

Why production theory and the Second Law of Thermodynamics support high energy taxes

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Technological constraints on the combinations of production factors result in new conditions for economic equilibrium: output elasticities must be equal to a modification of the usual factor cost shares, where shadow prices due to the constraints add to factor prices. The shadow prices depend on the output elasticities. Consequently, output elasticities must be computed independently from the equilibrium conditions. This is done by observing the twice differentiability requirement on production functions and econometric analysis of output growth in Germany, Japan and the USA. This reveals energy conversion as a powerful factor of production and routine labor as the weakest one. Furthermore, all energy conversion is coupled to entropy production and the associated emissions of heat and particle currents. Therefore, a shift of the burden of taxes and levies from labor to energy may mitigate the trend towards increasing substitution of capital and energy for labor, and contribute to energy conservation and emission abatement.

1. INTRODUCTION

The oil price shocks 1973-1975 and 1979-1981 and the accompanying recessions known as the first and the second energy crisis prompted numerous investigations concerning the economic role of natural resources and energy. (Actually, it is energy *conversion* that matters in production. “Energy” is used as an abbreviation.) Solow (1974) expected: “The world can, in effect, get along without natural resources”, but cautioned that “if real output per unit of resources is effectively bounded – cannot exceed some upper limit of productivity which in turn is not far from where we are now – then catastrophe is unavoidable”. Analyses within the framework of neoclassical economics were performed by

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Stiglitz (1974), Hudson and Jorgenson (1974), Berndt and Jorgenson (1978), Berndt and Wood (1979), Jorgenson (1978, 1984, 1988), Denison (1979, 1984), and others.¹ Contributions to endogeneous growth theory by Barbier (1999) and Welsch and Eisenack (2002) also include energy, or a natural resource, in Cobb-Douglas-like production functions.

A quantitative measure of a production factor's economic weight and productive power is its output elasticity. Output elasticities equal factor cost shares in the equilibrium conditions of standard economics, as neoclassical profit optimization without technological constraints on factor combinations requires. On an OECD average factor cost shares have been roughly 25% for capital, 70% for labor, and only 5% for energy (before the oil price fluctuations that started in 2004). Therefore, the economic role of cost-share-weighted energy has been considered as marginal. For instance, Denison (1979) reasons: "Energy gets about 5 percent of the total input weight in the business sector ... If ... the weight of energy is 5 percent, a 1-percent reduction in energy consumption with no change in labor and capital would reduce output by 0.05 percent." According to this argument, the decrease of energy input in the US sector "Industries" by 7.3 percent between 1973 and 1975 could not possibly be related to the observed reduction of output by 5.3 percent. On the other hand, Jorgenson (1988) concludes "that there was a dramatic impact of energy prices on economic growth during the energy crisis."

The problem of energy, economic growth, and output elasticities appears in a new light, if one takes into account technological constraints on the combinations of production factors. Then, optimization of profit subject to these constraints yields new conditions for economic equilibrium: Output elasticities must be equal to a modification of the usual factor cost shares, where shadow prices due to the constraints add to factor prices.²

¹It needed the oil price shock that economic researchers finally did what Tryon (1927) had called for: "Anything as important in industrial life as power deserves more attention than it has yet received from economists. ... A theory of production that will really explain how wealth is produced must analyze the contribution of the element energy."

²If, in general, there are N factors of production $X_1 \dots X_i \dots X_N$, whose combinations are subject to a number of technological constraints, labeled by a and expressed by the equations $f_a(X_1 \dots X_i \dots X_N, t) = 0$, profit optimization results in N equilibrium conditions $\epsilon_i = X_i [p_i + s_i] / \sum_{i=1}^N X_i [p_i + s_i]$, $i = 1 \dots N$, relating the output elasticities ϵ_i of factors X_i to market prices p_i per factor unit and the factor shadow prices $s_i \equiv - \sum_a \frac{\mu_a}{\mu} \frac{\partial f_a}{\partial X_i}$. The ratios μ_a/μ of Lagrange multipliers are functions of the output elasticities, where μ is the Lagrange multiplier associated with the fixed-cost constraint.

