2$	20.66	–	14.36	–
	$r \leq 1$	37.20	–	16.54	–
	$r = 0$	61.26	*	24.06	–
USA I	$r \leq 3$	5.28	–	5.28	–
1960-93	$r \leq 2$	17.12	–	11.84	–
	$r \leq 1$	39.31	*	22.18	–
	$r = 0$	82.34	***	43.04	***

Table 4: Trace and λ_{max} statistics of the Johansen test based on eq. (15) with $k = 2$. Asterisks denote rejection of hypothesis H_0 on 90% (*), 95% (**), and 99% (***) level. Critical values are given in Table 5.

	trace			λ_{max}		
H_0	90%	95%	99%	90%	95%	99%
$r \leq 3$	10.49	12.25	16.26	10.49	12.25	16.26
$r \leq 2$	22.76	25.32	30.45	16.85	18.96	23.65
$r \leq 1$	39.06	42.44	48.45	23.11	25.54	30.34
$r = 0$	59.14	62.99	70.05	29.12	31.46	36.65

Table 5: Critical values for Table 4 from [31].

To understand the meaning of eq. (15), it is important to note that all terms with exception of the one containing Π are by definition (trend-)stationary, if the variables in $\mathbf{X}_t$ become stationary after differencing.[15] Thus, eq. (15) can only be valid if $(\Pi, \mathbf{g})(\mathbf{X}'_{t-1}, t)'$ is also stationary, that is, if cointegrating relations exist. The rank r of matrix Π is the number of cointegration vectors, which is the number of linearly independent stationary linear combinations of the variables $\tilde{q}$, $\tilde{k}$, $\tilde{l}$, and $\tilde{e}$.

The Johansen procedure, which is described in detail in [28, 29, 30, 31, 32], provides a test for the rank r of Π. The test statistic "trace" tests the hypothesis H_0, that n or less cointegrating relations exist ($r \leq n$) for $n = 0, 1, 2, 3$ against the alternative of full rank of Π ($r = N$), which means that the process is stationary rather than cointegrated. The maximal eigenvalue statistic "λ_{max}" tests the same hypothesis H_0 against the alternative that $r = n + 1$.

The results of the Johansen test are shown in Table 4. We choose $k = 2$ (only one Γ-term in eq. (15)) for the same reason as we chose $p = 1$ in Section 4: to reduce autocorrelation of the residuals without shortening the time series too much. For USA TE, the hypothesis of no cointegration ($r = 0$) can only be rejected at 90% confidence level for the "trace" statistic, so there is less evidence for cointegration than in the other cases. The case of Japan Industries includes the possibility of trend-stationarity of $\mathbf{X_t}$, as the hypothesis of $r \leq 3$ can not be rejected on 90% confidence level, which coincides with the results in Table 1 that three of the four variables might be trend-stationary. However, if we take the 95% level as our confidence level (as, for example, Johansen [30], and Johansen and Juselius [29] did in their original work), we conclude that for all systems with the possible exception of USA TE, exactly one cointegrating relation exists.

These results complement our cointegration analysis in Section 5 and provide further evidence for the assumption that not more than one cointegrating relation exists, which is a requirement for the validity of the Engle-Granger tests. However, in order to exclude the possibility of more than one cointegrating relations definitely, one would have to perform the Johansen test in a more systematic manner than we did in this short analysis, in order to exclude the possibility of misspecification of the regression eq. (15). On the other hand, with our analysis of the economically meaningful sub-space of cointegration vectors in Section 5, we are in accordance with Johansen's statement that "the determination of the cointegration rank should also be based on the interpretation of the estimated cointegrating relations" [30].

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