Intertemporal utility optimization yields similar or identical equations.³ In a model with the production factors capital, labor, and energy, the constraint equations result from technological limits to automation and capacity utilization. For the derivation of the new equilibrium conditions, the explicit constraint equations, and the shadow prices of capital, labor, and energy we refer to Kümmel et al. (2008).

In the presence of technological constraints on factor combinations one cannot simply read off the output elasticities from the equilibrium conditions, because the shadow prices are elasticity dependent. Thus, output elasticities and the pertinent production functions must be calculated otherwise. The capital-labor-energy-creativity (KLEC) model facilitates such calculations (Kümmel 1986; Kümmel et al. 2002). It is sketched in the Appendix. The model is based on the requirement that production functions are twice differentiable. This leads to a set of differential equations. Their technological boundary conditions result in special solutions for output elasticities and production functions. The latter are presented in Section 2. Three integration constants are the technology parameters of the model. These parameters must be determined from empirical data, e.g. by econometric estimations. Therefore we apply the model to economic growth in Germany, Japan, and the USA in Section 3. The resulting output elasticities are for energy much larger and for labor much smaller than the cost shares of these factors.

The other problem with standard growth theory – limits to growth because of environmental constraints – emerged shortly before the first oil price shock. The near coincidence of the Yom Kippur War with the publication of “The Limits to Growth” is accidental. But the energy and the pollution problem are two sides of the same medal. This medal is coined by the First and Second Laws of Thermodynamics, which say that nothing happens in the world without energy conversion and entropy production. Therefore, the analysis of how energy conversion drives economic growth is complemented in Section 4 by the equation for entropy production in irreversible processes. This rather general formulation of the the Second Law within the framework of non-equilibrium thermodynamics

³For decreasing marginal utility of consumption, $dU/dC < 0$, the shadow price of capital contains an additional term proportional to the time derivative of dU/dC . This term vanishes in linear approximation.

relates unavoidable entropy production to emissions and energy depreciation.⁴ Section 5 concludes that shifting the burden of taxes and levies from labor to energy should be the appropriate response to the social challenge that arises from the ongoing substitution of energy and capital for labor, on the one hand, and foster adaptation of society to the resource constraints that originate from the coupling of energy conversion to entropy production, on the other hand.

2. THE CAPITAL-LABOR-ENERGY-CREATIVITY MODEL

The KLEC model describes the generation of value-added q by the Linex production function

$$q_{Lt} = q_0 e \exp \left[a \left(2 - \frac{l+e}{k} \right) + ac \left(\frac{l}{e} - 1 \right) \right]. \quad (1)$$

This function depends *linearly* on energy e and *exponentially* on ratios of capital k , labor l and energy. Economic output q , its theoretical representation q_{Lt} , and production factors k, l, e are in dimensionless units, normalized to a base year. The technology parameters q_0, a , and c may depend on time t , when human creativity changes the technological state of the system via inventions, innovations and value decisions. They are modeled by Taylor series and logistics. The basic equations of the KLEC model are derived in the Appendix.

The output elasticities of capital, $\alpha \equiv (k/q)(\partial q/\partial k)$, labor, $\beta \equiv (l/q)(\partial q/\partial l)$, and energy, $\gamma \equiv (e/q)(\partial q/\partial e)$, that pertain to the Linex function are

$$\alpha = a \frac{l+e}{k}, \quad \beta = al \left(\frac{c}{e} - \frac{1}{k} \right), \quad \gamma = 1 - \alpha - \beta \quad . \quad (2)$$

The output elasticity of capital α incorporates the law of diminishing returns: Machines don't run without energy and (still) require people for handling, supervision, and maintenance; thus, labor and energy are complementary to capital; as their ratio to capital decreases, the α -dependent output of an additional unit of capital decreases, too. The output elasticity of labor β incorporates the substitution of energy and capital for labor: If capital approaches the magnitude $k_m(q)$ required for the *maximally* automated production of a given output quantity q at time t , and if energy approaches the corresponding

⁴Georgescu Roegen (1971) had called the attention of economists to the relevance of thermodynamics. Later, among others, Ayres and Nair (1984), van Gool and Bruggink (1985), Söllner (1996, 1997), Faber and Proops (1998), and Hall et al.(2001) took up the issue of thermodynamics and economics in some depth and breadth.

amount $e_m = ck_m(q)$, then the output of an additional unit of labor approaches zero; c may be interpreted as the energy demand parameter of the fully employed capital stock at the technological state of time t . The output elasticity of energy γ reflects constant returns to scale: If at a given time t all production factors double (or increase by λ), output must double (increase by λ), too. Complementarity is limited by the technological constraint that capacity utilization cannot exceed 100%, and substitutability is limited by the constraint that at a given time t the degree of automation cannot exceed a technologically given limit $\rho_T(t) \leq 1$. Since entrepreneurs will avoid a state of the economy where an additional unit of input will decrease output, the output elasticities must be non-negative. The output elasticity of creativity, δ , is defined as

$$\delta \equiv \frac{t - t_0}{q} \frac{\partial q}{\partial t} = \frac{(t - t_0)}{q_{Lt}} \left[\frac{\partial q_{Lt}}{\partial a} \frac{da}{dt} + \frac{\partial q_{Lt}}{\partial c} \frac{dc}{dt} + \frac{\partial q_{Lt}}{\partial q_0} \frac{dq_0}{dt} \right]. \quad (3)$$

The Linex function is fitted to the empirical time series of output by minimizing the sum of squared errors according to the Levenberg-Marquardt method, subject to the constraints of non-negative output elasticities of k, l, e .⁵ Then the time averages of the output elasticities, $\bar{\alpha}$, $\bar{\beta}$, $\bar{\gamma}$, and $\bar{\delta}$, are computed.

3. ECONOMIC GROWTH AND OUTPUT ELASTICITIES

The reproduction of economic growth in Germany, Japan and the USA since 1960 is presented in Figures 1 – 4.

The left part of the figures shows empirical growth (squares) and theoretical growth (circles) of the normalized output $q = Q/Q_{t_0}$ in Germany, Japan and the USA. The right part shows the empirical time series of the normalized factor inputs capital $k = K/K_{t_0}$, labor $l = L/L_{t_0}$, and energy $e = E/E_{t_0}$. The empirical data have been obtained in constant currency, annual workhours, and petajoules consumed per year from the national accounts, labor statistics, and energy balances. The base year t_0 is 1960 for Germany and the USA and 1965 for Japan.

The empirical time changes of inputs differ significantly in Figs. 1– 4, especially

⁵The elasticity constraints $\alpha \geq 0, \beta \geq 0, \gamma \geq 0$ limit factor combinations in the Linex output elasticities of eq. (2) in the sense – but not with the precision – of the technological constraints on capacity utilization and degree of automation.

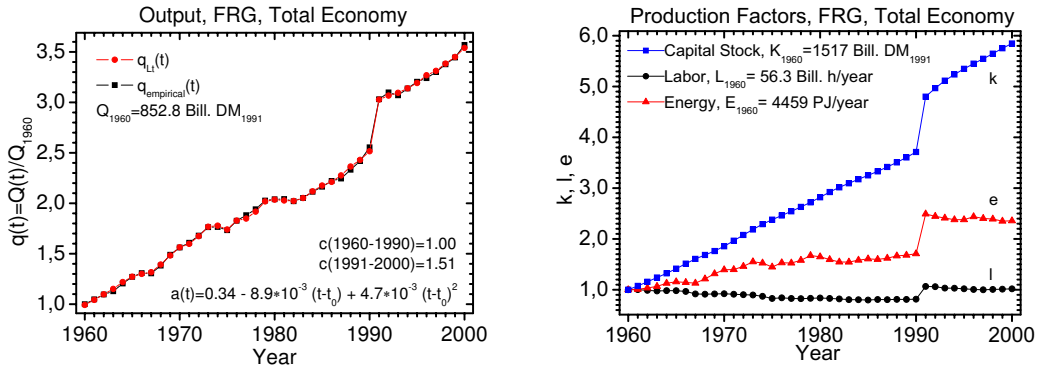


Figure 1: Growth of GDP in the Federal Republic of Germany 1960–2000

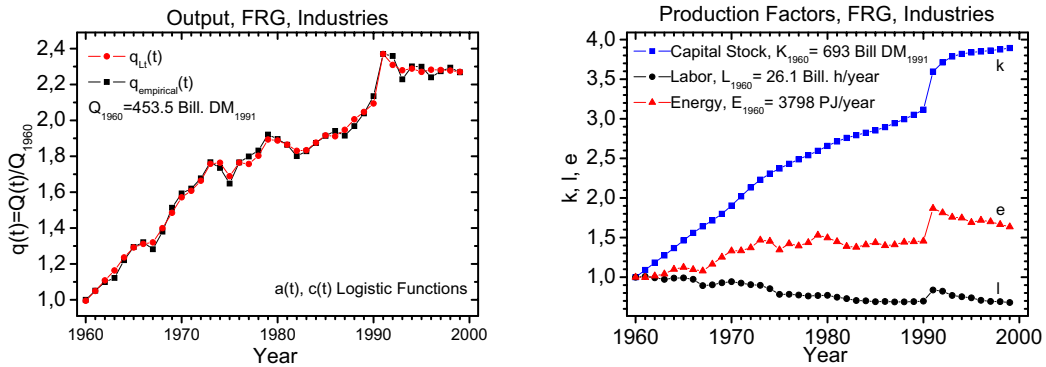


Figure 2: Output growth in the German industrial sector “Warenproduzierendes Gewerbe” 1960 – 1999

for routine labor l . The number of hours worked per year decreased almost continuously during 40 years in Germany’s total economy and in its industrial sector “Warenproduzierendes Gewerbe”, save for the small jumps after reunification on October 3, 1990. It stayed nearly constant in Japan, and almost doubled in the United States. In Germany the introduction of the 35-hours week in the 1980s had the effect of a “rationalization whip”, as the boss of the German labor unions, Michael Sommer, once admitted himself. This has re-enforced the trend to increasing automation, which had already existed before, when Germany’s capital stock, destroyed in WW II, was being rebuilt and expanded with the most modern machinery. (There was no political trouble, as long as people who lost their jobs to automation fell into the tightly-knit social net.) Japan has a similar post-war history. But here part of the firm culture was not to fire employees but maintain life-long employment. This produced the enormous loyalty of Japanese workers towards their firms. In the USA the labor-intensive service sector expanded early and vigorously.

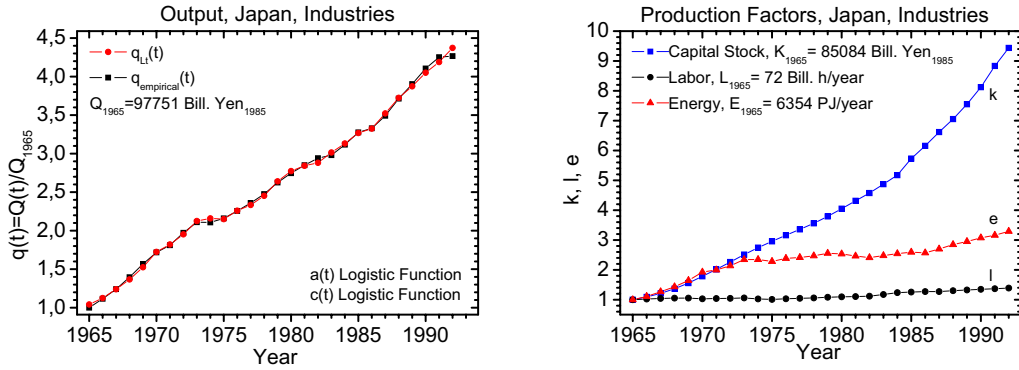


Figure 3: Output growth in Japanese “Industries” 1965-1992

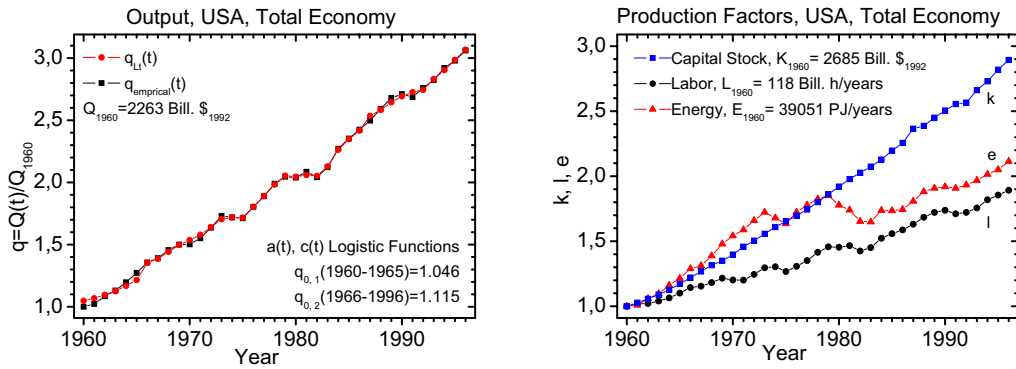


Figure 4: Growth of GDP in the USA 1960 – 1996

Furthermore, the number of “working poor”, who must hold more than one poorly paid job in order to make a decent living, has grown in the USA more rapidly than elsewhere; it reached about 25 percent of the total US work force by the year 2004 (Shulman, 2004). Despite of the rather different evolutions of the factor labor, which is considered as the most important production factor by standard growth theory, real GDP has grown in the Federal Republic of Germany and the USA between 1960 and 1989 by about the same factors 2.5 and 2.7, respectively. Apparently, growth of capital stock and energy input are more important than growth of routine labor. (The output elasticities in Table 1 are consistent with this observation.) The data on energy inputs indicate that after the first oil price shock incentives for energy conservation were stronger and worked better in Germany and Japan than in the USA; the former two countries do not have domestic oil, whereas the USA does.

In Figs. 1 – 4 the theoretical outputs q_{Lt} closely follow the empirical ones. They also reflect the ups and downs of the energy inputs during the energy crises 1973-1975

and 1979-1981. The strong energy-output coupling in the wake of the oil price shocks has several reasons. One is the worry of producers that energy, without which capital lies idle, may become scarce, so that they invest less.⁶ This complements industry’s strategy of minimizing its exposure to financial risk. Another one is that the buying power of domestic consumers is transferred to the oil producers, so that people can buy less. Consequently, reductions in the production of investment and consumption goods reduce the degree of industrial capacity utilization via reduced energy inputs; often, employment decreases subsequently, and consumer confidence collapses. Economic recessions result.

The sudden enlargement of the system “Federal Republic of Germany” (FRG) at reunification in 1990 is satisfactorily reproduced, too.⁷ The “Solow residual”, which accounts for that part of output growth that cannot be explained by the weighted input growth rates, is absent. In neoclassical growth theories with factor-cost-share weighting of production factors this residual, attributed to “technical progress”, amounts to more than 50 percent of total growth in many cases.

Table 1 shows the time-averaged output elasticities (productive powers) of capital ($\bar{\alpha}$), labor ($\bar{\beta}$), energy ($\bar{\gamma}$), and creativity ($\bar{\delta}$), together with the adjusted coefficient of determination R^2 and the Durbin-Watson coefficient d_W for the systems in Figs. 1 – 4 (FRG TE: total economy of the Federal Republic of Germany before and after reunification; FRG I: German industrial sector “Warenproduzierendes Gewerbe”, which produces about 50% German GDP; Japan I: Japanese sector “Industries”, which produces about 90% of Japanese GDP; USA TE: total economy of the USA). The best possible value of d_W would be 2, indicating no autocorrelation at all. Cointegration analysis by Stresing et al. (2008) has confirmed the orders of magnitude of the k, l, e output elasticities .

Ayres and Warr (2005), using exergy⁸ inputs, multiplied by appropriate

⁶During the first oil price shock, for example, investment in the West German industrial sector “Warenproduzierendes Gewerbe” fell from about 90 billion DM in 1973 to less than 50 billion DM in 1975.

⁷The energy demand parameter $c(t)$ rises steeply in 1991 for the German systems, reflecting that the energetically rather inefficient capital stock of the former German Democratic Republic was added to the capital stock of the Federal Republic of Germany.

⁸The First Law of Thermodynamics says that conserved energy consists of a useful part, called exergy, and a useless part, called anergy. Exergy can be converted into any form of useful physical work. Anergy is, e.g., heat at the temperature of the environment. Our principal primary energy carriers are practically 100 percent exergy.

Table 1 Time-averaged Linex output elasticities and statistical quality measures.

System	FRG TE 1960-2000	FRG I 1960-99	Japan I 1965-92	USA TE 1960-96
$\bar{\alpha}$	0.38±0.09	0.37± 0.09	0.18± 0.07	0.51± 0.15
$\bar{\beta}$	0.15±0.05	0.11±0.07	0.09±0.09	0.14±0.14
$\bar{\gamma}$	0.47±0.1	0.52±0.09	0.73±0.16	0.35±0.11
$\bar{\delta}$	0.19±0.2	0.12±0.13	0.14±0.19	0.10±0.17
R^2	>0.999	0.996	0.999	0.999
d_W	1.64	1.9	1.71	1.46

conversion efficiencies, (and physical work by animals) as energy variable, have fitted the Linex function with two constant technology parameters to the gross domestic product of the US economy between 1900 and 1998. They obtain good agreement between the empirical and theoretical growth curves. Their time-averaged output elasticities are similar to ours. Their exergy data (Ayres et al., 2003) already include most of the efficiency improvements that have occurred in energy converting systems during the 20th century. If, on the other hand, one uses primary energy input as energy variable, as it is done in the Linex functions that yield Figs. 1 – 4, one needs the time-dependent technology parameters, which model the efficiency improvements in primary energy conversion and information processing.

4. ENTROPY PRODUCTION, ENERGY DEPRECIATION, AND ENVIRONMENTAL POLLUTION

Energy conversion in the furnaces, heat engines and information processors of the capital stock is a powerful driver of growth. This is everyday’s engineering experience, and the high output elasticities of energy in Table 1 are consistent with that. Unfortunately, energy conversion and entropy production are twins. The Second Law of Thermodynamics says that entropy production is never negative, and that in all changes of real life, which occur not infinitely slowly and are therefore irreversible, it is always positive. In a system containing N different sorts of particles k , and where local thermodynamic variables such as temperature and pressure are well-defined during all changes, non-equilibrium thermodynamics (Kluge, Neugebauer 1993) yields the density of dissipative entropy production

as

$$\sigma_{S,dis}(\vec{r}, t) = \vec{j}_Q \vec{\nabla}(1/T) + \sum_{k=1}^N \vec{j}_k [-\vec{\nabla}(\mu_k/T) + \vec{f}_k/T] > 0 \quad . \quad (4)$$

In words: entropy production density $\sigma_{S,dis}$ in the space-time point (\vec{r}, t) consists of the heat current density \vec{j}_Q , driven by the gradient ($\vec{\nabla}$) of temperature T , and particle (diffusion) current densities \vec{j}_k , driven by gradients of chemical potentials μ_k and of temperature, and by specific external forces \vec{f}_k .⁹ Thus, all processes of industrial production are associated with emissions of particles and heat. These emissions change the composition of and the energy flows through the biosphere. If these changes are so big and occur so rapidly that adaptation deficits of the living species and their societies develop, they are perceived as environmental pollution. Furthermore, whenever heat is generated and emitted, exergy – the valuable part of energy – is converted into useless energy, e.g. heat at the temperature of the environment. This is what “energy consumption” really means.

Equation (4) also shows that dissipation of matter is included in the Second Law of Thermodynamics. There is no need for a Fourth Law (Georgescu-Roegen, 1986). Since only the sum of the contributions from heat and particle emissions to entropy production must be positive, particle emissions can be converted into heat emissions by sufficiently large exergy inputs. Calculating the heat equivalents of noxious substances one can estimate, how the approach to the “heat barrier” of about 3×10^{14} Watts is accelerated by environmental protection measures that transform particle emissions, like SO_2 , NO_X , CO_2 , and radioactive waste into heat emissions (Kümmel and Schüssler, 1991). If anthropogeneous energy flows through the biosphere exceed the heat barrier, climate changes have to be expected, independently from the anthropogeneous greenhouse effect.

5. CONCLUSIONS

Table 1 shows that cheap energy has a high productive power, while expensive labor has a low productive power. This results in the pressure to increase automation, substitut-

⁹Equation (4) holds, if there are no chemical and physical transformations. Chemical reactions contribute an additional entropy production density, given by products of scalar “forces” and “currents”. This, however, cannot interfere with $\sigma_{S,dis}$, because scalar forces and currents cannot interfere with vectorial forces and currents.

ing cheap energy/capital combinations for expensive labor. It also reinforces the trend towards globalization, because goods and services produced in low-wage countries can be transported cheaply to high-wage countries. Thus, if the disequilibria between productive powers and cost shares of labor and energy are too pronounced, there is the danger that newly emerging and expanding business sectors cannot generate enough new jobs that can compensate for the ones lost to progress in automation and globalization. This, then, will result in the net loss of routine jobs in high-wage countries and increasing unemployment in the less qualified part of the labor force. A slow-down of economic growth, as natural constraints may cause, or economic recessions for whatever reasons, will aggravate the problem of unemployment. There are also many jobs that can best be done by the human hand, heart and brain, which, however, individuals and/or society cannot afford in sufficient quantities because of the high taxes and levies on labor. These jobs are in social services (intensive training and education for the socially or physically handicapped, care for children, sick and old people), in handicraft services (repair instead of throw away and new purchase), and in high quality research and education. Furthermore, the coupling of energy conversion to energy depreciation and emissions by the Second Law of Thermodynamics calls for innovations (Fri, 2003) and investments in energy conservation and in those non-fossil-fuel technologies that emit less of the most bothersome pollutants. At low energy prices the return on these investments is usually smaller than the return on the traditional combustion of fossil fuels.

Therefore, in order to fight increasing unemployment, reduce state deficits due to payments of unemployment benefits, and stimulate energy conservation and emission mitigation, the disequilibrium between the productive powers and cost shares of labor and energy should be reduced by a substantial decrease in the taxes and levies on labor, and a substantial increase in energy taxation.

The tax per energy (exergy) unit should increase at the same rate as it succeeds in stimulating energy conservation (Verbruggen 2006, 2008). And as long as hard and dangerous work is performed by energy-powered machines the tax basis will never shrink to zero, thanks to the laws of thermodynamics. Of course, in order to minimize problems

in the competitiveness of energy intensive industries and to minimize energy leakage, such energy taxes should ideally be introduced in an internationally harmonized manner. An alternative may be border tax adjustments (Baron, 1997). Social hardship can be avoided by refunding the energy tax to low-income consumers via existing transfer channels.

In the 1990s the Commission of the European Union had proposed several energy taxation schemes to its member states. They were initially well received, but then successfully torpedoed by powerful special interest groups. Nevertheless, the idea of the ecological tax reform is alive. And, as a message of this paper, the idea of shifting financial burdens from the production factor labor to the production factor energy is not only founded in the most fundamental laws of physics and their consequences for ecological stability, but also in production theory and its consequences for social and fiscal stability.

A recent BBC World Service Poll (2007), in cooperation with the Program on International Policy Attitudes, asked over 22 000 citizens all over the world, whether they would support higher energy taxes on the most harmful types of energy so that individuals/industry use less.¹⁰ The proportion of Chinese favoring higher taxes is 24 points greater than the next largest majorities in Australia and Chile (61% in both). This is followed by Germans (59%), Canadians (57%), Indonesians (56%), Britons (54%), and Nigerians (52%). In other countries like France and India the opinion is mixed, and in seven countries, among them the USA, a majority is initially opposed to higher energy taxes. The overall percentage in favor is 50 percent, if the tax revenue is not explained. However, if the energy tax revenue were dedicated to clean/efficient energy or to reducing other taxes so that the total tax bill stayed the same, about 75% would favor higher energy taxes. This may encourage politicians to present well founded and explained energy taxation schemes to their constituency. People seem to be ready for a change from wasteful to thoughtful energy use.

¹⁰A total of 22,182 citizens in Australia, Brazil, Canada, Chile, China, Egypt, France, Germany, Great Britain, India, Indonesia, Italy, Kenya, Mexico, Nigeria, the Philippines, Russia, South Korea, Spain, Turkey, and the United States were interviewed face to face or by telephone between May 29 and July 26, 2007. Polling was conducted for BBC World Service by the international polling firm GlobeScan and its research partners in each country. In eight of the 21 countries, the sample was limited to major urban areas. The margin of error per country ranges from +/-2.4 to 3.5 percent.

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APPENDIX: THE BASIC EQUATIONS OF THE KLEC MODEL

Infinitesimal changes of output, dq , capital, dk , labor, de and time, dt are related to each other by the *growth equation* (which is obtained from the total differential of the production function $q(k, l, e; t)$) :

$$\frac{dq}{q} = \alpha \frac{dk}{k} + \beta \frac{dl}{l} + \gamma \frac{de}{e} + \delta \frac{dt}{t - t_0} \quad . \quad (5)$$

The *output elasticities*

$$\alpha(k, l, e) \equiv \frac{k}{q} \frac{\partial q}{\partial k}, \quad \beta(k, l, e) \equiv \frac{l}{q} \frac{\partial q}{\partial l}, \quad \gamma(k, l, e) \equiv \frac{e}{q} \frac{\partial q}{\partial e}, \quad \delta \equiv \frac{t - t_0}{q} \frac{\partial q}{\partial t} \quad (6)$$

give the weights, with which relative changes of the production factors k, l, e , and of time t contribute to the relative change of output. In this sense they *measure the productive powers* of capital, labor, energy, and creativity.

We follow standard economics in assuming that production functions, at a fixed time t , are twice differentiable, lineary homogeneous state functions of the variables k, l, e within accessible factor space. Thus,

$$\alpha + \beta + \gamma = 1, \quad (7)$$

and from the equality of the second-order mixed derivatives of $q = q(k, l, e; t)$ one gets the differential equations

$$\begin{aligned} k \frac{\partial \alpha}{\partial k} + l \frac{\partial \alpha}{\partial l} + e \frac{\partial \alpha}{\partial e} &= 0, \\ k \frac{\partial \beta}{\partial k} + l \frac{\partial \beta}{\partial l} + e \frac{\partial \beta}{\partial e} &= 0, \\ l \frac{\partial \alpha}{\partial l} &= k \frac{\partial \beta}{\partial k}. \end{aligned} \quad (8)$$

The most general solutions of these equations are:

$$\alpha = A(l/k, e/k), \quad \beta = \int \frac{l}{k} \frac{\partial A}{\partial l} dk + J(l/e), \quad (9)$$

where $A(l/k, e/k)$, and $J(l/e)$ are any differentiable functions of their arguments $l/k, e/k$ and $l/e = (l/k)/(e/k)$.

The trivial solutions of the three coupled differential equations are the constants $\alpha_0, \beta_0, \gamma_0 = 1 - \alpha_0 - \beta_0$. Cost-share weighting of standard economics uses $\alpha_0 \approx 0.25$, $\beta_0 \approx 0.70$, $\gamma_0 \approx 0.05$.

We use the simplest non-trivial solutions that satisfy asymptotic technical-economic boundary conditions. They are :

$$\alpha = a \frac{l+e}{k}, \quad \beta = al \left(\frac{e}{l} - \frac{1}{k} \right), \quad \gamma = 1 - \alpha - \beta. \quad (10)$$

α satisfies the Law of Diminishing Returns: If the increase of the capital stock k exceeds the increase of labor l and energy e , an additional unit of k will contribute less and less to the growth of output. β results from inserting α into the second of eqs. (9) and determining $J(l/e)$ from the condition that an additional unit of labor will contribute less and less to the growth of output, if k approaches the magnitude $k_m(q)$ required for the maximally automated production of a given output quantity q , and if energy approaches the corresponding amount $e_m = ck_m(q)$ at full capacity utilization. γ follows from eq. (7).

Inserting the output elasticities into the growth equation (5) and integrating along any convenient path from the initial values $(q_0; 1, 1, 1)$ to $(q; k, l, e)$ one obtains the energy-dependent Cobb-Douglas production function $q_{CDE} = q_0 k^{\alpha_0} l^{\beta_0} e^{\gamma_0}$, if one uses the trivial constants, whereas the Linex production function

$$q_{Lt}(t) = q_0 e \exp \left[a(t) \left(2 - \frac{l+e}{k} \right) + a(t) c(t) \left(\frac{l}{e} - 1 \right) \right], \quad (11)$$

results, if one uses the elasticities from eq. (10). The Linex function depends *linearly* on energy and *exponentially* on quotients of capital, labor and energy. It contains the technology parameters a and c , which may become time dependent, if creativity acts. The capital-efficiency parameter $a(t)$ and the energy-demand parameter $c(t)$ are modeled by logistic functions or Taylor series and are determined by non-linear OLS fitting of $q_{Lt}(t)$ to $q_{empirical}(t)$, subject to the constraints: $\alpha \geq 0, \beta \geq 0, \gamma \geq 0$.

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